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Combined modelling of production of merchantable phosphoric ores using parallel processors and their transportation by pipeline

Mouna BAMOUMEN^{a, c}, Vincent HOVELAQUE^{a, c}, Vincent GIARD^{b, c, *}

^a Univ-Rennes, CNRS, CREM - UMR 6211, F-35000 Rennes, France

^b Paris-Dauphine, PSL Research University, 75016, Paris, France

^c Mines-Université Mohammed VI Polytechnique, Benguerir, Maroc

* Corresponding author

Mouna.Bamoumen@emines.um6p.ma, Vincent.Hovelaque@univ-rennes.fr, Vincent.Giard@dauphine.psl.eu

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Highlights

- Linked modelling of pulled production and pipe transport of merchantable ores (MOs)
- Models sharing constraints of schedule, batch sizes and quality constraints of MOs
- MO produced by blending n selected ores & routing on n lines working alongside
- Regular internal demand to meet continuously and splittable irregular export demand
- Joined performance criteria of SC links involving a possible explicit trade-off

Combined modelling of production of merchantable phosphoric ores using parallel processors before blending and their transportation by pipeline

Abstract: This paper focuses on managing the production and transfer of phosphoric merchantable ores (MOs) in a pull-flow mode of a supply chain that is currently addressed with limited effectiveness. MOs are obtained by washing extracted source ores in parallel lines, using alternative routings that change their weight and composition, before blending in tanks to achieve the required quality defined by a set of compositional constraints. The pipe transfer program defines the batches to be transported and their schedule, considering downstream demand (a roughly stable demand from the chemistry and irregular export demands (export orders of large quantities of MOs)), preventive maintenance of pipe or washing lines and prevention of defusing and saturation in the tanks of the delivery pipe station. A daily used Decision Support System (DSS) encapsulates two models used sequentially and sharing technical, availability, capacity, and quality constraints; then, the optimal solution of the transfer model is adapted to create the order book processed by the original blending model to obtain an optimal joint production and pipe transfer program. It also provides ways to improve SC performance while considering its impact on the chemical production performance.

Keywords: Supply Chain Performance, Flow Synchronisation, Pipe Transportation, Blending, Penalty Costs, Decision Support System

1 Introduction

This paper presents an integrated approach to short-term production and transport decisions in one of the phosphate Supply Chains (SC) of the OCP Group. The studied part of the phosphate North Axis SC is described in Figure 1 which provides some technical information used as parameters in the problem modelling, which will be explained later. The proposed methodology makes use of an innovative Decision Support System (DSS) that operates in a rolling mode. It includes two original generic models focused on pipe transfer and blending production, corresponding to the decisions made on links 2 and 3 of the SC described in Figure 1. The SC consists of an upstream link responsible for ore extraction from five mines (spread over several hundred km², see SC link 1 in Figure 1), according to tactical planning (push mode), and a downstream link geared towards meeting customer demand (pull flow, see link 4 in Figure 1). These models can be used separately but only in a pull-driven SC with intermediate stocks. The DSS provides consistent production and transportation decisions, considering all SC constraints.

In the specific context of the SC segment under study, the production of various phosphoric Merchantable Ores (MOs) is of considerable economic importance, accounting for about 70% of OCP production (whose revenues represent more than 8% of the Morocco's Gross National Revenue). The operations in this SC involve the use of five parallel washing lines and the sequential transport of products through a 187 km slurry pipe. This transport method, which interleaves batches of water between successive product batches (SC link 4 in Figure 1) meets for both a fairly regular internal demand (from nearby phosphoric acid production plants) and a highly irregular export demand (by boat). The challenge arises from *i*) the variety of MOs and the importance of the volumes involved, which makes a production-to-stock approach (push mode) impractical for reasons of space (ore storage along more than 3 km corresponding to only 7 days of production) but also of equipment (conveyor, bucket wheel, etc.) and *ii*) the temporal heterogeneity of the raw materials (Source Ore, SO), whose availability (in terms of volume, date and intrinsic chemical quality) depends on the mining extraction programs (link 1 in Figure 1). Therefore, this part of the SC has no decoupling point (Sun et al., 2008; Van Donk et al., 2016) before or after transport.

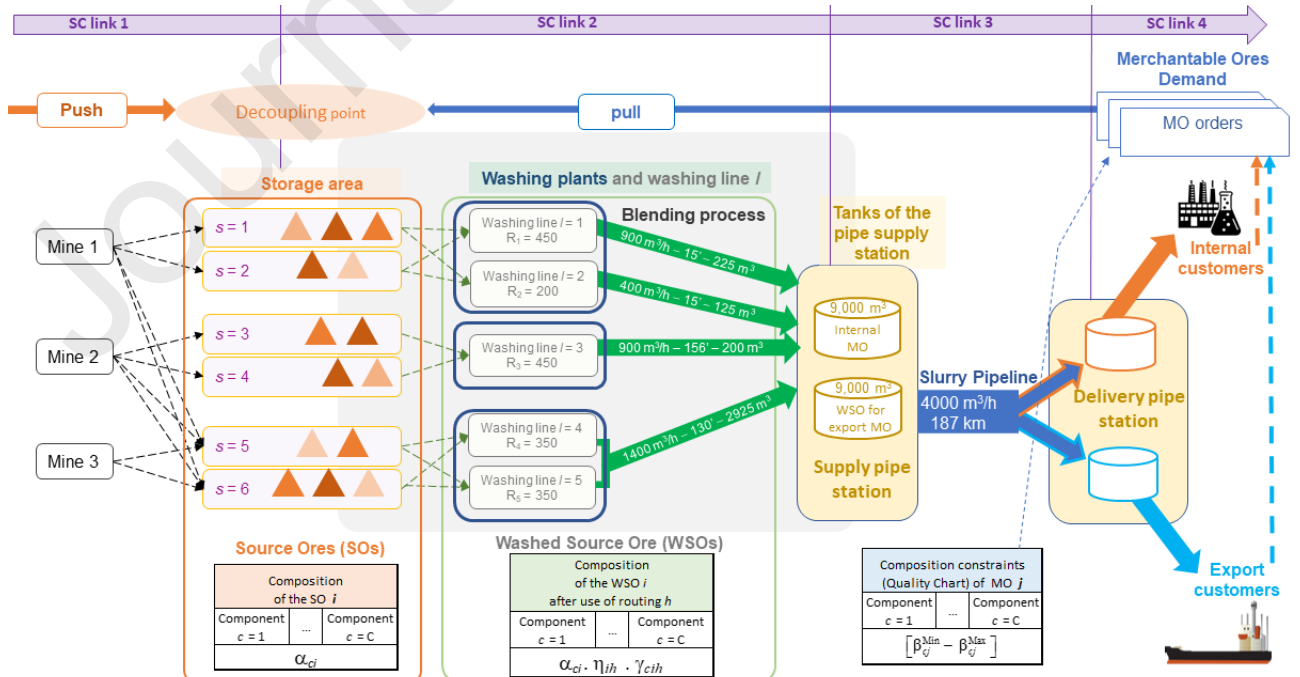


Figure 1 Studied part of the phosphoric supply chain.

The paper addresses two particular problems that require innovative approaches:

- The first is a blending production problem, a novel endeavour in practice, which consists in mixing n intermediate outputs (Washed Source Ores, WSOs) obtained simultaneously from the treatment of n inputs (extracted Source Ores, SOs) in parallel processors (washing lines). This process changes the composition and reduces the weight of the outputs differently depending on the routing used (whose change may trigger a setup time). Although our blending problem has unique characteristics, it shares compositional constraints with other blending problems: for each output, a Quality Chart (QC) specifies, for a few specific components (in our case, BPL (Bone Phosphate Lime), cadmium, etc.), the acceptable range within which the weight percentage of the output component must lie. The features that characterise our blending problem include: *i*) the mechanically defined input proportions by the production rates, the routing used and the number of parallel processors; *ii*) the consideration of residues in processors during production changes; *iii*) the potential setup times in case of routing changes; *iv*) the daily variations in input availability and characteristics (Azzamouri et al., 2021; Hilali et al., 2022); *v*) and the interdependence of SOs consumption in limited quantities within the chosen horizon.
- The second problem tackled is a transfer problem of an equally original nature. It combines the definition of shipping dates for batches selected from predetermined periods with the determination of quantities to be shipped from a predetermined set of possible quantities. Unlike traditional scheduling problems (where the "objects" to be scheduled are predetermined), the production and transfer processes occur continuously through a sequence of batches whose sizes are not predetermined. The transfer problem considers five sets of constraints induced by the production systems upstream and downstream of the slurry pipe, highlighting the need to: *i*) avoid defusing and saturation in the tanks allocated to the internal customers (phosphoric acid plants usually working at full capacity); *ii*) manage flow desynchronisation between incoming (production), transfer (pipe), and outgoing flows (internal demand); *iii*) account for tank capacities (which mitigate the flow desynchronisation problem); *iv*) ensure batch sizes meet quality chart requirements; and *v*) satisfy two types of downstream demand, one roughly regular and the other irregular.

The originality of the approach adopted is twofold. The first is to achieve consistency between production and transfer decisions not through global modeling, which is numerically challenging, but by sharing constraints between the problems. In fact, the use of a production order book, that incorporates the optimal solution to the transfer problem, improves the synchronisation between the two decision domains. Furthermore, the discretisation of time and flow helps to formulate problems that seamlessly mix batch and flow considerations. The second original feature introduces the possibility of incorporating penalty costs into the production problem (SC link 2) to mitigate the impact on chemical plants (SC link 4) of fluctuations in the MO composition, which can lead to costly adjustments.

From a management perspective, this approach offers double synchronisation—between processing lines for MO production and between production and transfer—to effectively meet the two types of downstream demand. From a sustainability perspective, the approach contributes to the prudent use of non-renewable resources, in particular phosphorus ore, the reserves of which are finite.

The remainder of this paper is organised as follows. Section 2 reviews previous blending studies in the mining context. Section 3 presents a literary foundation of the transfer and blending models and discusses their interactions, with detailed mathematical description provided in Appendix C. Section 4 presents a case study demonstrating the practical application and effectiveness of the proposed integrated approach. Finally, Section 5 draws meaningful conclusions and suggests future research.

2. Literature review

By combining production problems (of the blending type) and transport problems (of the pipeline type), it seemed appropriate to first examine the issue of decision coherence (§2.1). This examination led us to focus on scientific papers dealing with structured operational decision-making in two categories: one dealing with pooled interdependency approaches (§2.2) and the other dealing with sequential interdependency approaches (§2.3). This analysis leads us to believe that our formulation of these interdependent production and transport problems is original due to the absence of a decoupling point.

2.1 *Decision-making coherence*

Decision-making in several successive SC links, based on operational research models, can be analysed by combining three decision typologies that help to circumscribe the literature used. Anthony's classic classification (Anthony, 1965) distinguishes three types of managerial decisions (strategic planning, management control and operational control). We're concerned here with operational decisions. Gorry and Scott-Morton's classification (Gorry and Scott-Morton, 1971) distinguishes between structured and unstructured decisions, which they explicitly combine with Anthony's classification (Anthony, 1965). We're interested here in structured decisions. Finally, Thompson's classification (Thompson, 1967) of interactions between decisions taken by different entities distinguishes between pooled interdependence, in which several entities share constraints or resources, sequential interdependence, in which two entities whose decisions constrain each other's decisions, and reciprocal (or intensive) interdependence, which implies the existence of bidirectional constraints. We are interested in these last two forms of interdependence, which we develop in the following two subsections. It could be added that this consistency problem is part of the horizontal alignment problem (Kathuria et al., 2007), which refers to the coherence of decisions taken between the different production units in the supply chain for each decision level.

2.2 *Pooled interdependence approach*

For many years, practitioners and academics have studied the problems of production and transport planning extensively but individually. Conway et al. (1967) noted that, in most cases, sequencing cannot be separated from other decisions. Over the last two decades, a number of factors, such as increasingly global competitive pressures, market globalisation and customer expectations, have led practitioners to develop methods and strategies that enable supply chain members to behave and coordinate as part of a single system (Kumar et al., 2020). This has been made possible by the development of information technology (Feng et al., 2018), which has increased coordination and benefits between departments. Several authors (Azadian et al., 2015; Chen, 2010; Arshinder et al., 2008) have pointed out that coordinating manufacturing and distribution operations significantly increases manufacturers' chances of survival in today's competitive market environment.

For more extensive review studies on the joint production and transportation planning problems, see Arshinder et al., 2008; Mula et al., 2010; Chen, 2010; Fahimnia et al., 2013 and Díaz-Madroñero et al., 2015. The most recent one (Kumar et al., 2020) reviewed articles from 2000 to April 2019 based on a classification framework with eight dimensions: decision level, problem structure, nature of demand, supply chain structure, solution approach, modelling approach, software used and application domain. The authors point out several gaps in the literature, including the lack of studies focusing on the integration of production and distribution planning without inventory and the lack of consideration of the different modes of transport in the mathematical modelling, including the transition times, routing constraints and different cost patterns... It appears that no article deals with a joint problem of continuous production-to-order of multiple items directly linked to transport by pipeline.

2.3 *Sequential interdependence approach*

In this subsection, we successively consider the production (§2.3.1) and transport (§2.3.2) as two independent problems, which is relevant in the case of a push-flow structure with a decoupling point. In a pull-flow context, the integration these two models in a DSS that considers their interdependencies, in particular by sharing parameters, allows obtaining a global solution that performs better than that obtained by considering two problems as independent and treating them sequentially.

2.3.1 The blending problem in integrated Supply Chains

In this subsection, we are interested in planning determination for integrated SCs that include a blending activity. The blending problem was first mathematically formalised by Dantzig in 1947 (Dantzig, 1982) and has been the subject of numerous extensions. Given a set of inputs of known composition, the blending problem aims at defining the inputs to be used and their quantities in order to obtain an output requested by a customer, considering that the composition of this output must respect certain constraints. In our case, the inputs are SOs, and the outputs are MOs; our blending problem has the six specific characteristics listed in section 1. The papers dealing with the problem of mineral blending can be classified according to the following characteristics retained in the grid of Table 1.

- The characteristics of the ore produced: i.e., its nature (coal, metallic, non-metallic); whether it is a single grade of ore (mono-product) or several grades of the same ore (multi-product).
- The planning problem dealt with in these articles usually combines a production problem with another problem (supply, transport, etc).
- As blending is most often combined with complementary processing, three main configurations can be distinguished: *i*) "blending only" where there are no processing activities, only the blending of raw ores is operated to produce products; *ii*) "blending then processing", where raw ores are blended before being processed by one or more alternative processing routes; and *iii*) "processing then blending" consists of first enriching raw ores and then blending them to meet customer specifications. If processing and/or blending activities are not organised as described above (maybe some raw ores are blended and others processed at the same time), we refer to them as "Other". In addition, processing activities are characterised here by: *i*) the possible existence of alternative routings for the processing of extracted ores to produce either an intermediate or a finished product, depending on whether blending takes place before or after processing; *ii*) the possible change in the composition of an ore as a result of the processing operations; and *iii*) the possible existence of setup times associated with to processing operations.

- The decision-making level considered: operational, tactical, and strategic.
- The modelling approach: optimisation (exact methods, approximate methods), simulation, hybrid methods.

Table 1: Analysis grid of mineral blending articles

ID	Author(s)	Ore characteristics		Planning activities										Decision-making level **	Modelling approach	Case study
		Ore nature*														
					Processing			Blending								
					Alternative	Composition	Setup time		Blending only	Blending then	Processing then	Other				
1	Johnson (1968)	C	x		x								x	T	Linear programming	
2	Candler (1991)	C	x					x	x						Linear programming	
3	Sherali and Puri (1993)	C						x	x				x	T	Mixed-integer linear programming	x
4	Sherali and Saifee (1993)	C	x	x		x		x	x				x	T/S	Non-linear programming / Heuristic	x
5	Shih and Frey (1995)	C				x							x		Chance-constrained	x

14	Koushavand et al. (2014)	N M	x		x													T/S	Mixed-integer linear stochastic programming	x			
15	Montiel and Dimitrakopoulos (2015)	M		x		x	x	x											O/T	Non-linear stochastic programming	x		
16	Blom et al. (2016)	M		x			x	x											O/T	Non-linear programming / Heuristic	x		
17	Jélvez et al. (2016)	M	x																	Heuristic	x		
18	Lamghari and Dimitrakopoulos (2016)	M		x			x													Meta-heuristic	x		
19	Arigoni et al. (2017)	C																		S	Mixed-integer linear programming	x	
20	Chen et al. (2020)	M	x																		Non-linear programming	x	
21	Haonan et al. (2021)	M				x		x												O/T	Heuristic	x	
22	Azzamouri et al. (2021)	N M		x		x	x	x													T	Mixed-integer linear	x

The literature review did not identify any study that combined the following characteristics: *i*) variability of supply (ores available on stock and pushed by the extraction program) and demand over time (continuous and discrete); *ii*) constraints on storage and transport of ores to the blending area; *iii*) presence of multiple routings to produce a finished product; *iv*) evolution of composition in a processing unit; *v*) consideration of setup time in case of a routing change.

2.3.2 The transport (transfer) problem

At first sight, the problem of determining the shipping program has some analogies with that of lot-sizing under demand constraints and with that of multi-product transfer by pipeline. The analysis shows that this complex problem, which requires the consideration of production, storage, transport and demand constraints for several products, appears to be original, as will be shown.

- Wagner and Whitin (1958) were the first to take an interest in determining a delivery schedule that would satisfy a schedule of irregular demand for a given product without shortages while minimising the sum of inventory holding and setup costs. This seminal work gave rise to the vast literature on lot-sizing, considering single or multiple items, capacity constraints and different supply contexts. Several recent surveys have analysed the characteristics of existing models (Drexl & Kimms, 1997; Andriolo et al., 2014; Brahimi et al., 2017; Glock & Grosse, 2021)
- A recent synthesis of multi-product pipeline transport problems has been proposed by Kirschstein (2018) but research in this area continues (Abdellaoui et al., 2021). Five papers deal with the rolling schedule of pipeline transport of ordered batches of different products available in stock over several days, using an efficiency criterion (e.g., maximising pipeline utilisation).

Several characteristics of the problem under study prevent it from falling into either of the two classes of problem discussed above.

- Products transported by pipeline have different characteristics and cannot be mixed. They are necessarily produced to order because it is technically impossible to build up sufficient stocks upstream and downstream of the pipeline. As a result, production and transfer problems are highly interdependent.
- All lot-sizing models aim to minimise an efficiency criterion (cost). In our case, the transport rate is higher than the production rate of the washing plants, which is higher than that of the chemical plants, whose demand must be satisfied. Therefore, the remaining transport and production capacity can be used to satisfy the export demand. The transfer problem studied favours an effectiveness criterion maximising the quantities exported. This criterion implicitly maximises the pipeline use and minimises the water separating the shipments of granule batches of different products and can, therefore, be considered efficient. As the production costs are slightly affected by the transfer program thanks to the use of judiciously chosen common constraints, this effectiveness criterion used by the transfer model in the DSS guarantees that the chosen solution is globally efficient.

- In the transfer problem considered here, some possible expedition dates are given but not the quantities to be transported, which differs from conventional batch-sizing problems (where lead times are generally ignored).
- Multi-item lot-sizing models consider grouped deliveries of items that share a launch cost, which makes no sense in this case.

Multi-product pipe transfer models aim to find optimal solutions under steady-state conditions. The problem studied here deals with the case of deterministic variable demand and integrates the additional constraints induced by the preventive maintenance of the pipeline or washing lines.

3 Models foundations

The two formally independent models are linked by sharing common features of operation, time representation, and batch calibration to be considered in the definition of transfer and production order books involving a general organisational design of production and transfer (§3.1). The detailed foundations of the pipe transfer and blending models are presented in §3.2 and §3.3. The approach adopted here is to give a literary presentation of the basic principles of these models and

to provide their mathematical descriptions in Appendices A and B. Some additional considerations are added (§3.4) about the interaction between these models and their use embedded in a Decision Support System (DSS).

3.1 Overall design and parameter sharing by both models

The production and transport systems have to meet (1) a continuous demand (internal demand) with small fluctuations and (2) irregular boat exports of important batches (external demand). This is technically possible because the flow rate of the internal demand, which has priority to avoid stock shortages, is significantly lower than the production flow rate of the washing plants, which is significantly lower than the pipe transfer flow rate (see level 1 in Figure 2).

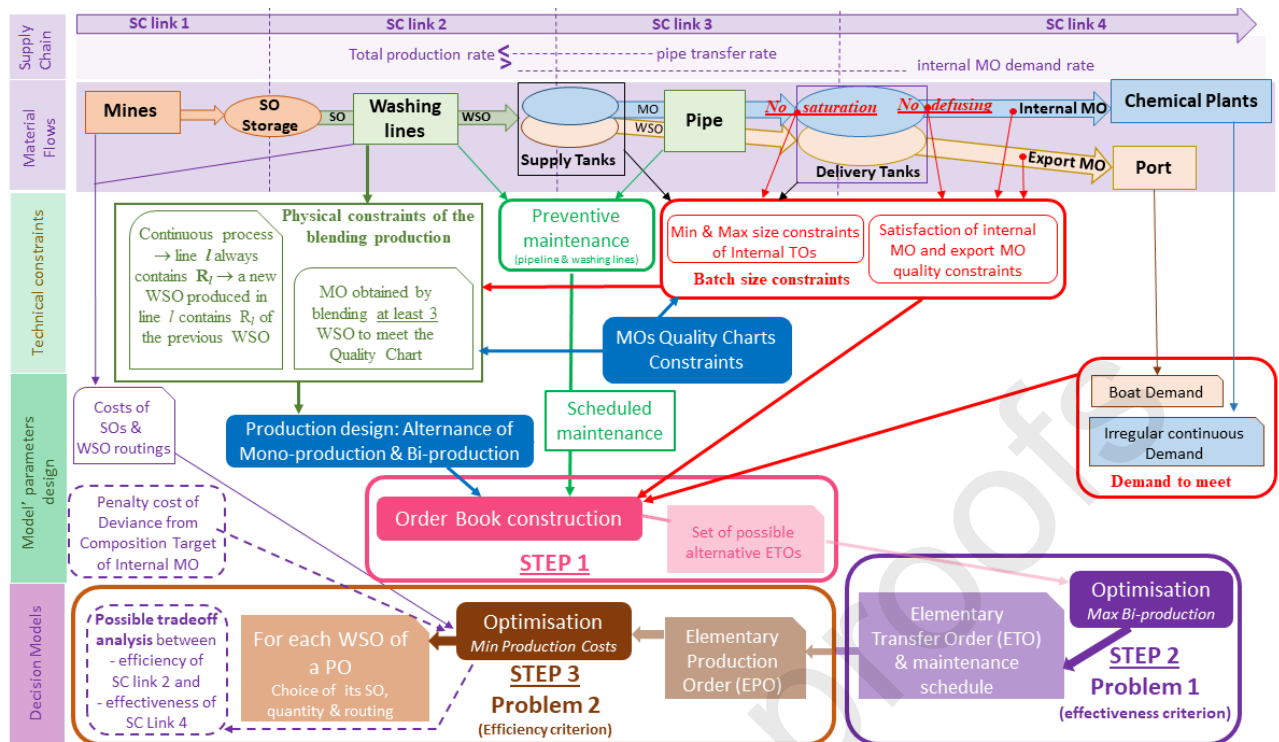


Figure 2 Description of the modelling approach.

Production and transport constraints imply that it is impossible to solve both problems by correctly satisfying both types of demand, starting from the raw expression of continuous and discrete demand, bearing in mind that production and transfer can only take place in batches. In addition, technical constraints (quality issues, production and storage capacities...; see the horizontal levels 2 and 3 of Figure 2) prevent the use of this raw demand information. The first step in this scheduling process is then to 'translate' the raw demand information into a transfer order book to obtain an optimal transfer program (step 2 in Figure 2) which can easily lead to a production order book to be used as an input for optimising the production program (step 3 in Figure 2).

The simultaneous consideration of continuous and discrete demand, as well as batches in production and transfer, requires a sufficiently fine discretisation of time to allow discrete optimisation models to properly account for the effects of the decisions to be made (§3.1.1). We will then detail the physical flow and inventory constraints to be considered in the modelling before addressing those resulting from the constraints of the MO composition (summarised in Figure 2) leading to a general organisation of the production and transfer of MOs (§3.1.2) based on parameter sharing between the two optimisation models.

3.1.1 Time specifications

The time specifications deal with the time division and the horizon used.

Demand, transfer, and production are continuous, but the modelling uses a discrete representation of time. The retained discretisation of time must allow a reasonably accurate definition of production and transport times as well as fluctuations in internal demand and fluctuations in the upstream and downstream pipe tanks. The choice of 15' as a unit of time is a good compromise in the modelling: when the production routing in a washing line is changed,

the set-up time can be zero, or 15' or 30', and with a transfer rate of 1,000 m³/15', the quantities transported are multiples of 1,000 m³.

The horizon of the production and the transfer programs is the same: two or three days, due to the knowledge of chemical and weight characteristics of the available SOs, bearing in mind that the models are embedded in a DSS that is launched every day.

3.1.2 General organisation of production and transfer principles

There are many ways to organise the production and transfer of MOs. The one chosen here is based on the simultaneous consideration of flow constraints and product constraints.

Meeting flow constraints

Three sets of physical constraints strongly structure the two decision problems.

- *Demand satisfaction constraints.* The continuous and fluctuating demand for internal MO from the chemical plants must be met without interruption. The required quantity of an export MO to be loaded onto a ship is known in advance and can be produced discontinuously in several batches. In both cases, quality requirements must be met.
- *Flow constraints.* The average flow rate of the internal demand (800 m³/15') is lower than that of the consolidated production of the washing plants (900 m³/15'), which is lower than that of the pipeline transfer (1,000 m³/15'). It follows that: *i*) the average utilisation rate of the pipeline cannot exceed 90%, the remaining transfer capacity being used to send water for separate deliveries of batches of different composition; *ii*) the remaining production capacity (100 m³/15') can be used to produce up to 9,600 m³ of export MO per day.
- *Stock constraints.* The pipeline is connected upstream to two *supply tanks* (see Figure 2), with an individual capacity of 9,000 m³ and downstream to two delivery tanks, with an individual capacity of 18,000 m³. In both cases, one of the stocks is dedicated to internal MO and the other to export MO, making it easy to switch between internal MO and export MO. Pipe delivery easily solves the potential problem of stock saturation upstream of the pipeline. Downstream, the stock receiving the external MO is connected to a stock located in the port, eliminating the saturation problem. An empty delivery stock of internal MO will be saturated on average after $18,000/(1,000-800) = 90$ quarter hours (22.5 hours), assuming that the maximum transfer rate (1,000 m³) can always be used; then an internal MO batch of more than 90,000 m³ cannot be shipped. A full tank that is not being fed will be emptied in $18,000/800 = 22.5$ quarter hours (= 5,625 hours), which is the maximum time during which an export MO can be transferred through the pipeline.

In view of these constraints, it would be possible to use all the washing lines alternately to produce internal MO and export MO. This production organisation is difficult to implement because: *i*) the difficulty of considering the preventive maintenance of the pipe or washing lines; *ii*) the variability of the quantities of export MO to be produced; *iii*) the weak fluctuation of the demand for internal MO. The MO quality constraints (see below) invalidate this type of rigid organisation in favour of a more flexible organisation that alternates the allocation of all washing lines to the production of internal MO (known as *mono-production*) with a split allocation of these lines between internal MO and export MO (known as *bi-production*). During the feeding of the delivery pipe station dedicated to the internal MO, its level increases in mono-production mode and decreases in bi-production mode (these variations are only possible without causing the tank to defuse or saturate). In bi-production, it seems advisable to allocate three washing lines to the production of internal MO to ensure that there is not a too rapid

decrease in the level of the internal MO pipe delivery stock and to guarantee that the output blend is of the required quality. In bi-production, internal and export MOs are co-produced simultaneously.

Meeting product quality constraints

The introduction and Figures 1 and 2 briefly describe the production of a MO. Here we examine the impact of the required Quality Charter (QC) compliance on the chosen production organisation. Since internal MO is required continuously, QC compliance must be ensured as the pulp enters the pipe. As the export MO is ultimately stored in a tank at the port where its various WSOs are blended, QC compliance only needs to be ensured at the end of this final storage.

Experience has shown that the QC of a MO is respected in an optimised solution where at least three WSOs are blended, which is the case in mono-production. An export order must be split into four (or more) batches of WSOs over two (or more) bi-production periods, bearing in mind that the two export WSOs produced in the same bi-production period are mixed in the export pipe supply tank.

At the start of a new production run, residues from the previous production are still present in the tanks of the washing lines. The QC of an internal MO can only be respected when the tank of the pipe supply station contains a certain amount of product, which is the lower limit of the internal MO batch in both production modes. The corresponding lower limit for the export MO in the bi-production mode is determined from the ratio of the production rates of the group of lines assigned to each product. Experience shows that this is achieved when the tank of the pipe supply station reaches 7,000 m³, after a filling time ε of $7,000/900=7.78$ quarter hours in mono-production, before the tank starts to be emptied. In bi-production, with a production rate of 575 m³/15' for internal demand and 325 m³/15' for export, this filling time is $7,000/575=12.17$ quarter hours, and the corresponding lower limit for an export batch is $7,000 \times 325/575 = 3,957$ m³.

The relative scarcity of certain SOs argues in favour of a MO batch size that is not too large in order to make the best use of the variety of SOs available, as some of those available in small quantities may no longer be eligible for the production of a WSO. Finally, in mono-production, the size of an internal MO batch used in the modelling must be between 7,000 and 50,000, corresponding to a production time of 2 to 14 hours, which is in line with current practice. In bi-production, with an internal demand flow rate of 800 m³/15' and a production flow rate of 575 m³/15' for the internal demand, a full-up delivery tank of 18,000 m³ is empty in $18,000/(800-575) = 80$ quarter hours, corresponding to an oversized internal MO batch produced in bi-production ($80 \times 575 = 46,000$ m³).

3.2 Foundations of the Pipe Transfer Model

This model (detailed in Appendix A) is based on an original approach of defining the number, size, rank, and arrival dates of internal MO ETOs (Elementary Transfer Order) to meet demand over the chosen time horizon. This problem is modelled by a linear program using binary variables. The DSS can automatically support these principles (see Figure 4). The ETO sizes must respect the previously defined upper and lower bounds, ensuring the articulation between the production and transfer models. The rules for defining TOs (Transfer Orders) and ETOs are detailed in §3.2.1. The effect of desynchronisation between production and transfer on the transfer problem is discussed in §3.2.2. The model has to take into account maintenance that

prevents the use of pipe or washing line for a certain period of time (§3.2.3). The optimisation criterion used is justified in §3.2.4.

3.2.1. Splitting Transfer Orders into Elementary Transfer Orders

Transfer planning focuses on internal MO transfer orders (TOs) generated in mono or bi-production mode. Their transfer dates are not predetermined but are defined over periods of several hours, with the final date defined by optimisation. Each TO is divided into several elementary transfer orders (ETOs).

- In *mono-production* mode, these TOs are *not exclusive*; only some are retained in the optimal solution. The ETOs of a TO are exclusive and of different sizes, covering the acceptable range of sizes. The idea is to provide a surplus of supply to ensure that a solution is possible. It is possible to systematically use the same set of TOs for all problems without any loss of effectiveness, since each TO is identically divided into several ETOs, provided that the scheduling horizon is correctly covered (Table 3 shows 3 TOs that can be used for each of the 2 scheduling days, each TO being decomposed into 9 exclusive ETOs ranging from 10,000 to 50,000 in steps of 5,000).
- In *bi-production mode*, these TOs correspond to *exclusive* variants of splitting an export order into ETOs to be produced and transported. The ETO sizes of an export TO are freely defined, subject to the bound constraints, with the sum of ETO sizes equal to the TO size. As export MO is co-produced with internal MO, the selected export ETO size determines the size of the internal ETO produced in bi-production mode. At most, one TO can be retained; its selection depends on the size of its ETOs and the remaining capacity after internal demand has been met. For example, in Table 3, we consider the production of all or part of an export order of 16,000 m³, to which we arbitrarily associate 3 possible exclusive TOs to cover this demand, each TO being subdivided into ETOs whose size respects the volumetric constraints described in §3. 1.2 (here, the first TO is split into 2 ETOs (5,000 m³ and 11,000 m³), the second TO is also split into 2 ETOs (6,000 m³ and 10,000 m³), and the last TO is split into 4 ETOs of the same volume (4,000 m³)). The volume of an export ETO (column 5 of table 3) automatically determines the volume of the corresponding internal ETO (column 3 of table 3), given the systematic allocation of washing lines in biproduction. While in mono-production it is possible to always use the same dataset, in bi-production the dataset depends on each export order, with simple rules being used to define a few TOs to associate with an export order.

As the transfer model is used in a rolling mode, one must consider the case of a bi-production TO that has already begun. In this case, no other variant of this TO can be retained, and only the remaining ETOs of this TO and possibly an ETO in progress, are considered. If the remaining capacity is enough, other export orders can be considered, involving possible additional bi-production TOs and requiring the introduction of an export rank to avoid mixing different export MOs in tanks.

Once an optimal solution has been found for the sizes and schedules of the internal ETOs, the export transfer schedule is defined "manually" by "filling" the periods when the pulp is not sent, considering the constraints of water shipment after each transfer in the pipeline. Including the export ETOs in the optimisation model is possible, but this would not be of operational interest.

3.2.2 Implications of the desynchronisation between production and transfer.

The transfer problem defines, for each ETO, the start and end dates of arrival at the pipe delivery station. Quality considerations prevent a new batch of internal MO from being sent to the pipe until the tank level reaches the required threshold (7,000 m³). In the meantime, the pipe cannot accept any more pulp, and this interval (ϵ) named *filling slot*, is used to send water (for cleaning) or export WSO. The transport then starts during a period called *transportation slot*, whose length (δ) is the quotient of the quantity transported by the pipe transfer rate. We call the *transfer slot*, the concatenation of the filling slot, and the transportation slot. Figure 3 illustrates the synchronisation between arrivals at the tank of the pipe delivery station, departures from the tank of the pipe supply station, and arrivals in this tank, using a simplified example.

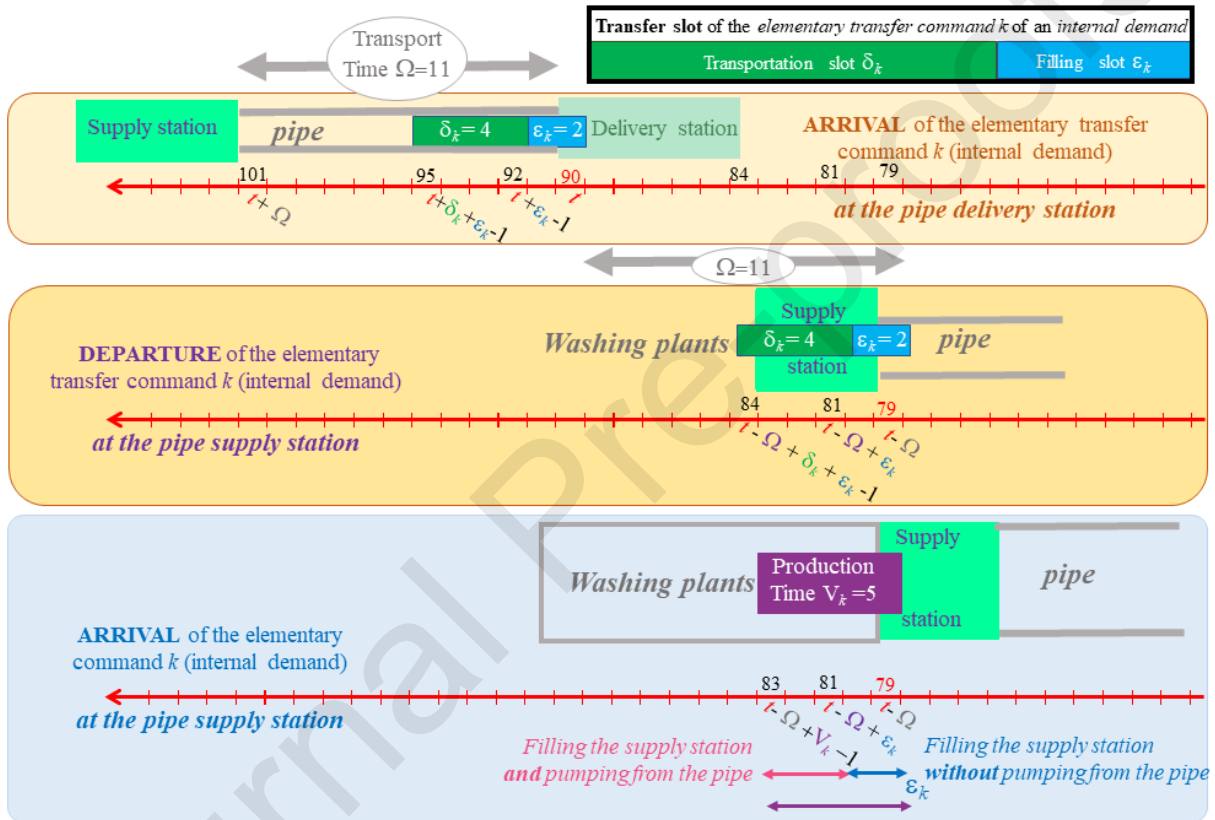


Figure 3. Synchronisation between pipe transfer, filling the pipe supply tank and production.

3.2.3 Consideration of maintenance in the pipe transfer program

This maintenance can be carried out on the pipe or on a washing line.

- The *pipe maintenance* is modelled by a "dummy" TO, including a single ETO, whose transfer slot contains only a filling slot corresponding to the maintenance duration and no transportation slot (with pulp feeding at the delivery station). Wherever the maintenance has to be carried out, the flow is interrupted. This means that TOs whose arrival date at the delivery station is later than the start of the maintenance will take longer than the transport time Ω to reach their destination. Therefore, in order to meet the arrival date at the delivery station, TOs arriving after the maintenance period must be injected into the pipeline at a date reduced by the transport time (Ω) and the maintenance duration (illustration given at §4.1.2). The completion interval of the arrival date corresponds to the degree of freedom

available to perform this maintenance operation. If the maintenance date is imposed, the earliest and latest arrival dates at the delivery station coincide. This dummy TO must be executed, unlike other TOs, which may not be executed.

- The *washing line maintenance program* assumes that there is no overlap in the maintenance of several washing lines. The maintenance reduces the production throughput of the group of lines working for internal demand (in mono or bi-production) and/or for export demand (bi-production). The usual duration of this maintenance is relatively long (in the order of 7 hours), which leads to defining an internal TO batch calculated over this period with a single mandatory ETO. These dates are determined considering the transfer times from the washing plant to the pipe supply station to ensure the simultaneous arrival of the WSOs at the supply station. As for pipe maintenance, it is also possible to leave a degree of freedom in the execution of this maintenance, with the model determining the exact dates of this maintenance.

3.2.4 Performance criterion of transfer programming.

The use of the pipe to transport the quantities required by internal or export customers is non-negotiable; the maintenance of the pipe results from this mandatory use and is not affected by the chosen transfer program. A cost criterion for the water used between different product transfers can be used, but it is closely linked to the amount of pulp transferred, which is considered in the effectiveness criterion. Therefore, cost considerations are not very useful.

The use of an effectiveness criterion is preferred. It takes into account two elements: *i*) from the moment internal demand is satisfied, it is advisable to produce (and therefore transfer) as much of the WSO produced for export demand as possible, seeking to maximise the co-production of internal MO, which leads to maximising the export production (and limiting the production of internal MO in mono-production mode); *ii*) the importance of internal MO transfer depends on the initial stock in the delivery station tank; it is then advisable, for an identical amount of MO transferred, to favour the programming in which the final stock of internal MO is the highest to safeguard the future and maximise the use of the pipe. The effectiveness criterion is therefore a weighted sum of the transferred amount of co-produced internal MO and the final stock at the delivery station, with the weights used favouring export maximisation.

An *optimal solution* may not exist if some punctual requirements are too high and/or the SO variety is too low (this influence is illustrated in §4.4), combined with maintenance constraints, in which case the corresponding parameters must be modified, which requires negotiation with the stakeholders of the problem (customer, maintenance team...).

3.3 Foundations of the Blending Production Model

The optimal transfer program depends on ETO parameters (line assignment and transfer dates) and results in the production order book. A production order (PO) of an internal MO corresponds to an ETO, except in the case of mono-production, where its size can be halved to allow better use of SOs and facilitate QC compliance. This transfer program also defines the maintenance of washing lines and the production of exported WSOs. Based on this data, the blending problem defines the routing and the quantity of SO for each line implied by the mono- or bi-production of each PO. Some details need to be added to justify certain relationships in the model (§3.3.1) before detailing the optimisation criterion (§3.3.2).

3.3.1 Description of the blending process

A PO is processed by a set of washing lines operating synchronously. Each washing line processes a unique SO taken from a nearby storage facility connected to it (see Figure 1), where SOs of known characteristics are stored; conveyor constraints prevent two lines from being fed simultaneously from the same storage area. Each stockpile, of limited capacity, is connected to several mines. The production program is based on the SOs' initial availability, which creates a strong interdependence in the SOs' assignments to the ETO in order to produce within the planning horizon.

The five washing lines have two or three *routings* among the following: scrubbing, scrubbing + flotation, scrubbing + flotation + grinding. The dwell time in a washing line is the same regardless of the routing used. In other words, using a higher quantity of SO to obtain a given quantity of WSO will change the rate at which SO enters the washing line, but not the rate at which WSO leaves.

The choice of routing has five consequences: *i)* the washing treatment results in a dry weight of WSO that is significantly lower than that of the used SO; *ii)* the chemical composition of WSO is different from that of the used SO; *iii)* the routing change may involve a significant set-up that prevents the washing line concerned from producing during the set-up time, resulting a desynchronisation and a loss of production that has to be compensated by increasing the production of the other washing lines; *iv)* a routing change involving a set-up time is not allowed at the start of a bi-production; *v)* the cost of a ton of WSO depends on the chosen routing, not only because of the energy and chemical products used, but also because of the quantity of SO used which varies from one routing to another, for an identical quantity of WSO; this must be considered because the extraction costs are high.

As the washing process is continuous, the new SO arriving on a washing line at the start of a new PO enters a line filled with the previous SO from the previous order. As the washing process takes about half an hour, the production of a WSO mixes a fixed quantity of previous SOs (washing line production rate multiplied by two quarter-hour periods) with a quantity that depends on the ETO quantity. This overlapping of flows must be considered in order to comply with the quality chart.

3.3.2 Optimisation criterion

The objective-function sums an efficiency criterion and an effectiveness criterion.

- The *efficiency criterion* is the sum of the extraction costs (considered the same, regardless of the SO) and the processing costs (energy, water, chemical product, waste disposal...) associated with the routing used.
- The *effectiveness criterion* does not refer to the studied scope of the SC (production and transport of MO) but to an extended SC scope, including the phosphoric acid plants, which are the customers of the internal MO. Differences in the composition from one MO batch to the next, while respecting the QC, degrade the performance of the acid production line for a while if the new composition is too different from the previous one. This loss of performance generates additional costs that are not currently evaluated. Therefore, from the point of view of the overall SC performance, it was decided to consider an effectiveness criterion calculated on the sum of the absolute values of the deviations in the MO composition compared to a target composition considered to be the best from a chemical point of view. This is introduced into the objective function, weighted by an arbitrary

penalty cost. *If this penalty cost is zero, the optimal solution found is based only on a local efficiency point of view*; otherwise, it leads to both an increase in cost and an improvement in the effectiveness criterion. Variations in the penalty cost trigger downstream variations in the effectiveness, providing a financial basis for a possible negotiation between these two SC links (SC links 2 and 4).

3.4 Encapsulation of the transfer and production models in a Decision Support System

The foundations of DSS were laid by Gorry & Scott Morton (1971) and developed into a proper system by Keen & Scott Morton (1978). Developments in Information Systems and Information Technology have enormously increased the potential of DSS without changing its rationale. Basically, a DSS consists of (Azzamouri et al., 2021): *i*) an interface to formally express a complex issue that is partially structured to define a structured problem; *ii*) one or more modules to solve the structured problem, typically based on optimisation or simulation models (Power & Sharda, 2007); *iii*) an interface capable of exploring all the consequences of an obtained solution and either marginally adjusting it or validating it; *iv*) if none of these solutions are acceptable, the DSS is then used to define a new structured problem using the feedback from previous formulations that did not provide a satisfactory solution.

In our case, the DSS to be used in a rolling planning mode would encapsulate the transfer and production models used sequentially, with two possible feedbacks to redefine a new structured problem (see Figure 4). It should be noted that the design of the models implicitly implies a periodicity of two days in the DSS use (a slight adaptation of the transfer model is to be made for a DSS daily use). From the perspective of the SC, this DSS would help to negotiate a trade-off between an efficiency gain downstream and an efficiency loss for the studied part of the phosphoric SC. Figure 4 describes the principles to be used in this DSS.

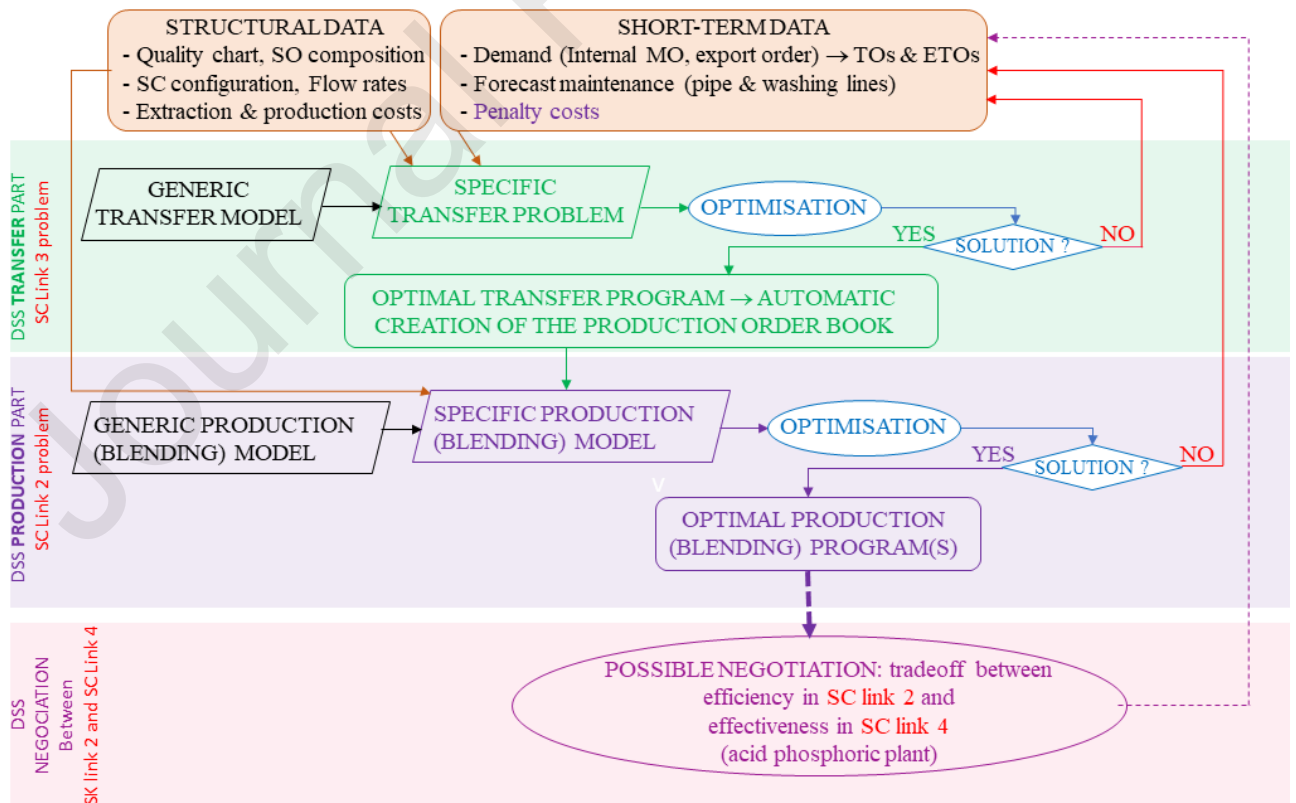


Figure 4. DSS architecture for a joint resolution of transfer and blending problems.

It should be emphasised that we are providing the basis for what could be a DSS that implements our approach in the classic POC (Proof of Concept) perspective. Data management on a relational database has been tested (but its presentation here is of no interest). We have verified that the solution times of the two models were short enough to be integrated into a DSS used for operational decision making (see Table 9). The routine for transforming the optimal transfer solution into an order book to be used as input for the production model is based on two very simple rules, which are taken into account by Excel. The use of the solutions was also tested in Excel. Our conclusion is that the translation of all this into a DSS designed by professionals on the basis of specifications derived from our work and refined by feedback from potential users is a simple additional task.

As part of this POC approach, the interactions between the virtual DSS and potential users have been tested on a number of occasions. However, it is clear that the DSS user interface can only be improved on the basis of a 'beta' version tested in the field.

4. Case study

In this section, we illustrate the approach presented in this paper by determining transfer programs through several scenarios (§4.1) and then examining the combination of one of the scenarios with several blending instances (§4.2). Finally, we discuss the use of penalty costs as a negotiation tool between supply chain actors (§4.3).

4.1 Determination of the transfer program

4.1.1 Studied scenarios

The same demand pattern is used in all examples, knowing that any pattern based on the time unit can be used. The flow rates are given in Table 2. Table 3 describes the possible TOs offered for mono-production (TOs 1 to 6) and bi-production (TOs 7 to 10). For bi-production, the volumes of both internal and external demands are rounded to the nearest miller. This is technically consistent because the flow rates in the real process can be easily adjusted for each set of lines dedicated to internal or export production to obtain a consistent ratio of production rates of producing internal and export WSOs, slightly different from those calculated using average rate values. If necessary, one can work efficiently with a finer time division (e.g., 5' instead of 15') to limit this rounding problem associated with the discretisation of a continuous process by choosing that these decisions can only be taken every n periods (for example, $n = 3$) and by smoothing their effects over n periods (Bouzekri et al., 2021 and 2023). At the start of the program, the export TO 7 is in progress, with a single ETO still to be produced (5,000 m³, corresponding to an ETO of 9,000 m³ of internal MO). A further export demand of 16,000 m³ is considered with three split variants (TOs 8, 9 and 10). The constraints on the TOs dates (at the delivery station) are shown in Table 3.

Table 2. Internal MO Demand

Day 1	Day 2
-------	-------

	[0:00 - 8:00]	[8:00 - 24:00]	[0:00 - 8:00]	[8:00 - 24:00]
période p	1 to 32	33 to 96	97 to 128	128 to 192
Demand rate (m ³ /15')	728	808	756	835

Table 3. Available TOs and ETOs

	TO p	Internal ETO (m ³)	Export TO rank	Export ETO (m ³)	V_{k_p}	ε_{k_p}	δ_{k_p}	restricti on in selecting dates
					Producti on time (unit 15')	Filling slot (unit 15')	Transportat ion slot (unit 15')	
Mono-production	[1 to 6]	1	10,000		12	8	10	[1 to 96] for TOs 1,2,3 & 7
		2	15,000		17	8	15	
		3	20,000		23	8	20	
		4	25,000		28	8	25	
		5	30,000		34	8	30	
		6	35,000		39	8	35	
		7	40,000		44	8	40	
		8	45,000		50	8	45	[96 to 192] for TOs 4,5 & 6

		9	50,000			56	8	50	
Biproduction	7	1	8,846	1	5,000	15	12	9	[96 to 192] for T0s 8 to 10
	8	1	8,846	2	5,000	15	12	9	
		2	19,462		11,000	34	12	19	
	9	1	10,615	2	6,000	18	12	11	
		2	17,692		10,000	31	12	18	
	10	1	7,077	2	4,000	12	12	7	
		2	7,077		4,000	12	12	7	
		3	7,077		4,000	12	12	7	
		4	7,077		4,000	12	12	7	

Four scenarios are examined:

- *Scenario A*: without maintenance
- *Scenario B*: scenario A + a pipe maintenance of 9 quarter hours, starting between periods 96 and 140. A mono-production must then be placed whose filling time (8) occurs during the maintenance, thus increasing the transferred production. The pipe being under maintenance does not prevent the Supply Station tanks from being filled. Therefore, of the 9 maintenance periods available, 8 are used to fill the Supply Station tanks so that QC can be maintained afterwards. This means that the filling slot of the TO after the pipe maintenance is $\text{Max}(0, 8 - \text{maintenance duration})$. Since the maintenance duration is 9, the maximum here is 0, a value assigned to the TO 6. Line maintenance necessarily starts at the beginning of a mono-production run; as production on all lines must be finished at the same time, the production not completed during line maintenance must be made up by delaying the production finish date that would have been reached without maintenance.
- *Scenario C*: scenario A + a maintenance of 24 time-units on line MEA LG to be achieved between periods 1 and 88 (during this maintenance, the four remaining lines will produce an internal MO at a rate of 675 m^3 per time-unit; so, this TO corresponds to $24 \times 675 = 16,200$ rounded to $16,000 \text{ m}^3$)
- *Scenario D*: fusion of scenarios B and C.

None of these scenarios use a penalty cost (discussed in §4.3), and initial stocks are arbitrarily set to 0 at the Supply Station and to 15,000 m³ at the Delivery Station for internal MO.

4.1.2 Transfer program solutions

Table 4 shows an optimal solution (based on the available TOs and ETOs in Table 3) for each of the four scenarios considered. Computational information is provided in §4.4.

Table 4. Optimal solutions for the four scenarios.

Scenario A					Scenario B				
TO - ETO	MO Quantity (m ³)		Transportation dates		TO - ETO	MO Quantity (m ³)		Transportation dates	
	Internal	Export	Start	End		Internal	Export	Start	End
7 - 1 (Export)	9,000	5,000	13	21	7 - 1 (Export)	9,000	5,000	13	21
2 - 9	50,000		30	79	3 - 9	50,000		30	79
3 - 9	50,000		88	137	2 - 5	30,000		88	117
9 - 2 (Export)	18,000	10,000	150	167	Pipe maintenance	0		118	126
9 - 1 (Expot)	11,000	6,000	180	190	6 - 6	35,000		127	161
					8 - 2 (Export)	19,000	11,000	174	192

Total (Export)	38,000	21,000							
Final stock level at the delivery station (m ³)				676	Total (Export)	28,000	16,000		
					Final stock level at the delivery station (m ³)				5,676
Scenario C					Scenario D				
TO - ETO	MO Quantity (m ³)		Transportation dates		TO - ETO	MO Quantity (m ³)		Transportation dates	
	Internal	Export	Start	End		Internal	Export	Start	End
11 - 1 (line 1 maintenance)	16,000		9	24	11 - 1 (line 1 maintenance)	16,000		9	24
1 - 9	50,000		33	82	3 - 9	50,000		33	82
7 - 1 (Export)	9,000	5,000	95	103	1 - 3	20,000		91	110
5 - 9	50,000		112	161	<i>Pipe maintenance</i>	0		111	119
8 - 2 (Export)	19,000	11,000	174	192	6 - 3	50,000		122	171
					7 - 1 (Export)	9,000	5,000	184	192

Total (Export)	28,000	16,000			Total (Export)	9,000	5,000		
Final stock level at the delivery station (m ³)			6,676		Final stock level at the delivery station (m ³)				7,676

In scenario B, chosen for illustrative purposes (whose Gantt is given in Figure 5), the first ETO transferred (7-1) is the last ETO of a TO started the previous day. The last ETO (8-2) is fully transferred at the end of period 192. It can be seen that before the pipe maintenance, there was no transfer for nine periods, allowing possible maintenance anticipation (for the same optimum). On the $(2 \times 24 \times 4 =) 192$ periods included in the retained horizon, the maintenance lasts 9, and the transportation time lasts $(9 + 50 + 30 + 35 + 19) = 143$ 143, leaving $192 - (143 + 9) = 40$ periods for water or export batches. The cumulated filling time (excluding maintenance overlap) involved by quality considerations is $(2 \times 12 + 2 \times 8 + 0) = 40$. The pipe occupancy rate is $((143 + 9) / 192 =) 79\%$, not too far from the theoretical limit of 90% (which does not consider the filling slots involved by quality considerations). Finally, it should be noted that the ETO 8-2 is not completed at the end of the first day, which means that if the DSS is used on a rolling daily basis, the following day, the pipe will continue to receive ETO 8-2 for a quantity equal to 11,000 m³.

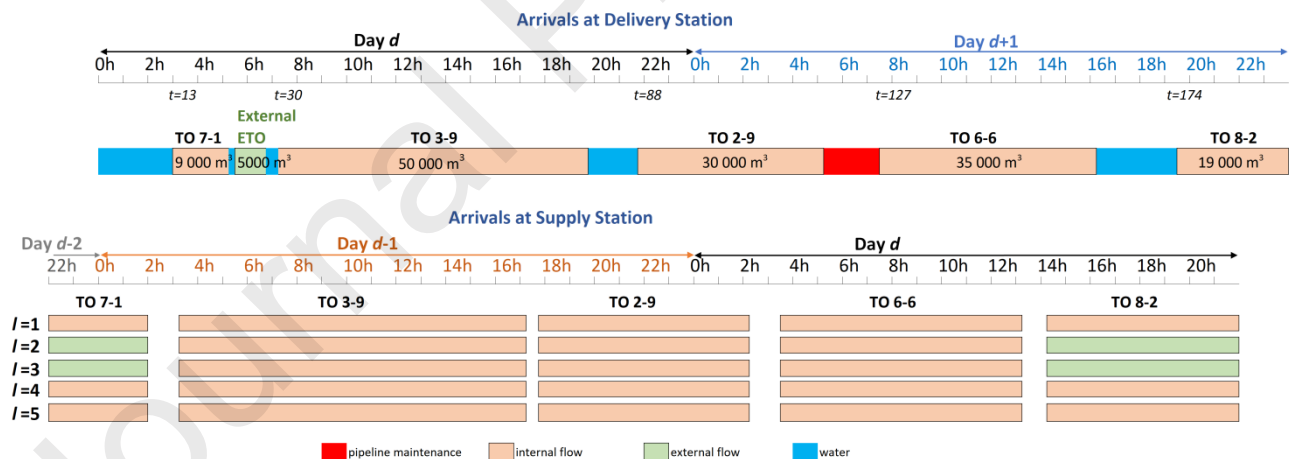


Figure 5. Pipeline transfer program for scenario B

Pipe maintenance and, to a lesser extent, washing line maintenance delay occurrence of saturation and then the subsequent need for bi-production, thus reducing export production. This explains why Scenario A yields the highest quantity of export MO shipped (38,000 m³) against 28,000 m³ in Scenarios B and C, due to the pipe *or* washing line maintenance and 9,000 m³ in Scenario D due to the pipe *and* washing line maintenance.

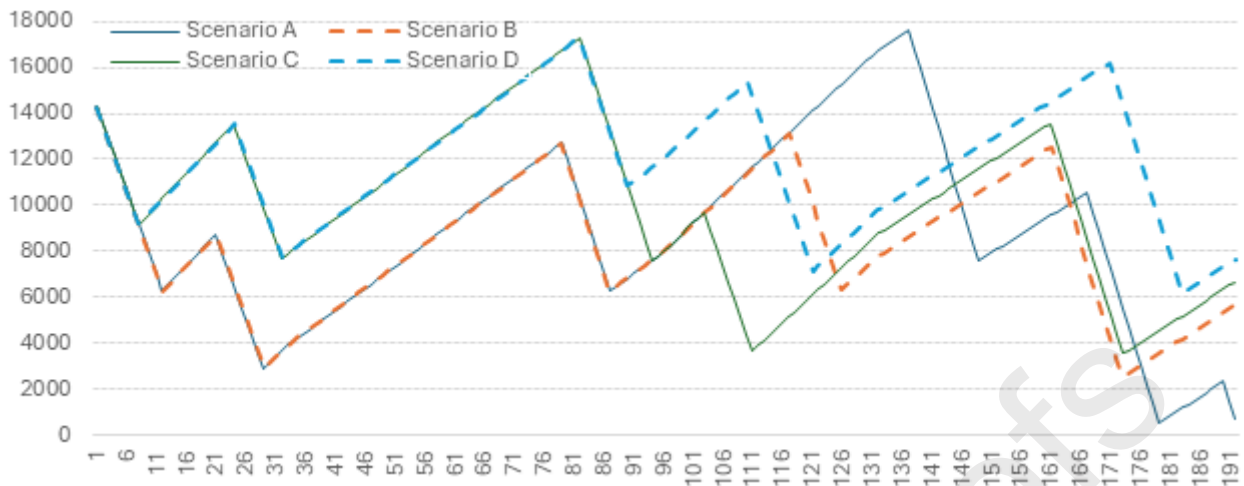


Figure 6. Stock evolution in the tank of the pipe delivery station according to the studied scenarios

Figure 6 illustrates the evolution of the internal MO in the pipe delivery stock. In all scenarios, there is neither defusing nor saturation. It should be remembered that these bounds are parameters that can easily be adjusted to take account of potential risks. The number of orders received is lower in scenario A than in scenarios B to D because maintenance leads to the use of smaller batches in order to comply with the non-saturation and non-defusing constraints. Note that the downward slopes of the curves are lower in the case of simultaneous supply and withdrawal and steeper in the absence of supply.

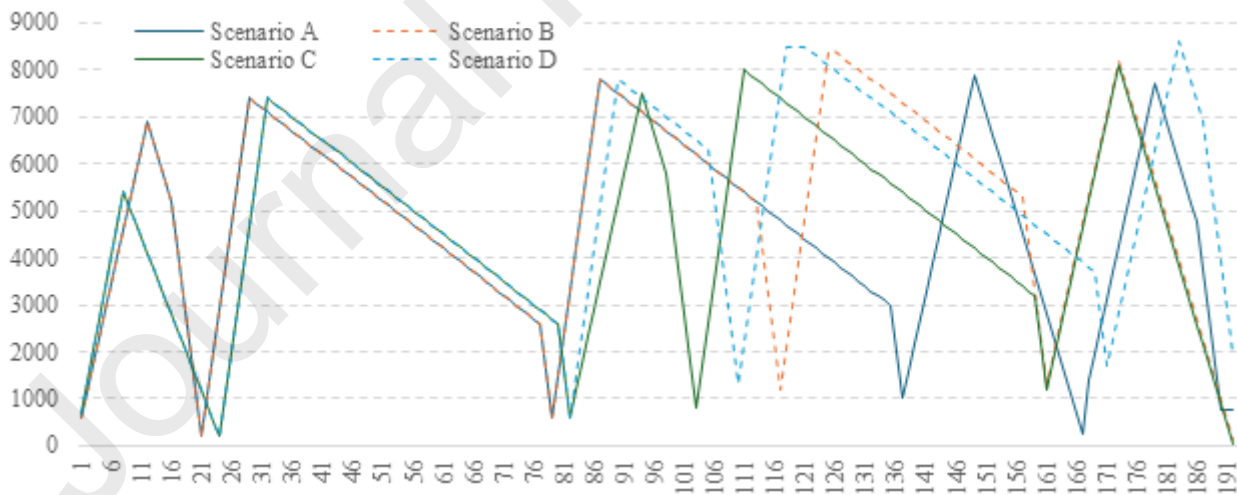


Figure 7. Stock evolution in the tank of the pipe supply station according to the studied scenarios

For each Scenario, Figure 7 shows a succession of irregular cycles starting with a phase of feeding the tank without delivery through the pipe (segment with increasing slope), followed generally but not always, by a phase of simultaneous filling and emptying of the tank (segment with slightly decreasing slope), followed by a phase of emptying without feeding (segment with strongly decreasing slope). For example, the first order feeds the stock for 9 periods, after which

the next order cannot arrive until this stock has been emptied, to avoid mix-ups and correctly manage the quality problem. With the removal rate of the pipe, this stock is emptied in 5 periods, during which the stock decreases sharply, before being replenished in period 22

4.2 Determination of the production order book

The optimal solution of Scenario B (see Table 4) is retained in this section for illustrative purposes. As explained in §3.1.3, all TOs arriving at the pipe delivery stock at a time t before pipe maintenance leave the pipe supply stock at that date minus the transport time in the pipe ($t-\theta$); those following a pipe maintenance leave earlier with an additional delay equal to the duration of this maintenance. In the production model, only the order of the ETOs, and not their due dates, matters to satisfy the transfer program, which is justified by two considerations. The routing changes, not considered in the parameters of the transfer problem, may lead to start-up times, implying a slight increase in mono-production duration. On the other hand, the difference between the production and consumption rates of the internal demand implies some short production stoppages that can absorb the excess production time caused by routing changes.

In mono-production, some PO quantities are too high (arbitrary threshold of 25,000 m³) to allow effective use of the available SOs, which involves splitting them into elementary production orders (EPOs spotted by k_p). In scenario B, the internal batch of the PO $p=1$ (ETO 7-1) produced in bi-production has an acceptable size. The PO $p=2$ corresponds to the export batch produced along the previous PO. The following POs are too large and must be split into 3 EPOs ($\rightarrow k_p = 1$ to 3): {7,000; 21,500; 21,500} for PO $p=3$ and {7,000; 14,000; 14,000} for PO $p=5$ (ETO 2-9 and 6-6). While PO $p=4$ and $p=6$ are split into two EPOs ($\rightarrow k_p = 1$ to 2): {7,000; 23,000} and {7,000; 12,000} respectively. Finally, PO $p=7$ corresponds to the export batch produced along PO $p=6$.

4.2.1 SOs availability

In this test, 28 SOs are available, distributed over the 6 storage areas. For each SO, the chemical characteristics, the reduction, and the distortion coefficients of the components according to the three possible routings are known (Table 5).

Table 5: SO initial stock and chemical composition

SO designation	SO i	SO stock	Initial quantity	Chemical composition		
				BPL (%)	MgO (%)	Cd (ppm)
BT 320-490	$i=1$	$s=1$	25,000	60.60%	1.36%	20.8
BT 210-310	$i=2$		15,000	61.90%	1.32%	18

BT 220-250	$i=3$		12,000	59.60%	1.45%	18.5
BT 140-210	$i=4$		12,500	61.92%	1.75%	19.6
BT manut1	$i=5$		9,000	60.13%	1.80%	19
BT manut2	$i=6$		25,000	61.61%	1.67%	21.3
TBT 570-590	$i=7$	$s=2$	10,000	56.43%	2.88%	17.5
TBT 500-540	$i=8$		15,000	55.48%	3.00%	18.25
TBT 590-640	$i=9$		11 000	56.11%	2.90%	17.6
TBT 440-500	$i=10$		13,400	53.24%	2.15%	18
TBT manut1	$i=11$		19,000	54.32%	2.04%	23.3
MT 160-210	$i=12$		$s=3$	12,300	66.10%	0.51%
MT 380-430	$i=13$	9,500		67.20%	0.48%	14.8
TBT 300-350	$i=14$	11,200		57%	1.33%	21
TBT 20-80	$i=15$	10,400		59.40%	1.23%	21
BT manut3	$i=16$	14,000		59.60%	1.99%	17.8
TBT manut2	$i=17$	13,500		59.10%	2.25%	19.8
TBT 370-450	$i=18$	$s=4$	29,400	59%	0.85%	20.5
TBT 160-200	$i=19$		13,000	55.40%	1.21%	23
TBT 250-320	$i=20$		10,200	57.36%	1.65%	25

BTR 30-80 /A	$i=21$	$s=5$	9,000	66.60%	0.69%	18.0
TBT 30-220 /B	$i=22$		9,800	55.50%	1.77%	20.7
MT 70 -130 /A	$i=23$		28,000	66.60%	0.80%	22
TBT 270-300 /D	$i=24$	$s=6$	27,800	56.63%	1.51%	24
TBT 390-450 / D	$i=25$		12,000	54.46%	2.00%	22
TBT 420-500 / C	$i=26$		13,400	55.49%	1.98%	19.43
TBT 160 - 300 / C	$i=27$		15,300	55.48%	1.80%	19
TBT manut3	$i=28$		12,800	55.80%	1.70%	20

ppm : particules per million

In this baseline scenario, the quantities of SO considered (Table 5) are limited, which may involve different blend structures of TOs of the same MO (see results in Table 7). To show the economic impact of SO availability, we provide some results at the end of this section, where SO availability is very high.

The study was carried out on two MOs that are currently shipped through the pipeline. The MOs $j = 1$ and $j = 2$ are ordered respectively by an internal and an export customer. The quality chart of these MOs for 3 considered chemical components is given in Table 6, along with the setup times.

Table 6: MO quality chart and setup times

Quality Charts

	MO j	Component c		
		BPL (%) ($c=1$)	MgO (%) ($c=2$)	Cd (ppm) ($c=3$)
Lower bound	Internal ($j=1$)	64	0	0
	Export ($j=2$)	65	0.45	18
Upper bound	Internal ($j=1$)	100	0.95	80*
	Export ($j=2$)	66	0.65	25
Target value	Internal ($j=1$)	65	0.8	75*
	Export ($j=2$)	65	0.55	25

ppm : *particules per million*

* in mg/P₂O₅

Set-up Time

		New routing		
		1	2	3
Previous Routing	1	0	40'	60'
	2	0	0	20'

	3	0	0	0
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The used production costs, varying according to the used routing, are close to the actual ones.

4.2.2 Production program solutions

In the mining literature, a feasible solution is considered of good quality if its deviation from the solution from the theoretical optimal solution is close to 5% (details given in §4.4). The objective function of the second model (see appendix B) is based on two partial costs: *i*) the production costs to extract ores and wash them; this cost, which is roughly proportional to the quantities of SO used, favours solutions that save the most on the use of non-renewable resources (SO) while meeting all the requirements of the demand to be satisfied (MO); *ii*) the penalty cost of the deviance of the ETOs' chemical composition from a preferred target value expected by the chemistry (internal MO) (see the model in Appendix B). In the current tests, the deviance penalty is zero, and only production costs are considered; the analysis of the trade-off between effectiveness and efficiency is presented in §4.3. The solution obtained for the scenario B is detailed in Table 7.

Table 7: SO withdrawals and routings to apply for each elementary production order (scenario B).

TO - ETO	7 - 1 (internal)	7 - 1 (export)	3 - 9	2 - 5	6 - 6	8 - 2 (internal)	8 - 2 (export)
PO p	$p=1$ (9,000 m ³)	$p=2$ (5,000 m ³)	$p=3$ (50,000 m ³)	$p=4$ (30,000 m ³)	$p=5$ (35,000 m ³)	$p=6$ (19,000 m ³)	$p=7$ (11,000 m ³)

Feed ing Supp ly Stati on	Startin g period*	13	13	30 - θ			88 - θ			127 - θ			174 - θ		17	
		- θ	- θ												4 - θ	
S O st o c k	S O i	EPO k_p	$k_p=$	$k_p=$	$k_p=$	$k_p=$	$k_p=$	$k_p=$	$k_p=$	$k_p=$	$k_p=$	$k_p=$	$k_p=$	$k_p=$	$k_p=$	
			1	1	1	2	3	1	2	1	2	3	1	2	1	
			(9,000 m ³)	(5,000 m ³)	(7,000 m ³)	(21,500 m ³)	(21,500 m ³)	(7,000 m ³)	(23,000 m ³)	(7,000 m ³)	(14,000 m ³)	(14,000 m ³)	(7,000 m ³)	(12,000 m ³)	(11,000 m ³)	Total withdrawal (in tonnes)
s = 1	1	Withdra wals (in tonnes)					6,980						3,557	6,098		16,634
		Washin g line				$l=1$							$l=1$	$l=1$		
		Routing				$h=1$							$h=1$	$h=1$		
	2	Withdra wals (in tonnes)	4,573		2,370				7,467							14,410
		Washin g line	$l=1$		$l=1$				$l=1$							
		Routing	$h=1$		$h=1$				$h=1$							
	3	Withdra wals (in tonnes)													4,395	4,395

		Washing line											$l=2$	
		Routing											$h=1$	
		Withdrawals (in tonnes)												4,270
	4	Washing line											$l=2$	
		Routing											$h=1$	
		Withdrawals (in tonnes)												6,980
	5	Washing line											$l=1$	
		Routing											$h=1$	
		Withdrawals (in tonnes)												11,460
	6	Washing line											$l=1$	
		Routing											$h=1$	
		Withdrawals (in tonnes)												4,480
$s=2$	7	Washing line											$l=2$	
		Withdrawals (in tonnes)												4,480
		Routing											$h=1$	

	1 2	Withdra wals (in tonnes)							3,9 55			8,4 24	12, 379
		Washin g line							<i>l=3</i>			<i>l=3</i>	
		Routing							<i>h=1</i>			<i>h=1</i>	
$s = 3$	1 3	Withdra wals (in tonnes)		2,0 96					6,6 05				8,7 02
		Washin g line		<i>l=3</i>					<i>l=3</i>				
		Routing		<i>h=2</i>					<i>h=2</i>				
	1 5	Withdra wals (in tonnes)											7,4 09
		Washin g line											
		Routing							<i>h=2</i>				
$s = 4$	1 6	Withdra wals (in tonnes)											12, 161
		Washin g line											
		Routing							<i>h=2</i>			<i>h=1</i>	

		Withdra wals (in tonnes)						2,0 91		2,0 10					8,2 36
	2 1	Washin g line						$l=3$		$l=3$					
		Routing						$h=2$		$h=1$					
		Withdra wals (in tonnes)							5,8 07						5,8 07
	2 2	Washin g line							$l=5$						
		Routing						$h=1$							
$s=5$		Withdra wals (in tonnes)	2,8 74	1,4 89	4,3 86	4,3 86	1,4 28		1,4 28	2,9 17	2,8 56	2,2 35	3,8 32		27, 832
	2 3	Washin g line	$l=5$	$l=5$	$l=5$	$l=5$	$l=5$		$l=5$	$l=4$	$l=4$	$l=4$	$l=4$		
		Routing	$h=1$	$h=1$	$h=1$	$h=1$	$h=1$		$h=1$	$h=1$	$h=1$	$h=1$	$h=1$		
$s=6$		Withdra wals (in tonnes)		1,8 43		5,4 29	1,7 67	5,8 07		3,6 11	3,5 35		4,7 43		26, 735
	2 4	Washin g line		$l=4$		$l=4$	$l=4$	$l=4$		$l=5$	$l=5$		$l=5$		
		Routing		$h=1$		$h=1$	$h=1$	$h=1$		$h=1$	$h=1$		$h=1$		

	26	Withdrawals (in tonnes)			5,429										5,429
		Washing line			$l=4$										
		Routing			$h=1$										
	27	Withdrawals (in tonnes)	3,557						1,767						5,324
		Washing line	$l=4$						$l=4$						
		Routing	$h=1$						$h=1$						
	28	Withdrawals (in tonnes)									2,766				2,766
		Washing line									$l=5$				
		Routing									$h=1$				
		Total (in tonnes)	11,004	6,133	8,588	27,894	27,894	8,760	29,635	8,567	17,116	18,091	5,792	14,672	12,819

* the starting and ending periods of feeding Supply Station are derived from the starting and ending periods of feeding the Delivery Station by subtracting transportation duration θ

In the solution obtained, different blends of SOs and routings are used to produce the same amount of the same internal MO (elementary orders $k_p=1$ for POs $p=3, 4, 5$ and 6). This substantially influences the production costs, totalling 2,498,800 in this scenario. When SOs are highly available, the optimal solution is obtained in 10 seconds with a production cost of 2,135,21, allowing a significant gain of about 14.6% by reducing the consumption of a non-renewable resource through the systematic use of the first routing, illustrating the impact of the SOs scarcity (in volume and composition); in this case, an MO is always produced with the same blend of SOs.

4.3 The use of the penalty costs

The cost of the penalty-free solution reflects the best performance of the second SC link (MO production). Increasing the penalty cost (from 0) asymptotically reduces the deviation of the internal MO composition from its target values, with two consequences: *i*) an increase in the effectiveness of the fourth link in the supply chain (chemical plants, whose launch costs incurred by a batch change decrease), *ii*) an increase in the production cost of the MOs, due to the scarcity of certain SOs which forces the use of higher quantities of SOs and may require additional routing changes (table 8). In this study, the gradual increase of the penalty improves the quality of the MOs by generating a decrease of up to 60% in the deviance from target values $\sigma_c = 10,000$, compared to a zero penalty. However, this reduction in the deviance from target values is only possible thanks to an increase in the quantity of SOs withdrawn of up to 4.5% for $\sigma_c = 10,000$, which is equivalent to an increase in the production cost of up to 20%, which is only acceptable if it leads to a greater saving in downstream production costs. This example highlights a trade-off between the chemistry's requirements (a stable internal MO in terms of chemical composition) and the production efficiency.

Table 8: Trade-off between efficiency and effectiveness

Penalty σ	0	10	50	100	500	1000	5000	10000
Objective function (10^3)	2,660	2,585	2,918	3,071	4,213	5,139	13,249	23,708
Deviance (m^3)	5,553	5,069	5,055	4,404	2,308	2,040	2,027	2,058
Deviance %	6.53%	5.96%	5.95%	5.18%	2.72%	2.40%	2.39%	2.42%

Number of variables	6,226	14,524			5,961	18,931		
Number of constraints	562	27,030			563	35,551		
Resolution time	59.6"	300"	900"	7,235"	170"	300"	900"	11,257"
Objectif function	-	29,31,930	24,85,580	24,85,580	-	28,59,210	25,39,290	24,98,800
Optimum loss Time (5' or 15') vs Gap 5%	-	17.96%	0.00%	-	-	14.42%	1.62%	-

	Scenario C				Scenario D			
	Transfer	Production			Transfer	Production		
	Gap 5%	Time 5'	Time 15'	Gap 5%	Gap 5%	Time 5'	Time 15'	Gap 5%
Number of variables	6,314	17,247			5,855	21,555		
Number of constraints	563	32,335			564	40,717		
Resolution time	89.8"	300"	900"	5,683"	375"	300"	900"	4.691"
Objectif function	-	27,68,480	25,34,590	24,67,820	-	25,37,710	22,95,550	22,31,190
Optimum loss Time (5' or 15') vs	-	12.83%	2.71%	-	-	13,8%	2.88%	-

Number of variables								
Number of constraints								
Resolution time								
Objective function								
Optimum loss Time (5' or 15') vs 5%								
Gap 5%								

4.5 Benchmark

Prior to the introduction of the slurry pipeline in 2014, the OCP Group's production planning decision makers managed phosphate production and transfer to internal clients by experience or using tools that aggregate data and are not suitable for short-term planning (weekly and daily planning). Furthermore, the critical component they initially focused on in phosphate products was the phosphate content (expressed by BPL).

However, with the establishment of a new port in Jorf Lasfar (where internal clients were already located), planning production to satisfy both internal and external demand via the slurry pipeline became challenging. The problem became even more complex when specifications for other components had to be considered due to new quality regulations. The previous strategies for selecting SOs to be blended are less flexible and ignore the possibility of relaxing blending constraints at the washing plants and imposing them only at the Supply Station. Figure 8 shows that for all three components (BPL, MgO, and Cd) of the internal PO measured at the washing plant exit and Supply Station in November 2021 (a representative month), blending WSO produced at the washing plants improves overall product quality to be transferred. Even so, the compliance rate of internal PO related to component BPL (respectively component MgO) does not exceed 79% (respectively 86%). It should be added that the aggregation of daily data used to calculate these monthly statistics masks significant daily variations around these monthly averages. This violation of the QC specifications leads to costly adjustments in the phosphoric acid lines. Finally, benchmarking the results produced by our

model (which always provides solutions that respect all the constraints in all the tests carried out) with the company's results was not possible because the company lacks data tracking the exact quantities of SOs mixed in each order (it only has daily data on the SOs quantity withdrawn from stocks, but not on the SOs quantities transferred in each PO).

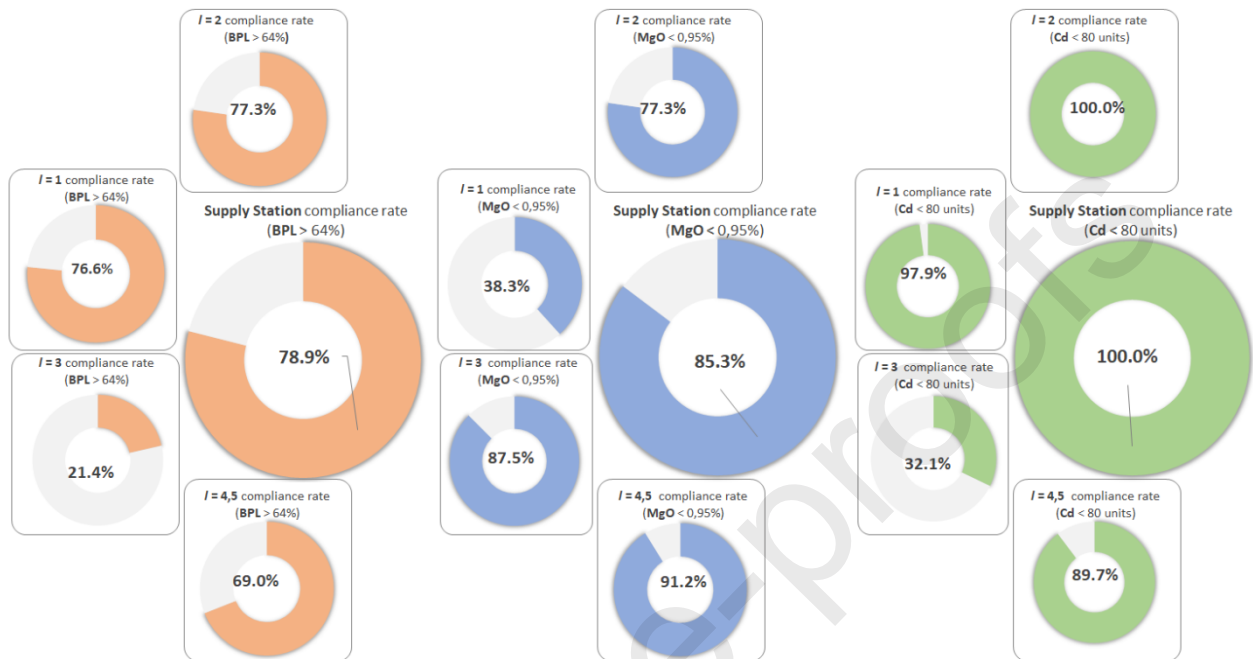


Figure 8: Compliance rate of internal orders at the exit of the five washing lines and Supply Station according to three components in November 2021.

We tested our blending model on several instances of real data. In every case, we managed to respect the QC of both internal and external PO, contrary to what existing statistics show. Under these circumstances, it can be said that comparing our approach with the existing one (as explained above) becomes irrelevant, as QC is systematically respected with our approach and not so often with the existing one.

Current practice dictates a pre-determined routing for the treatment of each SO based on its extraction location, a method that has been shown to be sub-optimal by our tests, which show that the consumption of energy and chemicals can be reduced by half if the routings are not pre-determined.

5. Conclusion

The first point to emphasise is that this approach is consistent with a sustainable development approach, as it allows demand to be met with much lower consumption of minerals and chemical products to produce the same final output and with less water used by the pipe for transport due to the maximisation of export MOs to deliver. We would now like to highlight the scientific contribution of the models that have been established to respond to the complex problem posed. We will then look at a number of possible ways to improve performance, before discussing the problems associated with implementing this innovative approach.

An in-depth analysis of the scientific literature leads us to believe that our modelling approach is original for several reasons.

- To the best of our knowledge, the problem of ore transfer by pipeline is only addressed in the scientific literature from the perspective of a downstream decoupling stock and not in a pull-flow context of diversified products such as the one studied. Our pipeline transfer model for determining the size and expedition date of batches to satisfy a relatively stable continuous demand and a highly irregular discrete export demand using a linear program with binary variables is innovative and particularly efficient, as the numerical result shows. This approach can be easily adapted to other problems involving the pipeline transport of multiple products in a pull-flow context with limited storage downstream of the pipeline, which constitutes an independent scientific contribution.
- The production model (formalised by a linearised quadratic program) is also original in that it deals with a blending problem of continuous synchronised production on a given number of processors using as many inputs, in quantities linked to the production rates of these processors and the routing used and considering the tank residues at the production changeovers and the set-up times. This model, which satisfies an order book that respects its production constraints, can be used regardless the production transport (train, etc.), since it depends only on the pipeline transport problem, by considering some specific constraints in the definition of the order book. This modelling of the blending problem through a synchronised production on several parallel processors is, therefore, another independent scientific contribution. It should be added that the depletion of phosphate deposits inevitably leads, as in the case of the problem studied, to the exploitation of distant mines of medium or low-quality ores, which require enrichment in washing plants close to these mines and whose production must be blended to meet the desired compositional specifications. From this point of view, the problem we are dealing with is likely to become widespread.
- The design of a DSS based on a possible sequential resolution of the transfer and production problems, thanks to the joint definition and sharing of parameters by these two problems, also seems to us to be an interesting scientific contribution from the point of view of the decision-making coherence in a SC, through an original treatment of the reciprocal interdependence of the decisions taken by the two SCs.

Two interesting tracks may be followed to improve the SC performance.

- The first is to move upstream the decoupling point, currently located in the storage areas near the washing plants fed from the mines in a push mode. The mining extraction process includes several operations (Azzamouri et al. 2021); the last one collects extracted SO of the same composition in some areas of the mine, providing significant SOs stocks from which quantities are taken daily to be transported by conveyor to the storage areas near the washing plants. At Centre Axe, the decision taken concerns both the blending and the supply of SOs from the mines to the washing plant storage area. (Azzamouri et al. 2021; Hilali et al. 2022). In the North Axe, this last decision can be encapsulated in the production problem, resulting in a dynamic SO supply, defined in a pull mode based on the predicted MO demand. This, which requires knowledge of the composition of the mined ores, allows the horizon to be shifted and the four links of the supply chain to operate in a pull-flow.
- The modelling approach may allow a trade-off between an efficiency criterion for one link in the supply chain (link 2 for the MOs production) and an effectiveness criterion for a distant link in the supply chain (link 4 for chemical production, which varies in opposite directions). which could serve as the basis for a negotiation making it possible to go beyond local visions and improve the overall performance of the SC. The current penalty system applies the same weight to the deviation of all the components, assuming that their impact is equal, which is certainly not the case. Further analysis needs to be carried out with the

phosphoric acid plant engineers to find a better weighting system and assess the cost of restoring production to a steady state. In order to improve the SC performance, several meetings with the managers in charge of the MO production and the managers in charge of the phosphoric acid production will be necessary.

Finally, the problem of implementing this approach must be addressed. The tests, carried out on field data, from the perspective of a DSS proof of concept, undeniably demonstrate the interest of the proposed modelling and the gains in effectiveness and efficiency achieved. In addition, the DSS is used daily to determine the order and quantities of batches to be produced and transferred, ensuring a robustness that is difficult to achieve in this context of production-to-order involving flows (Ivanov et al., 2016). The transition from a POC to a DSS presents several technical and organisational problems that may take some time to resolve.

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Appendix A. Description of the Transfer Model

All quantities (stocks, flows, etc.) are expressed in m^3 . The unit of time is a quarter of an hour. The time tracking is defined at the pipe exit (pipe delivery station). Rates are in m^3 per unit of time. Duration is defined as a number of time units.

Indexes

- t Index of time period ($t = 1, \dots, T$); t' can replace t ($t' = 1, \dots, T$).
- p Index of a Transfer Order (TO) ($p = 1, \dots, P$) whose production is carried out in mono- or bi-production. The internal batch is only considered in the transfer model, but, in the case of bi-production, some properties dealing with the co-produced export batch of TO p are introduced in the TO characterisation enforce the respect of some sequencing constraint in the export production.
- k_p Index of an Elementary Transfer Order (ETO) of the TO p ($k_p = 1, \dots, K_p$). These ETOs are exclusive if the TO p is produced in mono-production. In bi-production, k_p plays the role of the production rank of this ETO in the production of TO p .
- r Export rank index ($r = 1, \dots, R$) of the TO p of an export MO co-produced with an internal MO in bi-production mode ($\pi_p = 2$), which cannot be produced until TOs of lower rank are produced.
- n Index of a predicate ($n = 1, \dots, N$).

Parameters

- π_p $\pi_p = 1$ if the TO p is produced with the mono-production mode, $\pi_p = 2$ if the TO p is produced with the bi-production mode, and $\pi_p = 3$ if the TO p is a dummy and deals with a pipe maintenance intervention.
- E_p Execution rank of an alternative export TO p ($\pi_p = 2$); several export TOs may have the same rank.
- Ω Batch transport duration in the pipe ($\Omega = 26 \times 4 = 104$ quarter hours).
- ρ^p Pipe transfer rate.

- ρ^M Production rate of internal Merchantable Ore (MO) by the washing plants working with the **Mono-production** mode.
- ρ^B Production rate of the internal MO by the washing plants in the **Bi-production** mode (implicitly, the lines assignments in the bi-production mode are always the same in the considered horizon).
- O^D Initial level of the Internal MO in its dedicated tank in the pipe **Delivery** station.
- U^D Capacity of the tank dedicated to the internal MO in the pipe **Delivery** station.
- O^S Initial level of the Internal MO in its dedicated tank in the pipe **Supply** station.
- U^S Capacity of the tank dedicated to the internal MO in the pipe **Supply** station.
- Q_t Demand of the internal MO at the beginning of period t .
- δ_{k_p} Duration of the transportation slot of the ETO k_p (volume is therefore $\delta_{k_p} \cdot \rho^P$).
- V_{k_p} Duration of the production of the ETO k_p .
- ε_{k_p} Duration of the filling slot of the ETO k_p .
- o_{k_p} Earliest period of arrival of the ETO k_p at the delivery station.
- M_{k_p} Latest period of arrival of the ETO k_p at the delivery station.

Variables

- $x_{pk_p t}$ *Binary decision variable.* Equals 1 if the ETO k_p of TO p arrives at the pipe delivery station at the beginning of period t ; 0 otherwise. This variable existence is restrained to the earliest/latest period of arrival: $o_{k_p} \leq t \leq M_{k_p}$.

g_p *Binary auxiliary variable.* Equals 1 if the TO p produced with the bi-production mode ($p|\pi_p = 2$) is retained; 0 otherwise.

h_p *Binary auxiliary variable.* Equals 1 if *all* the ETOs of the TO p produced with the bi-production mode ($p|\pi_p = 2$) are retained; 0 otherwise.

a_t^D *Auxiliary variable.* Quantity of Internal MO arriving at the pipe **D**elivery station during the period t .

a_t^S *Auxiliary variable.* Quantity of Internal MO arriving at the pipe **S**upply station during the period t .

I_t^D *Auxiliary variable.* Inventory level in the tank of the **D**elivery station dedicated to the internal MO, at the end of period t .

I_t^S *Auxiliary variable.* Inventory level in the tank of the **S**upply station dedicated to the internal MO, at the end of period t .

Predicates

A mathematical program is made of a set of variables and a set of constraints made of a linear or non-linear combination of these variables, one of them being an objective function to optimise. Constraints usually restrain the variables' validity domain, each defined for a set of variables through a universal quantifier. The variables' validity domain may be narrowed by using an Algebraic Modelling Language (AML, available in some software like Xpress (used here), GAMS, *etc.*; Fourer, 2013), which rests on the separation of a generic description of the model, and data to use; after what, an instance of the model combining the generic model and data, which can be submitted to a solver. AML allows the usage of predicates to drive the creation of an instance of the problem. A predicate is a logical statement that returns either a value of "True" or "False" based on the parameter values used in the statement, which binds the existence of a variable, depending on the parameters' values. Predicates can be used to restrain the number of expanded constraints in relation using a universal quantifier (as used in the models' relations) and the validity domain of some variable without using a constraint, decreasing the number of constraints in a model and avoiding unnecessary calculations in the optimisation search. The following table deals with the second kind of predicate.

Γ_1	$\Gamma_1 \equiv (t \geq o_{k_p} \wedge t \leq M_{k_p})$: variable $x_{k_p t}$ exists only for $t \in [o_{k_p}, M_{k_p}]$.
Γ_2	$\Gamma_2 \equiv (t \geq o_{k_p} + \Omega \wedge t \leq M_{k_p} + \Omega)$: variable $x_{k_p t}$ exists only for $t \in [o_{k_p} + \Omega, M_{k_p} + \Omega]$.

Model of pipeline transfer

$$\sum_{k_p, t | \Gamma_1} x_{pk_p t} \leq 1, \forall p | \pi_p = 1 \quad (\text{A.1})$$

$$\sum_{t | \Gamma_1} x_{pk_p t} \leq g_p, \forall p | \pi_p = 2, \forall k_p \quad (\text{A.2})$$

$$\sum_{p | \pi_p = 2 \wedge E_p = r} g_p \leq 1, \forall r \quad (\text{A.3})$$

$$K_p \cdot g_p - \sum_{k_p} \sum_{t | \Gamma_1} x_{pk_p t} \leq K_p \cdot (1 - h_p), \forall p | \pi_p = 2 \quad (\text{A.4})$$

$$h_p \leq g_p, \forall p | \pi_p = 2, \forall k_p \quad (\text{A.5})$$

$$\sum_{p | E_p = r \wedge \pi_p = 2} g_p \leq \sum_{p | E_p = r - 1 \wedge \pi_p = 2} h_p, \forall r | r > 1 \quad (\text{A.6})$$

$$\sum_{t | \Gamma_1 \wedge E_p = r - 1} t \cdot x_{pk_p t} \leq T \cdot (1 - g_p) + \sum_{t | \Gamma_1 \wedge E_p = r} t \cdot x_{pk_p t}, \quad (\text{A.7})$$

$$\forall p | \pi_p = 2, \forall k_p, \forall p' | \pi_{p'} = 2, \forall k_{p'}, \forall r | r > 1$$

$$\sum_{k_p, t | \Gamma_1} x_{pk_p t} = 1, \forall p | \pi_p = 3 \quad (\text{A.8})$$

$$\sum_p \sum_{k_p} \sum_{t | (t' - (\delta_{k_p} + \varepsilon_{k_p}) + 1) \leq t \leq t' \wedge \Gamma_1} x_{pk_p t'} \leq 1, \forall t' \quad (\text{A.9})$$

$$a_t^D = \rho^P \cdot \sum_{p | \pi_p < 3} \sum_{k_p} \sum_{t | (t' - (\delta_{k_p} + \varepsilon_{k_p}) + 1) \leq t \leq t' - \varepsilon_{k_p} \wedge \Gamma_1} x_{pk_p t'}, \forall t' \quad (\text{A.10})$$

$$\begin{cases} I_0^D = O^D \\ I_t^D = I_{t-1}^D + a_t^D - Q_t, \forall t | t > 0 \\ 0 < I_t^D \leq U^D \end{cases} \quad (\text{A.11})$$

$$\begin{aligned} a_t^S = & \rho^M \cdot \sum_{p | \pi_{E_p} = 1} \sum_{k_p} \sum_{t | (t' - \nu_{k_p} + 1) \leq t \leq t' \wedge \Gamma_2} x_{pk_p t'} + \\ & \rho^B \cdot \sum_{p | \pi_{E_p} = 2} \sum_{k_p} \sum_{t | (t' - \nu_{k_p} + 1) \leq t \leq t' \wedge \Gamma_2} x_{pk_p t'}, \forall t' | t' > \Omega \end{aligned} \quad (\text{A.12})$$

$$\begin{cases} I_0^S = O^S \\ I_t^S = I_{t-1}^S + a_t^S - a_{t+\Omega}^D, \forall t | t > \Omega \\ 0 < I_t^S \leq U^S \end{cases} \quad (\text{A.13})$$

$$\text{Max} \left[\rho^P \cdot \sum_{p | \pi_{E_p} = 2} \sum_{k_p, t | \Gamma_1 \wedge (t \leq T - (\delta_k + \varepsilon_k) + 1)} \delta_{k_p} \cdot x_{pk_p t} + 0.1 \times I_T^D \right] \quad (\text{A.14})$$

Justifications of the relations

In the case of *mono-production* ($\pi_p = 1$), Relation A.1 forces to retain at most one of its alternative ETO k_p of the TO p (note that no ETO k_p is retained if the TO p is not retained).

In the case of *bi-production* ($\pi_p = 2$), Relation A.2 enforces the binary variable g_p to equal 1, meaning the TO p produced with the bi-production mode is retained, if at least one of its ETO (k_p) is retained (i.e., $x_{pk_p t} = 1$).

Relation A.3 enforces at most one of the TO p , made in bi-production ($\pi_p = 2$) and producing an export batch of rank r ($E_p = r$), to be retained.

In the case of bi-production ($\pi_p = 2$), the number of alternative ETOs of the TO p is K_p . Then, the number of the remaining ETOs to be produced is $K_p \cdot g_p - \sum_{k_p} \sum_{t \in \Gamma_1} x_{pk_p t}$. If the TO p is retained ($g_p = 1$), this number, corresponding to the left side of Relation A.4, becomes 0 if all the ETOs are produced and the binary variable h_p must be 1, otherwise it must be 0.

Relation A.5 enforces h_p to be null if $g_p = 0$ for a bi-production order p ($\pi_p = 2$)

Relation A.6 enforces retaining an export of rank i only if all the ETOs of the chosen export TO of the previous rank are retained.

Relation A.7 prevents placing an export ETO of rank r before an export ETO of rank $r-1$; without that relation, this permutation that would mix different MOs would be possible.

In the case of *pipe maintenance* ($\pi_p = 3$), Relation A.8 enforces the dummy TO corresponding to a pipe maintenance to be performed.

Relation A.9 is a disjunctive constraint that prevents overlapping of batch transfer slots (see figure 2) at any period t , at arrival at the delivery station tank (and consequently anywhere in the pipe). The batch transfer slot of the ETO k_p exits the pipe during period t if it arrives between $t - (\delta_{k_p} + \varepsilon_{k_p}) + 1$ and t and respects its arrival constraints $\left[o_{k_p}, M_{k_p} \right]$.

Relation A.10 defines the quantity of internal MO arriving at the pipe delivery station during the period t , inside the transportation slot of the concerned ETO.

Relation A.11 is a flow conservation relation at the delivery station, including non-defusing and non-saturation constraints ($0 < I_t^D \leq U^D$).

Relation A.12 defines the quantity of internal MO arriving in the supply station during period t , considering the two possible production modes. Figure 2 shows that for an ETO which starts coming in the delivery station at period t , the ETO batch k_p leaving the pipe at t' is produced between periods $t' - \Omega - V_{k_p} + 1$ and $t' - \Omega$; therefore, during the period $t = t' - \Omega$, the tank of the supply station receives a share of the ETO k_p arriving at the delivery station at

a period comprised between $t - V_{k_p} + 1$ and t . This translation modifies the existing condition of the variable, $x_{pk_p t}$ which must consider the same translation.

Relation A.13 is a flow conservation relation enforcing the pulp level in the tank of the internal MO of the supply station to stay between a lower and an upper bound. The incoming flow is given by Relation A.12. The outgoing flow is a translation by Ω of the incoming flow at the delivery station, as defined by Relation A.9.

Relation A.14 is the objective function. Among all the solutions that respect the problem constraints, the ones that provide the highest export production (bi-production) are preferable, which is considered in the first part of Relation A.14. In the subset of solutions that provide the highest export production, it is preferable to retain one providing the highest level of internal MO in the delivery station tank (also yielding the highest occupation of the pipe). The second part of relation A.14 fits this target with weights that consider the tank capacity and the smallest size of internal bi-production.

Production order book. From the production point of view, the set of production orders of internal MO is defined by a set of quantities $\rho^p \cdot \sum_{p|\pi_{E_p} < 3} \sum_t \delta_{k_p} \cdot \sum_{t|\Omega \geq 0_{k_p} \wedge t + \Omega \leq M_{k_p}} x_{k_p t}$ and periods of their first arrival in the tank of the supply station $\sum_{p|\pi_{E_p} < 3} \sum_{t|\Omega \geq 0_{k_p} \wedge t \leq M_{k_p}} (t - \Omega) \cdot x_{k_p t}$.

Appendix B. Description of the Blending Model

The unit of time is a quarter of an hour. SO stocks are expressed in tons; productions (WSOs, MOs) are expressed in m^3 ; rates are in m^3 per unit of time.

Indexes

- i Index of SO ($i = 1, \dots, I$).
- c Index of a chemical component considered in the Quality Chart ($c = 1, \dots, C$).
- s Index of a SO storage area ($s = 1, \dots, S$).
- h Index of a production routing ($h = 1, \dots, H$).
- l Index of a washing plant line ($l = 1, \dots, L$).
- p Index of a Production Order (PO) retained to produce ($p = 0, \dots, P$). A PO can be for an internal or external order. It follows that in bi-production, 2 POs are produced in parallel. A PO corresponds to a TO and can be broken down into several EPOs (see below) if its size is too large (to facilitate the compliance with the quality charter), or can comprise a single EPO. By convention, $p = 0$ relates to the elementary order in progress at the beginning of the production program (implicitly, assuming that all production lines were working). Note that this index also corresponds to a production rank of POs.
- k_p Index of an Elementary Production Order (EPO) to produce ($k_p = 1, \dots, K_p$). In bi-production, a PO is rarely split into several EPOs because an TO is already split into at least 2 ETOs to make it easier to comply with the quality charter in production ($K_p = 1$). This is often the case with mono-production ($K_p > 1$). The fact that the PO index is a production rank implies that the sequence $[p, k_p]$ at the EPO level defines a list of dated events that can be used to describe the dynamic behaviour of the production system in the model (e.g., relations of stock conservation).
- j Index of an output (WSO or MO) ($j = 1, \dots, J$).

Parameters

- θ Coefficient of conversion from weight (ton) to volume (m^3).

- ρ_l For line l , minimum between its production rate and its transfer rate to the pipe supply tank.
- δ_l Reference of the SO present in the washing line l before starting production.
- δ'_l Reference of the routing used by the washing line l before starting production.
- H_l Volume of residue (m³) in the washing line l before starting production.
- Π_{sl} Boolean parameter that equals 1 if the storage area s feeds the line l , 0 otherwise.
- Ψ_i Reference of the storage area of the SO i .
- S_{i0} Available amount (ton) of SO i in its storage area, before starting production.
- α_{ci} Share of component c in the weight of SO i .
- η_{ih} Coefficient of the weight reduction of the SO i by the routing h . If the routing h cannot be used to process the SO i , then $\eta_{ih} = 0$.
- γ_{cih} Distortion coefficient of the proportion of component c in the weight of SO i when using the routing h .
- λ_p Reference of the output j concerned by the PO p ($j = \lambda_p$).
- D_{k_p} Volume of the EPO k_p to produce.
- G_p Boolean parameter that equals 1 if the PO p relates to an internal MO, 0 otherwise.
- m_{clk_p} Volume of component c produced on line l in the production of EPO k_p of a PO p related to an export MO ($G_p = 0$) and partially produced the previous day; $m_{clk_p} = 0$ otherwise.
- $\beta_{c\lambda_p}^{\text{Min}}$ Lower bound of the proportion of component c in the weight of the output $j = \lambda_p$.

- $\beta_{c\lambda_p}^{\text{Max}}$ Upper bound of the proportion of component c in the weight of the output $j = \lambda_p$.
- $\tau_{c\lambda_p}$ Target percentage of component c in the output weight. This target value is only used if the output $j = \lambda_p$ is ordered by an internal customer ($G_p = 1$).
- $B_{k_p l}$ Boolean parameter that equals 1 if the ETO k_p mobilises the washing line l , 0 otherwise.
- A_p Boolean parameter that equals 1 if the TO p mobilises all the *available* washing lines (mono-production mode), 0 otherwise (bi-production mode).
- $W_{k_p l}$ Reference of the EPO that precedes the EPO k_p in the sequence of commands executed on line l (the EPO assignments to lines being already specified in the transfer model which is based on a predetermination of lines for external or export production in the bi-production mode).
- $F_{hh'}$ Setup time when switching from routing h to routing h' .
- σ_c Penalty cost (per m^3) associated with the absolute difference between the desired target quantity of component c of the output λ_p (calculated with $\tau_{c\lambda_p}$) and that obtained by the blend producing that output λ_p .
- χ Cost of extracting one ton of SO.
- φ_h Cost of processing one ton of SO with routing h .

Variables

- ω_{ihlk_p} *Binary decision variable*: equals 1 if the SO i is treated with the routing h by the washing line l to produce the EPO k_p , and 0 otherwise.
- $z_{hh'lk_p}$ *Binary intermediate variable*: equals 1 in case of routing change on line l between ETO k_p (using routing h) and its predecessor $W_{k_p l}$ (using routing h') on line l , with a positive setup time ($F_{hh'} > 0$), and 0 otherwise.

S_{ik_p} *Intermediate variable:* stock of the SO i (ton) in the storage area Ψ_i at the end of processing the EPO k_p .

T_{k_p} *Intermediate variable:* production time of the EPO k_p in the mono-production mode.

$v_{k_p,l}$ *Intermediate variable:* volume to be produced by line l for the constitution of the EPO k_p .

μ_{ck_p} *Intermediate variable:* weight of component c in the volume produced on line l for the constitution of the EPO k_p .

Δ_{ck_p} *Intermediate variable:* absolute deviation of component c from its target value in the EPO k_p of an internal customer ($G_p = 1$).

Predicates

Γ_3	$\Gamma_3 \equiv (\mathbf{B}_{kl} \cdot \Pi_{\Psi_i,l} \cdot \eta_{ih} = 1)$ The variable ω_{ihlk_p} exists under 3 conditions: the EPO k_p is processed by line l ($\mathbf{B}_{kl} = 1$), which can mobilise the SO i ($\Pi_{\Psi_i,l} = 1$), which can be treated with the routing h ($\eta_{ih} = 1$).
Γ_4	$\Gamma_4 \equiv (p \cdot F_{hh'} \cdot \mathbf{B}_{k_p,l} \cdot \mathbf{A}_p > 0)$: the variable $z_{hh'lk_p}$ exists under 4 conditions: the PO p is not fictitious ($p > 0$); the routing change (if it exists) between the EPO k_p and the previous EPO $W_{k_p,l}$ produced on that line l does not involve a positive setup time ($F_{hh'} > 0$); the EPO k_p mobilises the washing line l ($\mathbf{B}_{k_p,l} = 1$); the EPO k_p is processed in mono-production ($\mathbf{A}_p = 1$)

Model of blending production

$$\sum_{h,i} \omega_{ihlk_p} | \Gamma_3 = 1; \forall l, p | p > 0, \forall k_p \quad (\text{B.1})$$

$$\begin{cases} \sum_{i,h,l | l < 3 \wedge \Psi_i = s \wedge \Gamma_3} \omega_{ihlk_p} \leq 1, \forall s | s \in \{1, 2\}, \forall p | p > 0, \forall k_p \\ \sum_{i,h,l | l > 3 \wedge \Psi_i = s \wedge \Gamma_3} \omega_{ihlk_p} \leq 1, \forall s | s \in \{5, 6\}, \forall p | p > 0, \forall k_p \end{cases} \quad (\text{B.2})$$

$$\left\{ \begin{array}{l} z_{hh'lk_p} \geq \left(\sum_{i|\Pi_{\Psi_l, \eta_{ih}} > 0} \omega_{ih'lk_p} + \sum_{i|\Pi_{\Psi_l, \eta_{ih}} > 0} \omega_{ihlW_{k_p}} \right) - 1 \\ z_{hh'lk_p} \leq \frac{\sum_{i|\Pi_{\Psi_l, \eta_{ih}} > 0} \omega_{ih'lk_p} + \sum_{i|\Pi_{\Psi_l, \eta_{ih}} > 0} \omega_{ihlW_{k_p}}}{2} \end{array} \right., \forall h, \forall h' | h' \neq h, \forall l, \forall p | p > 0, \forall k_p | \Gamma_4 \quad (\text{B.3})$$

$$\sum_{i|\mathbb{B}_{kl} \cdot \Pi_{\Psi_l, \eta_{ih}} > 0} \omega_{ih'lk_p} + \sum_{i|\mathbb{B}_{kl} \cdot \Pi_{\Psi_l, \eta_{ih}} > 0} \omega_{ihlW_{k_p}} \leq 1, \quad (\text{B.4})$$

$$\forall h, \forall h' | \mathbb{F}_{hh'} > 0, \forall l, \forall p | p > 0 \wedge A_p = 0, \forall k_p \mathbb{B}_{k_p l} = 1$$

$$T_{k_p} = (D_k + \sum_{h, h' | \mathbb{F}_{hh'} \cdot \mathbb{B}_{k_p l} > 0} F_{hh'} \cdot z_{hh'lk_p} \cdot \rho_l) / \sum_l \rho_l \cdot \mathbb{B}_{k_p l}, \forall p | p > 0 \wedge A_p = 1, \forall k_p \quad (\text{B.5})$$

$$\left\{ \begin{array}{l} v_{k_p l} = (T_k - \sum_{hh' | \mathbb{F}_{hh'} > 0} F_{hh'} \cdot z_{hh'lk_p}) \cdot \mathbb{B}_{k_p l} \cdot \rho_l, \forall l, p | p > 0 \wedge G_p = 1, \forall k_p \\ v_{k_p l} = D_{k_p} \cdot \rho_l / \sum_{l'} \rho_{l'} \cdot \mathbb{B}_{k_p l}, \forall l, p | p > 0 \wedge G_p = 0, \forall k_p \end{array} \right. \quad (\text{B.6})$$

$$\left\{ \begin{array}{l} S_{ik_p} = S_{i0} - \sum_{h, l, k' | k'_p \leq k_p \wedge \Pi_{\Psi_l, \eta_{ih}} > 0} v_{k'_p l} \cdot \omega_{ihlk'_p} \cdot \theta / \eta_{ih} \\ S_{ik_p} > 0 \end{array} \right., \forall i, p | p > 0, \forall k_p \quad (\text{B.7})$$

$$\mu_{clk_p} = \sum_{i, h | \Pi_{\Psi_l, \eta_{ih}} > 0} H_l \alpha_{ci} \cdot \gamma_{cih} \cdot \omega_{ihlW_{k_p}} + \sum_{i, h | \Pi_{\Psi_l, \eta_{ih}} > 0} \alpha_{ci} \cdot \gamma_{cih} \cdot (v_{kl} - H_l) \cdot \omega_{ihlk_p}, \quad \forall c, \forall l, \forall p | p > 0, \forall k_p | \mathbb{B}_{k_p l} = 1 \quad (\text{B.8})$$

$$\beta_{c\lambda_k}^{\text{Min}} \cdot \sum_l v_{k_p l} \leq \sum_{l | \mathbb{B}_{k_p l} = 1} \mu_{clk_p} \leq \beta_{c\lambda_k}^{\text{Max}} \cdot \sum_l v_{k_p l}, \forall c, p | p > 0 \wedge G_p = 1, \forall k_p \quad (\text{B.9})$$

$$\beta_{c\lambda_p}^{\text{Min}} \cdot \sum_{k_p} D_{k_p} \sum_{h, i} \omega_{ihlk_p} | \Gamma_3 \leq \sum_{k_p, l | \mathbb{B}_{k_p l} = 1} (\mu_{clk_p} + m_{clk_p}) \leq \beta_{c\lambda_p}^{\text{Max}} \cdot \sum_{k_p} D_{k_p} \sum_{h, i} \omega_{ihlk_p} | \Gamma_3, \quad \forall c, \forall p | p > 0 \wedge G_p = 0 \quad (\text{B.10})$$

$$\left\{ \begin{array}{l} \Delta_{ck_p} \geq \sum_{l | \mathbb{B}_{k_p l} = 1} \mu_{clk_p} - \tau_{c\lambda_p} \cdot \sum_{l | \mathbb{B}_{k_p l} = 1} v_{k_p l} \\ \Delta_{ck_p} \geq \tau_{c\lambda_p} \cdot \sum_{l | \mathbb{B}_{k_p l} = 1} v_{k_p l} - \sum_{l | \mathbb{B}_{k_p l} = 1} \mu_{clk_p} \end{array} \right., \forall c, \forall p | p > 0 \wedge G_p = 1, \forall k_p \quad (\text{B.11})$$

$$\omega_{i, h=2, l=5, k} \leq 1 - \sum_{h > 1} \omega_{ih, l=4, k}, \quad (\text{B.12})$$

$$\forall i, k | k > 0 \wedge (\Pi_{\Psi_l=4, \eta_{ih=2}} > 0 \vee \Pi_{\Psi_l=4, \eta_{ih=3}} > 0) \wedge \Pi_{\Psi_l=5, \eta_{ih=2}} > 0$$

$$\text{Min} \left(\sum_{p | p > 0} \sum_{i, h, l, k_p | \Gamma_3} (\chi^+ \varphi_h) \cdot v_{k_p l} \cdot \omega_{ihlk_p} \cdot (1 - \eta_{ih}) \cdot (\theta / \eta_{ih}) + \sum_{p | G_p = 1} \sum_{c, k_p} \sigma_c \cdot \Delta_{ck_p} \right) \quad (\text{B.13})$$

Justifications of the relations

Relation (B.1) enforces the production of the EPO k_p on line l (defined by the transfer program hence the predicate Γ_3) to process a unique SO i and a unique routing h .

Relations (B.2) are disjunctive constraints in feeding washing lines from storage areas due to the conveyor constraints (see Figure 1). The line $l=2$ is fed by a single storage zone among $s=1$ and $s=2$, and the lines $l=4$ and $l=5$ may be fed by a single storage zone among $s=5$ and $s=6$.

A routing change on line l ($z_{hh'l k_p} = 1$) implies that $\sum_{i|\Pi_{\psi i} \cdot \eta_{ih} > 0} \omega_{ih'l k_p} + \sum_{i|\Pi_{\psi i} \cdot \eta_{ih} > 0} \omega_{ihl W_{k_p l}} = 2$ between EPO k_p and the precedent EPO on that line (EPO of reference $W_{k_p l}$). Relations (B.3) model this implication.

Relation (B.4) describes the interdiction of routing change resulting in a positive setup time in case of bi-production.

Relation (B.5) determines the production time T_{k_p} of the EPO k_p considering possible setup time $F_{hh'}$ caused by routing change ($z_{hh'l k_p} = 1$) in some production lines in the mono-production mode (the cumulated setup time being $\sum_{h,h',l|F_{hh'} \cdot B_{hl} > 0} F_{hh'} \cdot z_{hh'l k_p} \cdot \rho_l$) (note that no positive setup time is allowed in the bi-production mode).

Relation (B.6) defines the volume of EPO k_p produced by line l in case of mono-production, (upper relation) considering a possible positive setup time; in the bi-production mode (lower relation) no positive setup time is allowed.

Relation (B.7) calculates the stock of the SO i at the end of the production of the EPO order k_p by subtracting from the initial stock, the cumulative volumes consumed of this SO i , reduced to the ton by multiplying by the coefficient θ/η_{ih} . Note that: *i*) Within the model horizon, SO feeds are not considered due to the approximative knowledge of the composition of available extracted ores in the mine; *ii*) as explained in the table of notations, EPO ranks are used instead of their delivery dates; *iii*) relation (B.7) is non-linear due to the product $v_{k_p l} \cdot \omega_{ihl k_p}$; the intermediate variable $v_{k_p l}$ can be replaced by its expression as a function of $z_{hh'l k_p}$, showing the product of two binary variables that can be easily linearised¹.

Relation (B.8) calculates the weight $\mu_{cl k_p}$ of component c in the volume $v_{k_p l}$ to be produced by line l for the constitution of the EPO k_p , considering the volume of residue H_l left in the line tank by the previous EPO $W_{k_p l}$. Note that these relations are also non-linear but are easily linearised as for relation (B.7).

¹ A compact linearisation is obtained using the following property: x , w and z being three binary variables, then equivalent to the following inequalities: $z \geq x + w - 1$ and $z \leq (x + w) / 2$.

Relation (B.9) define the Quality Charter (QC) constraints, for the production of an internal MO (in mono-or bi-production) ($G_p = 1$). For each component c , the volume

$\sum_{l|B_{k_p l}=1} \mu_{clk_p}$ of this component in EPO k_p must be bounded by a lower/upper limit.

Relation (B.10) define the QC constraints for an export production (bi-production mode; $G_p = 0$). In the transfer problem, an export TO can only be retained if it can be produced within the horizon. In the production model used in a rolling mode, an export PO can be partially produced the day before, which leads to considering the possibility of starting the production problem with a partially produced export MO. In this case, the corresponding EPOs already produced must be considered, which is considered by relation (B.10). The quantity produced of PO p is $\sum_{k_p} D_{k_p} \sum_{h,i} \omega_{ihlk_p} \Gamma_3$ if the whole PO is produced, and the QC is applied to the set of produced WSOs. In the case of an export PO p partially produced the day before, the production already performed is $\sum_{k_p, l|B_{k_p l}=1} m_{clk_p}$, where m_{clk_p} is the volume of component c get by using line l in the production of the EPO k_p ($m_{clk_p} = 0$ in the TO p is a new export one). The relation (B.10) covers all situations.

Relations (B.11) calculate the absolute differences between the component weights in the EPO k_p and the target value preferred by the customer of an internal MO ($G_p = 1$) and operate a linearization of an absolute value.

Relation (B.12) forces the respect of conditions of routings operability on certain lines. In our case, if the line $l = 4$ uses the routing $h = 2$ or $h = 3$, then the line $l = 5$ can only operate with routing $h = 1$.

The objective function (B.13) is the sum of: *i*) the cost of SO extraction and processing in the washing plant using the routing h (first term); *ii*) the cost of the deviance of the ETOs composition from the preferred target values (second term). Due to the setup consequences (see section 3.2.1), this first part of the objective function is quadratic ($v_{k_p l} \cdot \omega_{ihlk_p}$) like in relation (B.8), and the same linearisation must to be used.

Appendix C. Abbreviations List

MO	Merchantable Ore
SO	Source Ore
WSO	Washed Source Ore
QC	Quality Charter
DSS	Decision Support System

TO	Transfer Order
ETO	Elementary Transfer Order
PO	Production Order
EPO	Elementary Production Order
BPL	Bone Phosphate of Lime
MgO	Magnesium oxide
Cd	Cadmium
BT	<i>Basse Teneur</i> (Low Grade)
TBT	<i>Très Basse Teneur</i> (Very Low Grade)
MT	<i>Moyenne Teneur</i> (Average Grade)