Spectral modelling approach for crowd-rhythmic activities performed on steel-concrete composite floors

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Abstract

During the last decades, building floors have become sensitive to vibrations caused by human activities, especially when a group of individuals perform rhythmic activities in a coordinated manner. Resulting effects on floor occupants can range from a mere perception, can pass through discomfort and can even go so far to cause panic. The present paper proposes a spectral modelling approach for crowd-rhythmic activities applied on floor structures. The proposed model comprises a frequency domain model for a single person, combined with coordination factors to account for the lack of synchronization between individuals. The identification of this model was based on vibration experiments carried out on a floor specimen specifically designed for that purpose. Two particular types of rhythmic activity were investigated: jumping and skipping, and performed by up to 16 individuals, where realistic coordination stimulus was adopted. Using least-squares identification procedures, the single person load parameters were determined and predicted load models are in good agreement with their measured counterparts. Resulting parameters are dependent on the motion style of each investigated activity. Coordination factors are also derived from crowd forces and present a hyperbolic decrease by crowd size for jumping activities against an exponential decrease for skipping activities. Based on a closed-form expression using the established crowd model, a design-oriented method is proposed for a simplified evaluation of floor response due to crowd-rhythmic activities and verified against floor response measurements. Slight differences were found between measured and numerical responses due to the variabilities either encountered during experiments or resulting from the load identification process.

Keywords: building floor, human-induced vibration, rhythmic load model, group effect, floor serviceability assessment.

1 Introduction

Recent developments in construction technologies have resulted in building floors with lower weights, larger spans and smaller amount of non-structural partitions. Such floors are characterized by lower natural frequencies and damping ratios, which make them sensitive to vibrations caused by human activities. Groups of individuals performing rhythmic activities in a coordinated manner are commonly considered as the worst load case scenario among these activities [1],[2]. Excessive vibrations mainly cause discomfort to the floor occupants, as demonstrated on a multi-panel composite floor devoted to a high-rise office subjected to normal jumping [3]. This effect could even turn to panic, as experienced in a 39-storey steel building in Seoul, South-Korea [4], where the upper floors shaked vertically for 10 min due to crowd rhythmic movements in an adjacent fitness centre. When a group of people perform rhythmic activities, corresponding forces are mainly acting vertically on the floor structure. In this case, each individual changes load parameters at each impulse of the activity ("intra-subjected variability") and has a specific motion different from others ("inter-subject variability"). Consequently, a reliable load model taking into account such variabilities is a pre-requisite for the vibration serviceability assessment of floors.

Experimental investigations have been conducted in order to characterize load models for crowd-rhythmic activities. Direct measurement of such forces can be accomplished by several techniques [5]. The most straightforward one consists of placing force plates on the position of each participant in order to measure individual loads while performing rhythmic activities [6]−[8]. Despite enabling a local experimental investigation of group effects for each person besides its variation analysis during motion, this procedure is not practical for a large crowd size [9]. Moreover, the motion of each individual is restricted by the area of the force plate, such that the activity would not be practiced in a comfortable way. Alternatively, human-induced forces can be measured by setting load cells at specific locations on the floor. The total load produced by large crowds could then be measured without restricting their motion [10],[11]. It is also possible to determine force parameters from measured vibrations by means of indirect identification techniques. This might be done by measuring responses of the floor [12],[13] or accelerations of the human body at specific locations [14].

In the vast majority of tests reported in literature [6]−[14], the main analysed activity was "normal jumping", where an individual is launching himself in the vertical direction and returning to the ground with an impact. However, other jumping-type activities are commonly encountered in several buildings (such as fitness centres, gymnasiums, sports venues, etc.). These activities could also produce higher loads, as found for stride jumps which had greater Dynamic Load Factors (DLFs) than "normal jumping" in some cases [7]. Furthermore, the movement of individuals was usually controlled by a metronome in these tests, which does not reflect real situations where people perform rhythmic activities under various audible and visual stimuli.

Most of load models representing rhythmic activities are expressed in the time domain, characterized by sharp peaks at each harmonic. In reality, a spread of energy is observed near each harmonic of the load due to "intra-subject variability" [2]. This leads to an overestimation of the floor response when resonance occurs and the opposite in non-resonant cases [15]. In

addition, time domain load models cannot excite multiple natural modes simultaneously which corresponds to many building floors encountered in real life (some have closely-spaced modes as for multi-span or multi-panel floors [16]−[18]). Hence, the previously mentioned load models would not provide accurate results for these structures. To circumvent these limitations, the rhythmic load is considered as a near-periodic narrowband process [2]. In this case, frequency domain modelling offers an excitation frequency window that could excite multiple closely spaced modes simultaneously [15]. However, only a few proposals of such model have been made for crowd-rhythmic activities expressed in a random field approach [19],[20].

In addition to the development of load models, design guidelines dedicated to the response prediction of floors due to rhythmic activities were established. Two design guidelines are commonly used: SCI P354 [21] and AISC/CISC DG11 [22] published by UK and US steel institute respectively. However, they are primarily applied to floors loaded by a single person performing rhythmic activity. The case of rhythmic activities performed by groups of people is not explicitly studied by these guidelines, although quite frequent in many floors. Other official documents dealing with the subject such as the UK Recommendations [23] and ISO 10137 [24] provide limited information on the vibrational response of floors for crowd cases.

This paper presents a spectral modelling approach characterizing crowd-rhythmic activities on floor structures. The proposed crowd load model comprises a frequency domain load model for a single person exciting the floor, combined with coordination factors to reflect the lack of synchronization between participants. Corresponding parameters were identified from force measurements carried out on a floor specimen designed for that purpose. Two particular types of rhythmic activities were investigated: jumping and skipping. Specific coordination stimuli were used to be as close as possible to real life conditions.

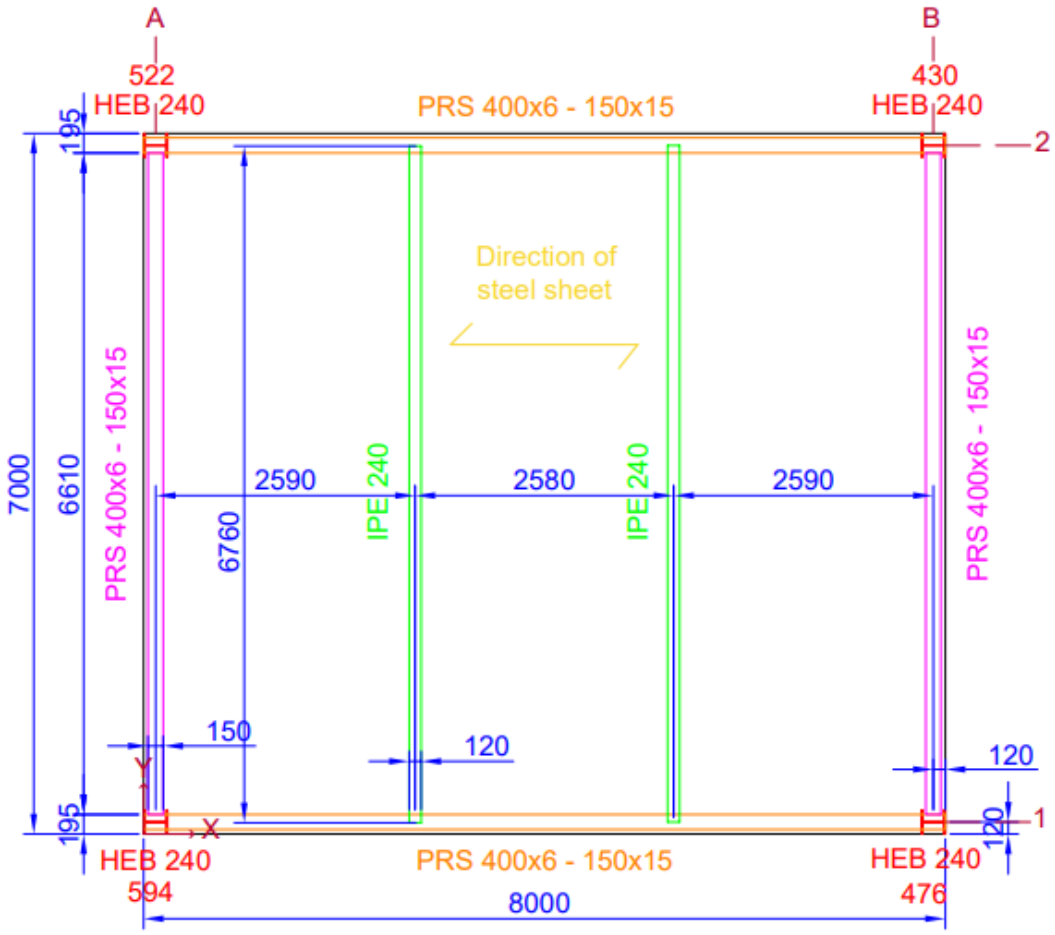
Experimental tests on the floor specimen are first described in Section 2, including Experimental Modal Analysis and vibration tests under crowd-rhythmic activities. The proposed load model is then presented for both a single person and multiple individuals in Section 3. The procedure to identify the crowd-rhythmic load model is detailed and applied to determine the single person load parameters together with coordination factors for crowds using recorded forces related to the investigated rhythmic activities. Finally, a simplified method for the prediction of floor response subjected to crowd-rhythmic activities is proposed in Section 4 based on the above-established model and verified against floor response measurements.

2 Experimental tests on a floor specimen

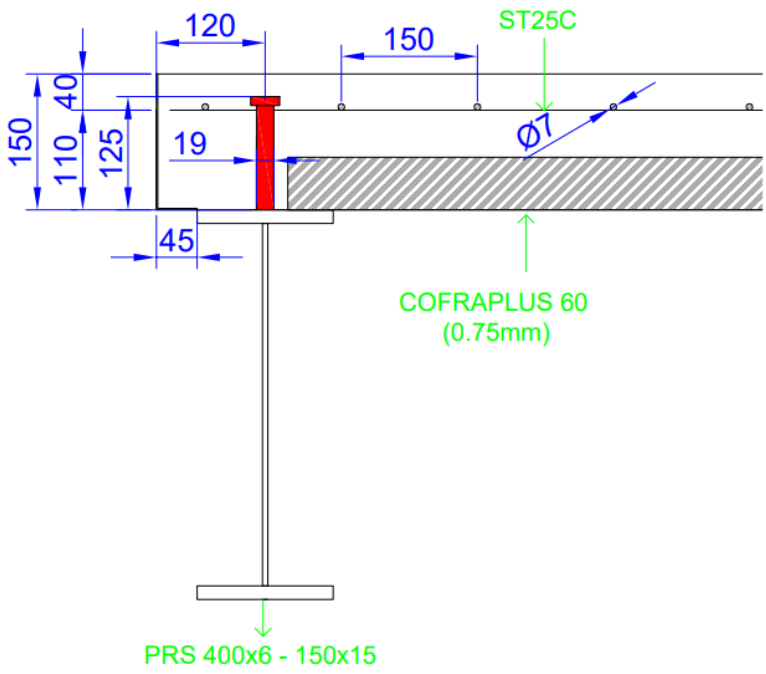
2.1 Tested structure

Vibrations due to crowd-rhythmic activities are a quite complex and random phenomenon as encountered in real life floor structures. Therefore, experimental tests were conducted on a steel-concrete composite floor specimen in order to have a first characterization of loads and responses resulting from such activities. The floor was tested at the structural mechanics laboratory of FCBA institute in Bordeaux and had an area of $8\times7m^2$. The floor characteristics were determined to be in line with current construction practice. The main floor components (slab, beams, columns) are presented in Figure 1(a), whereas reinforcement and shear stud

details are illustrated in Figure 1(b). The concrete class of the slab was C25/30. The steel grade was S275 for hot-rolled profiles and S355 for welded I-members, with a 350MPa tensile strength for shear studs (having a 207mm spacing for 7m span secondary beams and 200mm spacing for 8m span primary beams).



(a) Plan view (columns heights in red)



(b) Front view (steel sheet in grey, shear stud in red)

Figure 1: Floor specimen layout (dimensions in mm)

A preliminary dynamic design of the floor specimen was performed in order to obtain only one natural mode in the frequency range between 0 and 10Hz. Human activities have a frequency range between 1.5 and 3.5Hz [12],[21], leading to a maximum excitation frequency of 10Hz for the third harmonic. Hence, these activities would only excite the fundamental mode. The effects coming from the excitation of higher modes could then be eliminated which simplifies the floor response analysis. This condition was verified after performing a parametric study on the floor components with numerical modal analysis using an appropriate Finite Element Model of the floor [25]. It was concluded that the usage of large edge secondary beams along with rigid connections between beams results in a floor having a fundamental mode below 10Hz well separated from higher modes. Subsequently, the floor specimen was designed in accordance with Eurocode prescriptions to resist a total static load comprising a permanent load of 3.33kN/m² (self-weight of the floor) and a live load of 1kN/m² (amplified weight of 16 individuals and weight of the equipment). Figure 2 illustrates a side view of the floor specimen after construction.

Figure 2: Side view of the floor specimen

2.2 Experimental Modal Analysis

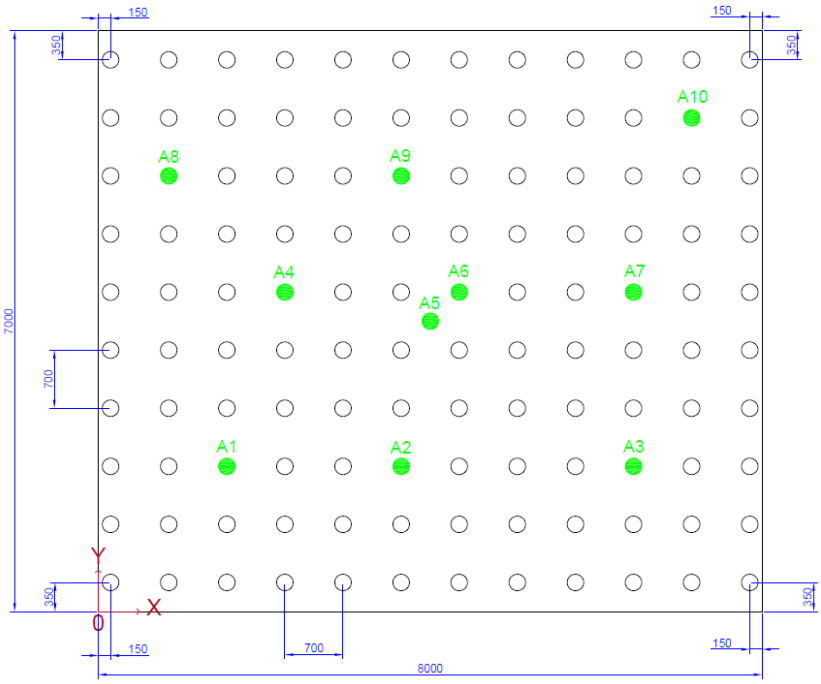
The modal parameters of the floor specimen were determined by means of Experimental Modal Analysis (EMA). Since the floor configuration is rather classical with a limited floor surface, the floor excitation was performed with an instrumented hammer.

2.2.1 Instrumentation

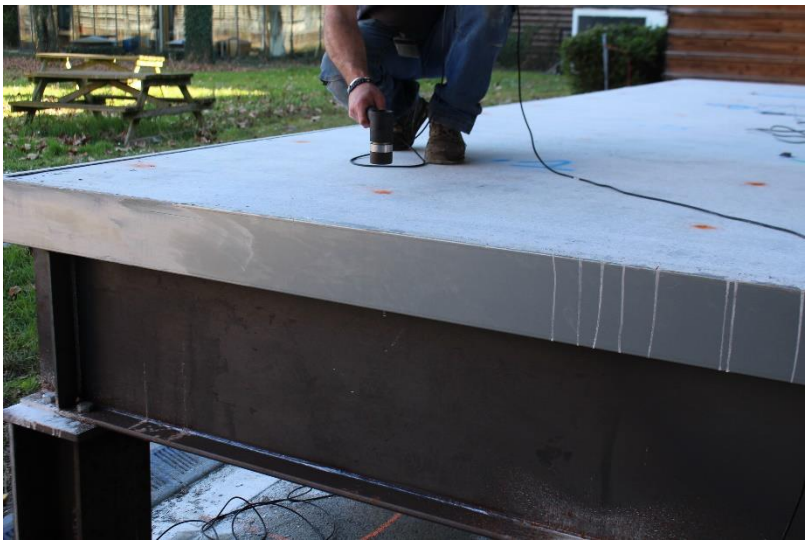
The hammer model used in the floor impacts was PCB 086D20, which could produce an impulsive load amplitude of up to 22.5kN within a wide frequency range. The response of the floor due to these impacts was measured by PCB 3711 accelerometers having a frequency limit of 1000Hz. Acquisition was made by a Bruël & Kjaer Lan XI data acquisition system.

2.2.2 Test procedure

The floor surface was initially meshed with 121 points (12×10 points, and the floor's centre). The distance between points was taken equal to 70cm in both directions, as shown in Figure 3(a). Accelerometers were placed at 10 locations over the floor (noted A1 to A10 in Figure 3(a)). At each mesh point, an impact was performed with the hammer (see Figure 3(b)) and the corresponding load and acceleration responses were measured. This lead to the calculation of a Frequency Response Function (FRF) for each response point. Successive impacts at all points of the mesh enable constructing a FRF matrix with dimensions 10×121 (10 response points, 121 excitation points) for each frequency, and the transpose of this matrix was used in modal extraction by virtue of being symmetric [26].



(a) Mesh and response points



(b) Hammer impact on the floor

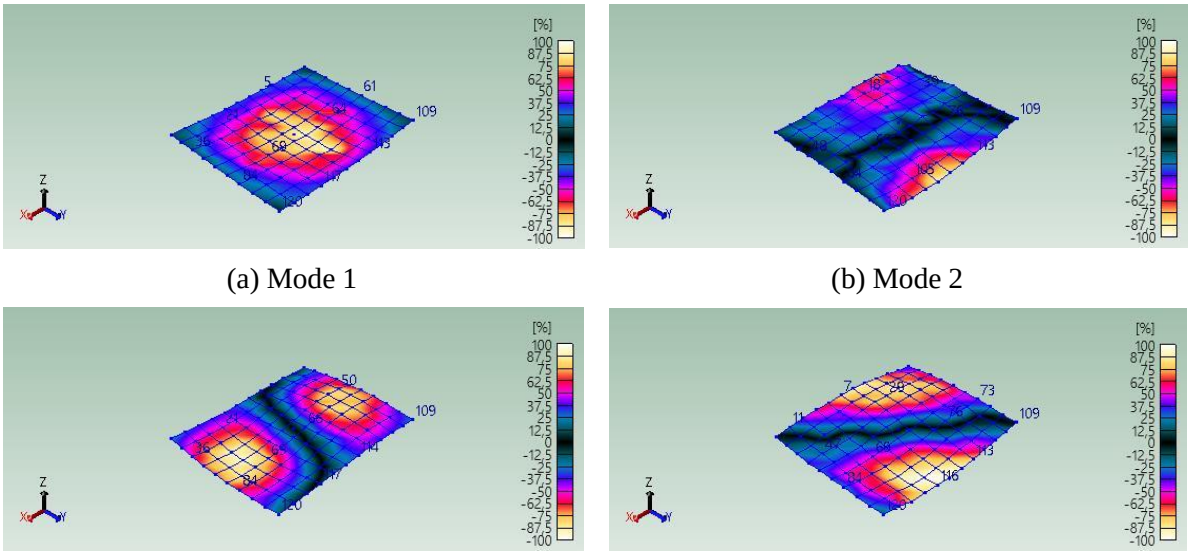
Figure 3: Experimental Modal Analysis setup (dimensions in mm)

2.2.3 Modal analysis results

The floor modal parameters were numerically extracted using BK Connect [25]. Investigated natural frequencies ranged between 0 and 20Hz, in which five natural modes were detected. Table 1 summarizes obtained natural frequencies and damping ratios for the first four modes, whereas Figure 4 illustrates corresponding modal shapes. Experimental results confirm that one natural mode (with natural frequency of 7.65Hz, clearly distinct from higher modes) is obtained in the frequency range between 0 and 10Hz (corresponding to human excitation [12],[21]) as expected in the design.

Mode	Natural frequency (Hz)	Damping ratio (%)
1	7.65	2.92
2	12.33	1.90
3	14.98	3.42
4	15.74	3.95

Table 1: Experimental floor modal parameters



(a) Mode 1 (b) Mode 2

(c) Mode 3 (d) Mode 4

Figure 4: Experimental floor modal shapes

2.3 Crowd-rhythmic vibration tests

A second test campaign was performed to investigate the vibrational behaviour of the floor specimen subjected to various crowd-rhythmic activities. Vertical human-induced rhythmic loads and displacement responses of the floor were measured while performing rhythmic activities by the participants. The scope of the study corresponds to a low density of participants (0.3 person/m²) leading to a restricted crowd size due to the limited floor surface.

2.3.1 Instrumentation

Load cells were selected to measure vertical rhythmic loads produced by individuals on the floor. Four load cells were placed at the bottom of each floor column, and the sum of the measured loads during rhythmic activities lead to the total group-induced load applied to the floor. Instrumentys LC-C110C load cells with a capacity of 10t were adopted for that purpose. Calibration was then made to have balanced measurements over the floor (approximately 25% of the total load measured by each load cell). The vertical displacement of the floor was also measured during rhythmic activities. One response point was assumed sufficient to capture the vibrational behaviour of such a classical structure. This was done by setting a Linear Variable Differential Transformer (LVDT) device (ACT1000C transducer) having a measurement range of ±25mm below the floor.

The load cells and the LVDT transducer were connected to a Bruël & Kjaer Lan XI data acquisition system for signal conditioning and digitalization. The sampling frequency was fixed at 128Hz for both devices.

2.3.2 Experimental protocol

Two particular types of rhythmic activities were investigated: jumping and skipping. Investigated jumping-type activities were "jumping jack" and "quick jumping", frequently performed in sport venues, fitness centres, stadiums, etc. Conversely, skipping-type activity was analysed in terms of two contact modes with the ground: on feet toes and on feet soles, which usually take place in gymnasiums, sports halls and other facilities. The adopted terminology for the investigated activities (throughout the paper) is presented in Table 2.

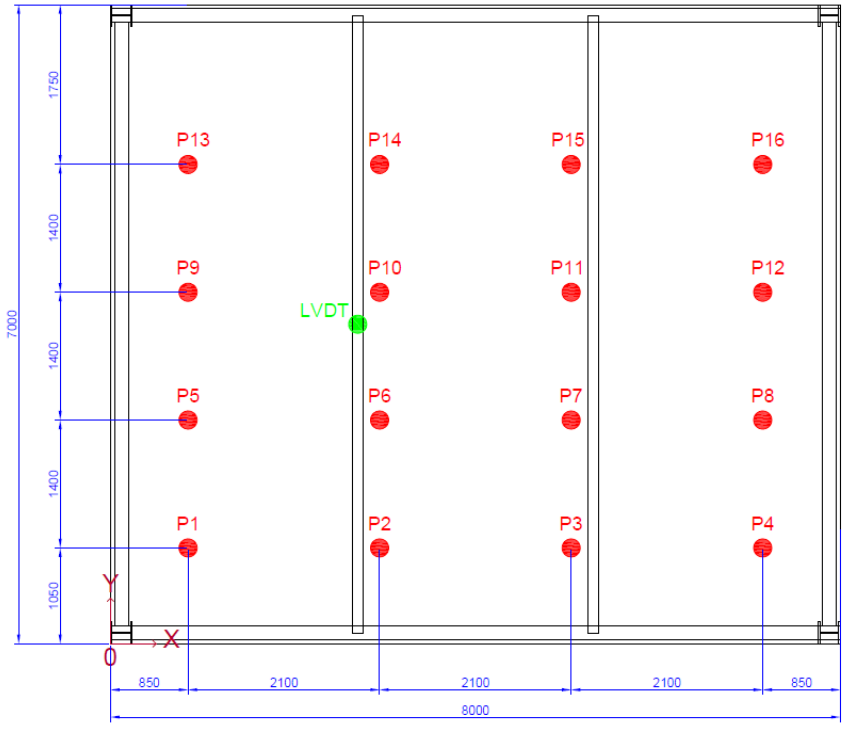
Activity	Terminology
Jumping jack	Jumping 1
Quick jumping	Jumping 2
Skipping on feet toes	Skipping 1
Skipping on feet soles	Skipping 2

Table 2: Terminology for rhythmic activities

Both audible and visual stimuli were used while performing rhythmic activities to achieve a coordination degree which is the most likely to occur in practice. Rhythmical songs were adopted as audible stimuli whilst the visual stimulus was ensured by an experienced sports coach standing in front of the participants.

The maximum crowd size allowed by the floor surface to have a moderate density (1 person/3m²) was 16 individuals. Therefore, tests were carried out with numbers of 1, 2, 4, 8 and 16 participants (illustrated in Figure 5(a)). A total of 33 participants were involved in the experiments. Their ages ranged between 19 and 60 years (mean: 37 years, standard deviation: 12 years) and their body masses varied from 58.6 to 108kg (mean: 73.5kg, standard deviation: 12.4kg). The participants were uniformly distributed over the floor according to positions shown in Figure 5(b), and adopted positions for each group size are listed in Table 3. Each individual participated in only one set of crowd size and had one specific position. For example, the individual who took part in the test of crowd size 1 did not participate in the tests of other crowd sizes (from 2 to 16) and was placed in position P11.

(a) 16 individuals performing “jumping jack”



(b) Positions of individuals (red circles) and LVDT (green circle)

Figure 5: Crowd-rhythmic test setup (dimensions in mm)

Crowd size	Positions
1	P11
2	P6, P11
4	P6, P7, P10, P11
8	from P5 to P12
16	from P1 to P16

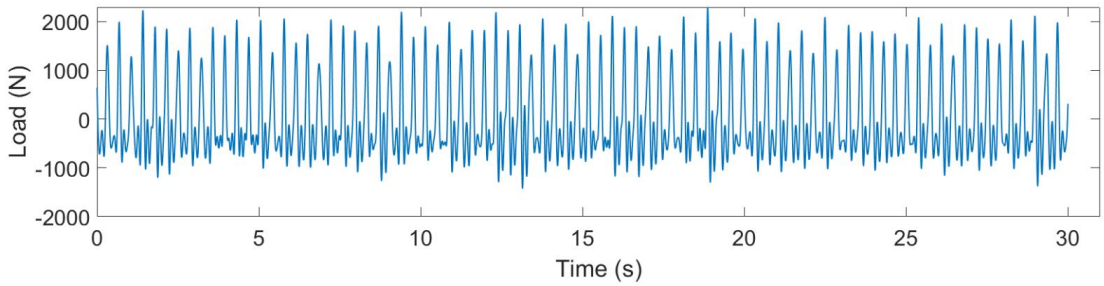
Table 3: Positions of participants by crowd size

Before each activity, a 30s preliminary test was done so that participants could warm-up and get used to the activity movements. Then, individuals performed continuous movements during 45s, given that people are likely to be tired after this duration [9]. For each investigated crowd size and rhythmic activity, the sequence was repeated three times, and the whole set was repeated two times. In total, the experimental campaign lasted about one and a half day.

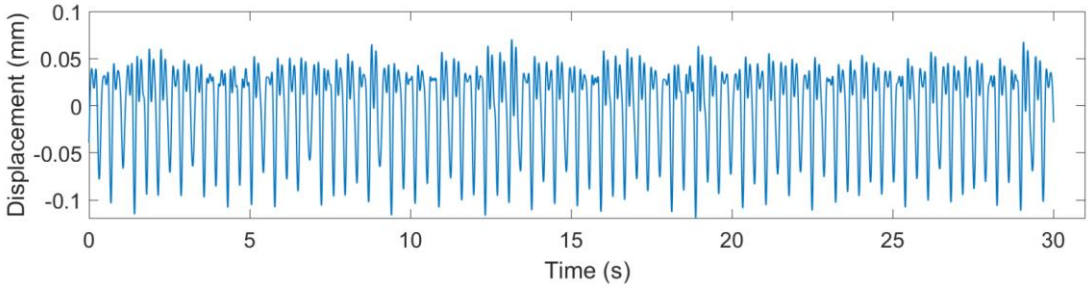
2.3.3 Test results

Vibration test results were the sum of forces recorded by the four load cells and displacements measured by the LVDT device. The total number of records was 120 forces and 120 displacements (6 measurements × 4 activities × 5 crowd sizes per parameter).

Force and displacement signals were processed using MATLAB. For each activity and crowd size, the relevant window during which participants performed rhythmic activities was determined, starting when a force amplitude reached 0.5% of the mean force and lasting 45s. All mean values (corresponding to static parameters) were also removed from test records in order to keep only the dynamical component of forces and displacements useful for further analysis. Furthermore, the signals were filtered to meet the usual frequency range of human activities (ranging between 0 and 10Hz [12]) using Fast Fourier Transform. The filtered signals were finally truncated by removing 10s at the beginning and 5s at the end in order to conserve the stationary response only and to match with the slot where all individuals were moving according to the protocol. An example of pre-processed force and displacement records corresponding to a single person performing “skipping 2” is shown in Figure 6.



(a) Load signal



(b) Displacement signal

Figure 6: Example of pre-processed signals for a single person performing "skipping 2"

3 Spectral load model for crowd-rhythmic activities

3.1 Proposed rhythmic load model

A frequency domain load model for crowd-rhythmic activities is proposed to account for the randomness of the motion during time due to intra and inter-subject variabilities. This model comprises a spectral model for a single person, combined with coordination factors for crowds.

3.1.1 Single person load model

Xiong and Chen [19] established a load model for a single person jumping in terms of a Power Spectral Density (PSD) function expressed for each harmonic *i* by:

$$S_{\mathrm{p,i}}(f) = (mg)^2 \left(\frac{\rho S_{\mathrm{i}}}{i f_{\mathrm{p}}}\right)\left[p_5 \exp\left(-\left[\left(\frac{f/if_{\mathrm{p}}-1}{p_6}\right)\right]^2\right) + p_7 \exp\left(-\left[\left(\frac{f/if_{\mathrm{p}}-1}{p_8}\right)\right]^2\right)\right] \quad (1)$$

where m is the body mass of the individual, g the gravity acceleration (9.81m/s²), f_{p} the excitation frequency, S_i the load energy of the i$^{\mathrm{th}}$ harmonic, ρ an energy compensation factor and p_5, p_6, p_7, p_8 are dimensionless model coefficients equal to 2.804, 0.079, 29.27 and 0.012, respectively [19].

This model is characterized by an exponential function enabling a gradual decrease in load amplitudes in the vicinity of each harmonic peak, which accurately represents the frequency content of the rhythmic load. Based on this observation, the proposed model simplifies Eq. (1) by assuming that each harmonic can be modelled by a unique exponential function given by:

$$S_{\mathrm{p,i}}(f) = (mg)^2 \alpha_{\mathrm{i}}^2 \exp\left(-\frac{\left(f - if_{\mathrm{p}}\right)^2}{\delta_{\mathrm{i}}^2}\right) \quad (2)$$

where α_{i} the i^{th} amplitude coefficient (related to the i^{th} harmonic peak) and δ_{i} the i^{th} bandwidth coefficient (controlling the spread of energy within the i^{th} harmonic).

The total PSD load model $S_{\mathrm{p}}(f)$ is then obtained for H harmonics by:

$$S_{\mathrm{p}}(f) = \sum_{i=1}^{H} S_{\mathrm{p,i}}(f) \quad (3)$$

The PSD load model given by Eq. (2) was further simplified in order to have a limited number of parameters for identification. Indeed, three harmonics (H = 3) were considered sufficient to

cover the expected frequency range of human activities (between 0 and 10Hz [12]). The coefficient δ_i was given by $\delta_i = i \times \delta_1$, as proposed by Xiong and Chen for jumping activity [19]. The ratio between amplitude coefficients $a_i = \alpha_i / \alpha_1$ was assumed the same as for Dynamic Load Factors (DLFs) of an equivalent time domain load model expressed by:

$$P(t) = mg\left[\sum_{i=1}^{3} DLF_i sin(2\pi i f_p t + \varphi_i)\right] \quad (4)$$

where DLF_i is the Dynamic Load Factor of the i^{th} harmonic and φ_i the corresponding phase angle.

Pernica [7] carried out an experimental investigation to extract DLFs for various human activities, including running-on-the-spot and stride jumps (see Table 4). These findings represent one of the most widely used models from the literature characterizing experimental activities (stride jumps for "jumping jack", running-on-the-spot for "skipping"). In addition, since activities of the same type (jumping or skipping) produce similar impacts on the floor, they were assumed to hold the same coefficients a_i (i={1, 2, 3}).

Noting $\delta_1 = \delta$ and $\alpha_1 = \alpha$, the PSD load model could then be rewritten by:

$$S_p(f) = (mg\alpha)^2 \sum_{i=1}^{3}\left[a_i^2 \exp\left(-\frac{(f - if_p)^2}{(i\delta)^2}\right)\right] \quad (5)$$

where coefficients a_i (i={1, 2, 3}) calculated from DLFs obtained by Pernica are provided in Table 4. Parameters to be identified for each rhythmic activity are then f_p, α and δ.

Activity	DLF_1	DLF_2	DLF_3	a_1	a_2	a_3
Skipping / running-on-the spot	1.57	0.58	0.26	1	0.4	0.15
Jumping / stride jumps	1.75	1.1	0.42	1	0.6	0.25

Table 4: Load parameters DLFi [7] and ai for jumping and skipping activities

3.1.2 Crowd load model

In the frequency domain, rhythmic activities practiced by a group of N individuals are usually modelled by a crowd load model, including an Auto-Spectral Density (ASD) model for a single person and a coherence function to consider the interaction between multiple persons [19]. The group effects noticed during movement should be included in the crowd model. In fact, individuals have different load amplitudes and frequencies while performing rhythmic activities due to "inter-subject variability". This feature is considered in the coherence function by introducing coordination factors quantifying the synchronization degree between participants. Coordination factors also indirectly include the interaction effects between the floor and the participants (termed human-structure interaction) where the motion of individuals is altered by the vibration of the floor (especially when the structure is perceptibly moving).

The crowd load model is then expressed by a PSD force matrix $[S_{p,N}(f)]$ ($N \times N$) defined by:

$$\left[S_{\mathrm{p,N}}(f)\right]_{\mathrm{k,l}} = \begin{cases} S_{\mathrm{p,N,k}}(f), & \mathrm{k=l} \\ \sqrt{S_{\mathrm{p,N,k}}(f) S_{\mathrm{p,N,l}}(f)}, & \mathrm{k} \neq \mathrm{l} \end{cases} \tag{6}$$

where $S_{\mathrm{p,N,k}}(f)$ and $S_{\mathrm{p,N,l}}(f)$ are the ASD models for the individual k and l present in the group of N individuals, respectively. The ASD function $S_{\mathrm{p,N,k}}(f)$ is given by:

$$S_{\mathrm{p,N,k}}(f) = \left[C(N)\right]^2 S_{\mathrm{p,k}}(f) \tag{7}$$

Here, $S_{\mathrm{p,k}}(f)$ is the PSD load model for the k^{th} individual with body mass m_{k} (obtained by Eq. (5)) and $C(N)$ the coordination factor related to crowd size N.

3.2 Rhythmic load model identification

The crowd-rhythmic load model presented above is identified based on force measurement recorded for each investigated rhythmic activity. To accomplish that, the procedure for the determination of load parameters of the single person model is first highlighted, followed by the calculation of coordination factors for the case of multiple individuals.

3.2.1 PSD load model identification

Consider $S_{\mathrm{p,exp}}(f)$ as the experimental PSD load corresponding to a rhythmic activity performed by a single person and $S_{\mathrm{p,th}}(f, \theta)$ the θ-dependent analytical PSD load model for the same activity. The objective is to determine the optimal parameters of vector θ (noted θ_{opt}), in order to have the closest analytical PSD model to its experimental counterpart.

The identification of PSD load parameters was performed by the least-squares method, more straightforward for such nonlinear optimization problems [27]. Unlike existing methods based on a local determination of load parameters (using Fourier spectrum [7],[10] or harmonic load energy [6],[11]), this technique enables the usage of the entire signal record. The intra-subject variability effects during the rhythmic action could then be taken into account. Hence, the optimal vector θ_{opt} was obtained by minimizing an objective function as follows [27]:

$$\theta_{\mathrm{opt}} = \arg\min_{\theta} \left[\left\| S_{\mathrm{p,th}}(f, \theta) - S_{\mathrm{p,exp}}(f) \right\|_2^2 \right] \tag{8}$$

A practical procedure was established to identify load model parameters collected in the vector $\theta = \{f_{\mathrm{p}}, \alpha, \delta\}^{\mathrm{T}}$ [25]. First of all, an initial set of parameters θ_0 was selected, and the analytical PSD load model vector $\{S_{\mathrm{p,th}}(\theta)\}$ was calculated using Eq. (5) for all analysed frequency values. The experimental PSD load vector $\{S_{\mathrm{p,exp}}\}$ was obtained from the time domain force signal using the periodogram method implemented in MATLAB. The optimal parameter vector θ_{opt} was then calculated by the least-squares method (see Eq. (8)) using *lsqnonlin* solver (with Levenberg-Marquardt algorithm) available in the Optimization toolbox of MATLAB.

The above procedure was applied to each force signal corresponding to the investigated rhythmic activity. The variability of all resulting optimal parameters f_{p}, α and δ was then characterized by their arithmetic mean values and standard deviations.

3.2.2 Crowd size effect

For each activity and crowd size, a total number of S records (called windows) was measured on the floor. The Root Mean Square (RMS) force was calculated for each window as a

representative load parameter. This parameter covers all the frequency content of the measured load and allows to have more accurate evaluation of the amplitude (omitting the effect of parasite peaks).

In order to eliminate the effect of variable body mass on the crowd load, RMS forces were normalized by the following expression:

$$F_{\mathrm{rms,s}} = \frac{N\bar{m}}{\sum_{k=1}^{N} m_{\mathrm{k}}} \hat{F}_{\mathrm{rms,s}} \tag{9}$$

where $F_{\mathrm{rms,s}}$ is the normalized RMS force of the s^{th} activity window, $\hat{F}_{\mathrm{rms,s}}$ the initial RMS force, N the crowd size, m_{k} the body mass of the k^{th} individual and $\bar{m}$ a nominal body mass for all individuals (usually taken as 75kg [21]). The statistical variability of the resulting S RMS forces was then evaluated by their mean values and standard deviations.

The variation of the mean RMS force $\bar{F}_{\mathrm{rms}}(N)$ against crowd size N would exhibit a growth with a decreasing rate due to "inter-subject variability". This variation was fitted by selecting an appropriate function that best describes this tendency, noted $F_{\mathrm{rms}}(N, \lambda)$ (λ being the vector of function parameters). Corresponding optimal parameters λ_{opt} were then determined by means of least-squares curve fitting using *lsqcurvefit* function available in the Optimization toolbox of MATLAB.

This lead to an equivalent RMS force describing a single person within a group of N individuals $F_{\mathrm{rms,eq}}(N)$ calculated by:

$$F_{\mathrm{rms,eq}}(N) = \frac{F_{\mathrm{rms}}(N, \lambda_{\mathrm{opt}})}{N} \tag{10}$$

where $F_{\mathrm{rms}}(N, \lambda_{\mathrm{opt}})$ is the best-fit function for measured crowd RMS forces.

The coordination factor for a group of N individuals $C(N)$ was then deduced by:

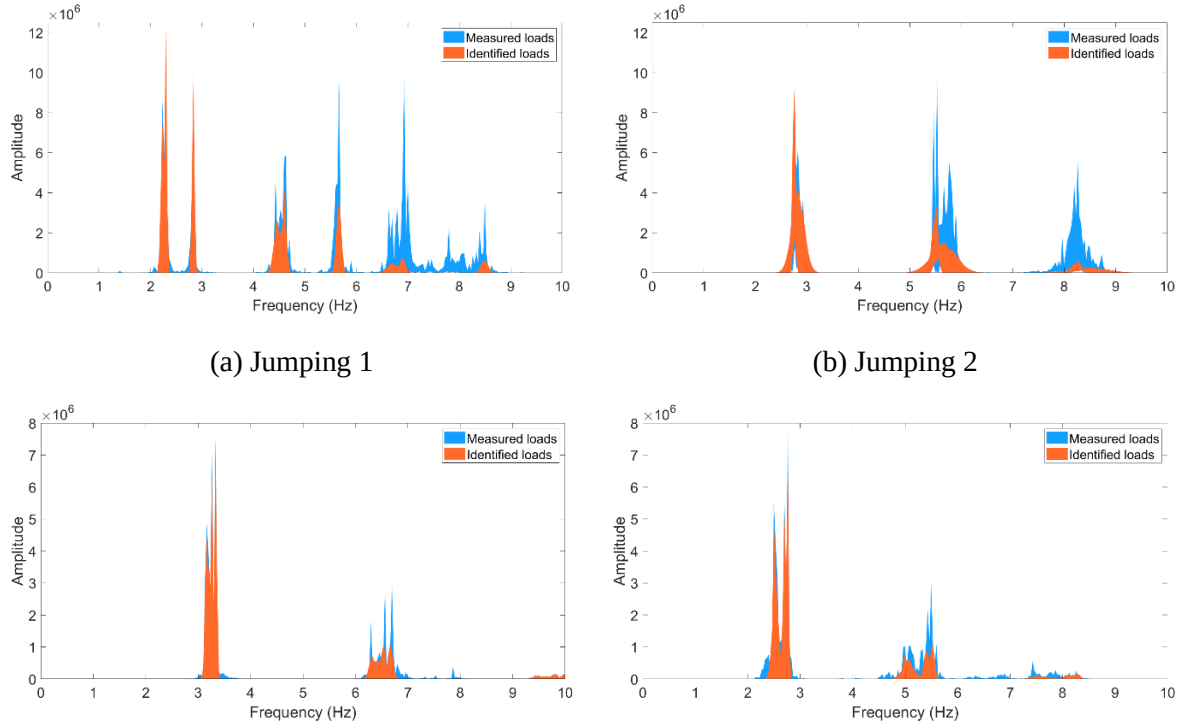
$$C(N) = \frac{F_{\mathrm{rms,eq}}(N)}{F_{\mathrm{rms}}(1)} \tag{11}$$

where $F_{\mathrm{rms}}(1)$ is the RMS force corresponding to the case of a single person on the floor and $F_{\mathrm{rms,eq}}(N)$ the equivalent RMS force for a single person within the group of N individuals.

3.3 Results

3.3.1 Identified PSD load parameters

For each activity, the PSD load parameters for a single person were identified using the method detailed in Section 3.2.1. Both measured and identified PSD loads are represented by their amplitude envelopes at each frequency (between minimum and maximum values of six activity windows). PSD results are plotted in Figure 7 for the four analysed activities. In general, a good agreement is found between measured and predicted PSD loads for the investigated activities especially for the first harmonic of the load.



(a) Jumping 1 (b) Jumping 2

(c) Skipping 1 (d) Skipping 2

Figure 7: Measured and identified PSD loads for the investigated activities

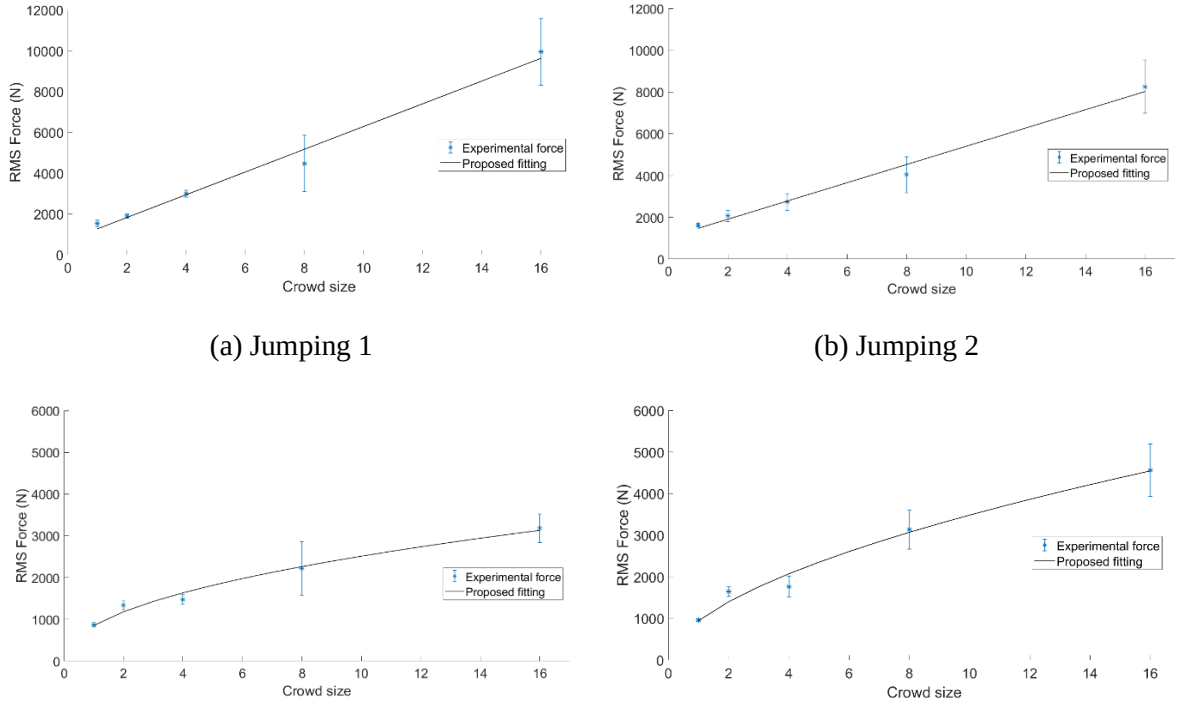
The variability between activity windows allows to determine a variation range (mean ± standard deviation) for each load parameter. Table 5 summarizes the mean and standard deviation of identified load parameters f_p, α and δ obtained from the six windows of each activity. Different sets of identified parameters are found for the investigated activities since each one is characterized by a specific style of motion. RMS forces were calculated using the corresponding mean load parameters. Obtained results by increasing order were 669N for "skipping 1", 693N for "skipping 2", 986N for "jumping 1" and 1308N for "jumping 2".

Activity	f_p (Hz)	α	δ (Hz)
Jumping 1	2.36 ± 0.21	4.64 ± 0.70	0.035 ± 0.01
Jumping 2	2.81 ± 0.05	3.64 ± 0.81	0.100 ± 0.05
Skipping 1	3.26 ± 0.06	3.80 ± 0.40	0.033 ± 0.01
Skipping 2	2.62 ± 0.10	3.53 ± 0.49	0.041 ± 0.02

Table 5: Identified PSD load parameters for the investigated rhythmic activities

3.3.2 Coordination factors

Crowd size effect was analysed for 2, 4, 8 and 16 individuals. For each activity and crowd size, crowd RMS forces were calculated using the procedure described in Section 3.2.2. The mean RMS forces are displayed as a function of the number of participants in Figure 8 for the four investigated activities. This trend is best described by a linear function for jumping activities and a power function for skipping activities. Regression has a high quality for all activities (with $R^2 \geq 0.98$).



(a) Jumping 1 (b) Jumping 2

(c) Skipping 1 (d) Skipping 2

Figure 8: Experimental RMS forces against crowd size (mean values in asterisk marks, standard deviation in error bars) and proposed fitting (black lines)

Coordination factors were then deduced from the best-fit functions representing the variation of RMS forces against crowd size (see Figure 8). Table 6 summarizes resulting factors for the investigated activities assuming a maximum number of 16 individuals.

Activity	Coordination factor
Jumping 1	$C(N) = 0.44 + \frac{0.56}{N}$
Jumping 2	$C(N) = 0.30 + \frac{0.70}{N}$
Skipping 1	$C(N) = N^{-0.53}$
Skipping 2	$C(N) = N^{-0.43}$

Table 6: Coordination factor C(N) by crowd size N for up to 16 individuals

3.4 Discussion

3.4.1 PSD load model identification

It can be seen from Figure 7 that differences between identified and experimental PSDs increase by harmonic order. This could be attributed to the potential errors resulting from the identification process as well as the variabilities observed during experiments (change of position while jumping, loss of synchronization with the audible stimulus, etc.). However, the identification results are considered valid as only slight differences occur at the first harmonic of the load having the maximum load energy. Compared to jumping activities, skipping activities have reduced amplitudes at the second and third harmonics besides more concentrated

load energy at each harmonic. On the other hand, larger frequency content is observed for jumping activities, with two distinct peaks for “jumping 1” and a relatively large energy distribution for “jumping 2”.

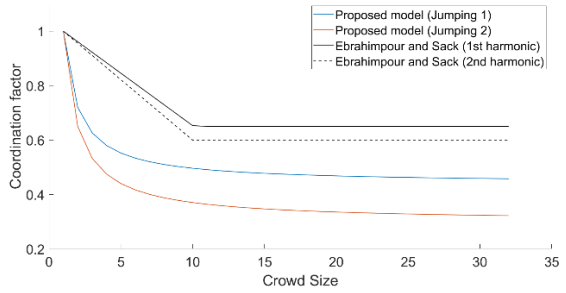
In terms of load parameters, various excitation frequencies are identified which all lie in the rhythmic activity frequency range between 1.5 and 3.5Hz [21]. Skipping activities are characterized by lower values of RMS forces and load coefficient α compared to jumping activities. This is due to the limited effort during motion required by skipping (similar to running). All activities have a narrow frequency band except “jumping 2” where a maximum value of the bandwidth coefficient δ is obtained (see Table 5), indicating a spread of energy within harmonics. The load coefficient α exhibits the highest variability among parameters due to the variable impact on the floor between pulses.

In general, the identified load models (having a limited frequency range) depend to some degree on the type of coordination stimulus adopted in the experiments. However, the existence of a variation range (mean ± standard deviation) for each load parameter enables an extended scope of rhythmic activities. The established PSD models could then be applied to similar situations encountered in practice.

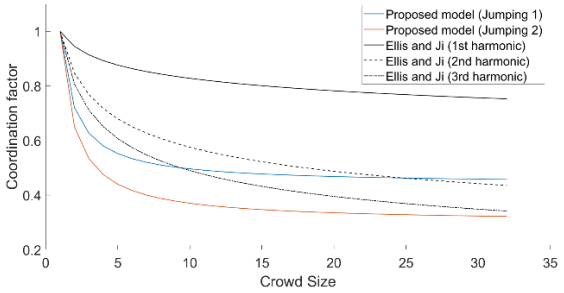
3.4.2 Crowd size effect

Figure 8 reveals that the standard deviation of RMS forces is increasing as a function of crowd size. In fact, individuals have more difficulties to coordinate their motion in larger groups compared to smaller ones. Each category of rhythmic activity has a specific trend of coordination factors against crowd size: a hyperbolic decrease for jumping activities against an exponential decrease for skipping activities (see Table 6). Skipping activities also have lower coordination factors than jumping activities due to more randomness in motion observed for the first type (similar to running) compared to the second. Moreover, the mode of leg contact to the ground has an effect on the synchronization of individuals since higher coordination factors are found for skipping on feet soles in comparison with those for skipping on feet toes.

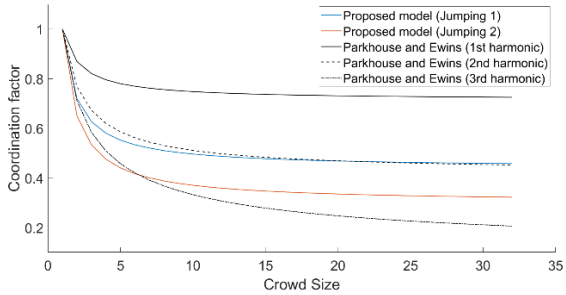
For jumping activities, coordination factors reported in the literature are divided into three categories according to their tendency: (1) linear decrease followed by a constant plateau (ISO 10137 [24], Ebrahimpour and Sack [11]), (2) continual power decrease (Faisca [8], Ellis and Ji [12]) and (3) gradual decrease for larger crowds (Parkhouse and Ewins [6], Li et al. [14]). Figure 9 illustrates the identified coordination factors with one model of each category cited above. The proposed factors belong to the third category with less conservative values. Furthermore, most of the existing factors are provided for each load harmonic (with decreasing values by harmonic order), whereas proposed factors account for a global crowd size effect to be applied to the entire PSD load model. For skipping activities, the unique coordination factor in the literature was proposed by ISO 10137 [24] for running (similar to skipping). The related variation against crowd size presented in Figure 9(d) has also a power tendency and corresponding values lie between the proposed factors for the two investigated skipping activities.



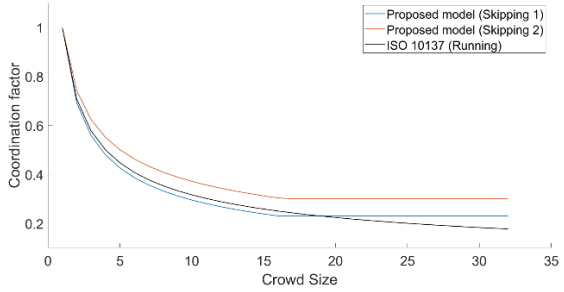
(a) Ebrahimpour and Sack [11]



(b) Ellis and Ji [12]



(c) Parkhouse and Ewins [6]



(d) ISO 10137 [24]

Figure 9: Several coordination factors for jumping and skipping activities

4 Simplified method for floor response prediction

A design-oriented method is proposed in order to perform a simplified response prediction of floors subjected to crowd-rhythmic activities. The method is based on the proposed spectral load model (see Section 3.1) and the random vibration theory [28].

4.1 Calculation procedure

The simplified method is available for floors satisfying two conditions:

- They are dominated by only one natural mode in the frequency range between 0 and 10Hz (corresponding to human excitation [12]);
- They are characterized by a regular modal shape (close to the classical sinusoidal one).

Consider a floor structure fulfilling the above conditions where a single person or a group of individuals ($N \geq 1$) perform rhythmic movements at fixed positions, as shown in Figure 10.

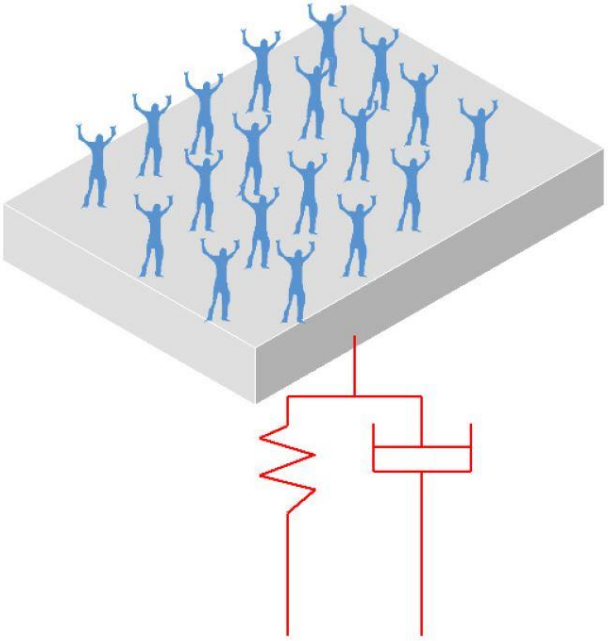


Figure 10: Single-mode floor subjected to rhythmic activities [20]

Each individual k is supposed to have a known position on the floor ($x_{0,k}$; $y_{0,k}$) and a known body mass m_k. The associated PSD load model $S_{p,N,k}(f)$ is computed by:

$$S_{p,N,k}(f) = \left[C(N) m_k g \alpha\right]^2 \sum_{i=1}^{3} \left[a_i^2 \exp\left(-\frac{\left(f - i f_p\right)^2}{(i\delta)^2} \right) \right] \tag{12}$$

An equivalent crowd load model derived from the above model is first established, based on the observation of experimental loads and responses against crowd size [25]. Indeed, instead of applying the PSD load model (given by Eq. (12)) at each of the excitation positions, the equivalent model is concentrated at a single specific location. The corresponding modal amplitude is the mean of the modal amplitudes at the N excitation positions.

The resulting equivalent load model is then expressed by:

$$S_{p,N}(f) = \left[N C(N) \bar{m} g \alpha\right]^2 \bar{S}_p(f) \tag{13}$$

where $\bar{m}$ is the mean body mass of individuals and $\bar{S}_p(f)$ is given by:

$$\bar{S}_p(f) = \sum_{i=1}^{3} \left[a_i^2 \exp\left(-\frac{\left(f - i f_p\right)^2}{(i\delta)^2} \right) \right] \tag{14}$$

The PSD of generalized forces $S_{p^*}(f)$ is obtained by:

$$S_{p^*}(f) = \bar{\Phi}_p^2 \, S_{p,N}(f) \tag{15}$$

Here, $\bar{\Phi}_p$ is the mean of the fundamental modal amplitudes at the N excitation positions.

This leads to the PSD of the modal coordinate $S_q(f)$ expressed as follows:

$$S_{\mathrm{q}}(f) = \bar{\Phi}_{\mathrm{p}}^2 \left|H_1(f)\right|^2 S_{\mathrm{p,N}}(f) \tag{16}$$

where $H_1(f)$ is the transfer function related to the fundamental mode, as follows [28]:

$$\left|H_1(f)\right|^2 = \frac{1}{M_1^2\left[16\pi^4\left(f_1^2 - f^2\right)^2 + \left(8\pi^2\xi_1 f_1 f\right)^2\right]} \tag{17}$$

In the above equation, f_1, M_1 and ξ_1 are the natural frequency, the modal mass and the damping ratio of the fundamental mode, respectively.

The PSD of the displacement response $S_{\mathrm{d}}(f)$ is then calculated by:

$$S_{\mathrm{d}}(f) = \left[NC(N)\bar{m}g\alpha\right]^2 \Phi_{\mathrm{r}}^2 \bar{\Phi}_{\mathrm{p}}^2 \left|H_1(f)\right|^2 \bar{S}_{\mathrm{p}}(f) \tag{18}$$

Here, Φ_{r} is the fundamental modal amplitude at the response location.

Eq. (18) is rewritten by:

$$S_{\mathrm{d}}(f) = \sum_{i=1}^{3} S_{\mathrm{d,i}}(f) \tag{19}$$

where $S_{\mathrm{d,i}}(f)$ is the PSD response due to the i^{th} harmonic of the load obtained by:

$$S_{\mathrm{d,i}}(f) = \left[NC(N)\bar{m}g\alpha\right]^2 \Phi_{\mathrm{r}}^2 \bar{\Phi}_{\mathrm{p}}^2 \left|H_1(f)\right|^2 a_{\mathrm{i}}^2 \exp\left(-\frac{\left(f - if_{\mathrm{p}}\right)^2}{(i\delta)^2}\right) \tag{20}$$

The RMS response $d_{\mathrm{rms,i}}$ due to the i^{th} harmonic is then expressed by:

$$d_{\mathrm{rms,i}}^2 = \int_0^{+\infty} S_{\mathrm{d,i}}(f)df = \gamma_{\mathrm{N}}\, a_{\mathrm{i}}^2 \int_0^{+\infty}\left[\left|H_1(f)\right|^2 \exp\left(-\frac{\left(f - if_{\mathrm{p}}\right)^2}{(i\delta)^2}\right)\right]df \tag{21}$$

where

$$\gamma_{\mathrm{N}} = \left[NC(N)\bar{m}g\alpha\right]^2 \Phi_{\mathrm{r}}^2 \bar{\Phi}_{\mathrm{p}}^2 \tag{22}$$

An approximation of the integral provided by Eq. (21) is performed using two principal simplifications.

Firstly, the exponential function of the PSD load model (see Eq. (20)) is replaced by a bilinear function $B_{\mathrm{i}}(f)$ expressed as follows:

$$B_{\mathrm{i}}(f) = \begin{cases} 1 + \left(\dfrac{f - if_{\mathrm{p}}}{2i\delta}\right), & i(f_{\mathrm{p}} - 2\delta) \le f \le if_{\mathrm{p}} \\ 1 - \left(\dfrac{f - if_{\mathrm{p}}}{2i\delta}\right), & if_{\mathrm{p}} \le f \le i(f_{\mathrm{p}} + 2\delta) \end{cases} \tag{23}$$

An illustration of the exponential and bilinear functions for the first harmonic with f_{p} = 2Hz and δ = 0.05Hz is displayed in Figure 11. It can be observed that the energy distribution is almost the same between the two functions. The obtained normalized load energy (variance) is 0.09 and 0.10 for the exponential and the bilinear function, respectively. Hence, a slightly conservative floor response is expected when adopting the bilinear function. The same remarks mentioned earlier are true for higher load harmonics.

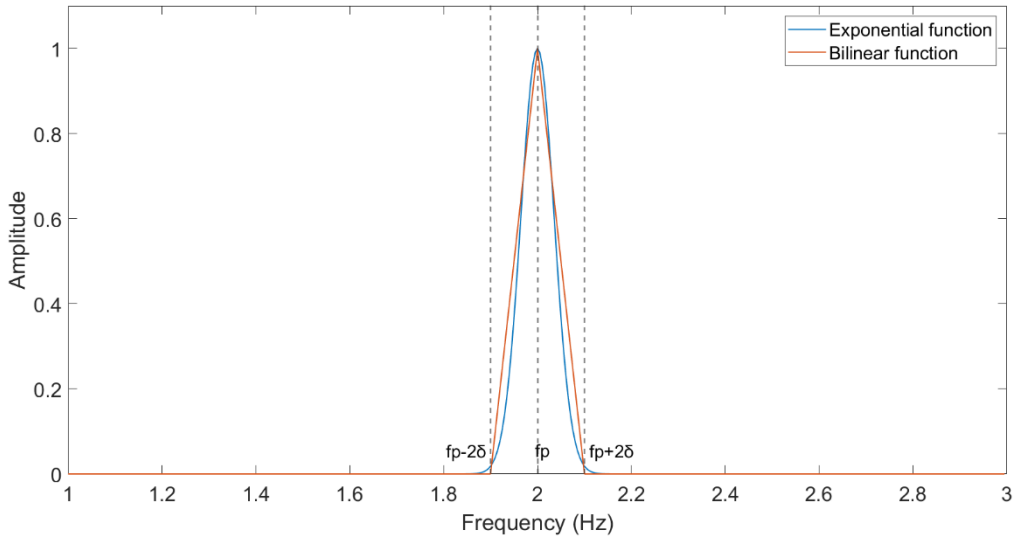


Figure 11: Comparison of exponential and bilinear functions for the PSD load model (first harmonic, excitation frequency of 2Hz)

Secondly, a closed-form expression of the bilinear function $B_i(f)$ is proposed. For all investigated activities, the identified values of the bandwidth coefficient δ are all below 0.15Hz (see Table 5), such that $\delta \ll f_p$. Consequently, it is assumed that the load energy of the i^{th} harmonic could be concentrated at the corresponding peak [29]. Since the enclosed area of the bilinear function curve equals $2i\delta$ (as illustrated in Figure 11), the following approximation is made:

$$\exp\left(-\frac{\left(f-if_p\right)^2}{(i\delta)^2}\right) \approx B_i(f) \approx \left(2i\delta\right)\,\Delta(f-if_p) \tag{24}$$

where Δ is the Dirac-delta function. This function has an integral property for a given function $Z(f)$ and a constant b expressed by:

$$\int_0^{+\infty} Z(f)\Delta(f-b)df = Z(b) \tag{25}$$

Using the approximation of the exponential function provided by Eq. (24), the application of Eq. (25) provides the following integral approximation:

$$\int_0^{+\infty}\left[\left|H_1(f)\right|^2 \exp\left(-\frac{\left(f-if_p\right)^2}{(i\delta)^2}\right)\right]df \approx \left(2i\delta\right)\left|H_1(if_p)\right|^2 \tag{26}$$

This leads to an approximate expression of Eq. (21) given by:

$$d_{rms,i}^2 \approx \gamma_N\, a_i^2\left(2i\delta\right)\left|H_1(if_p)\right|^2 \tag{27}$$

The PSD acceleration of the i^{th} harmonic $S_{a,i}(f)$ is obtained by:

$$S_{a,i}(f) = \left(2\pi f\right)^4 S_{d,i}(f) \tag{28}$$

Using the same procedure as for displacement, the RMS acceleration of the i^{th} harmonic $a_{rms,i}$ is approximated by:

$$a_{rms,i}^2 \approx \left(2\pi i f_p\right)^4 d_{rms,i}^2 \tag{29}$$

Finally, the RMS response of the floor w_{rms} (displacement or acceleration), widely used for human comfort assessment [21],[24], is deduced by:

$$w_{rms} = \sqrt{\int_0^{+\infty} S_w(f)df} = \sqrt{\sum_{i=1}^{3} w_{rms,i}^2} \tag{30}$$

4.2 Verification on the floor specimen

The floor specimen satisfies the two conditions presented in Section 4.1. Therefore, the simplified method is applied on this floor using the identified load models for the four investigated activities (see Section 3.3). Obtained responses are then compared with displacements measured during vibration tests.

4.2.1 Determination of floor response

All activities lasted 30s with a time step of 0.0078s. The positions of the individuals for each crowd size are given in Table 3 and illustrated in Figure 5(b). The mean body mass of individuals was taken equal to 75kg [21]. Modal properties of the fundamental mode of the floor (natural frequency, damping ratio, modal shape) are the result of Experimental Modal Analysis provided in Section 2.2.3. The total floor mass measured by load cells was M = 18.85t, corresponding to a fundamental modal mass M_1 = 7.73t [30].

For each activity and crowd size, the RMS displacement response of the floor was calculated at the position of the LVDT device (see Figure 5(b)) using the simplified method detailed in Section 4.1. The mean PSD load parameters (see Table 5) along with identified coordination factors (see Table 6) were used for that purpose.

Moreover, the RMS displacements were calculated from the experimental records on the floor specimen (see Section 2.3.3) for each of the six activity windows. To ensure a uniform body mass for all crowd sizes, the normalized RMS response was obtained by:

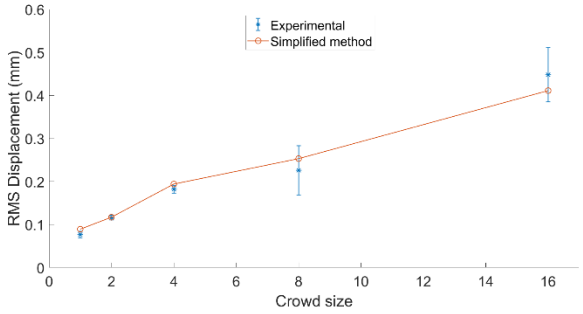
$$d_{rms,s} = \frac{N\bar{m}}{\sum_{k=1}^{N} m_k} \hat{d}_{rms,s} \tag{31}$$

where $d_{rms,s}$ is the normalized RMS displacement for the s^{th} activity window, N the crowd size, $\hat{d}_{rms,s}$ the initial RMS displacement, m_k the body mass of the k^{th} individual and $\bar{m}$ a nominal body mass for all individuals (taken as 75kg [21]).

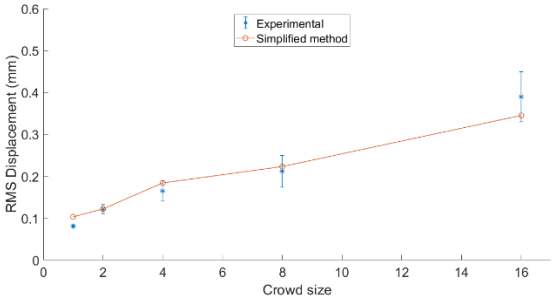
The six calculated displacements are then represented by their mean values and standard deviations for the investigated activity and crowd size N.

4.2.2 Comparison of RMS displacements

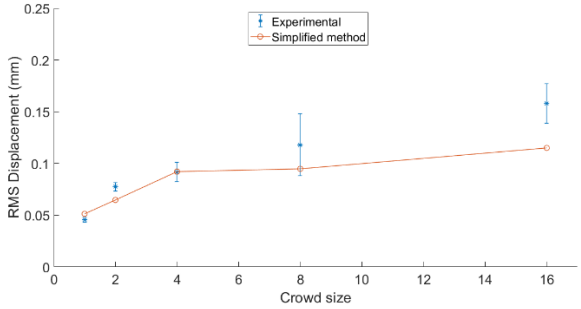
The experimental and numerical RMS displacements (using the simplified method) are plotted against crowd size in



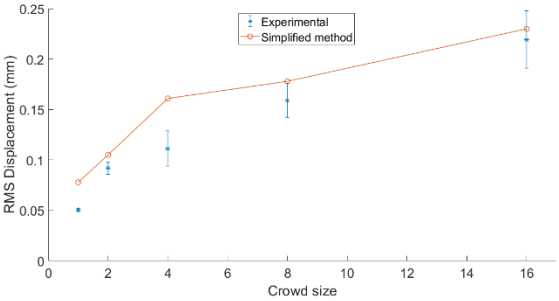
(a) Jumping 1



(b) Jumping 2

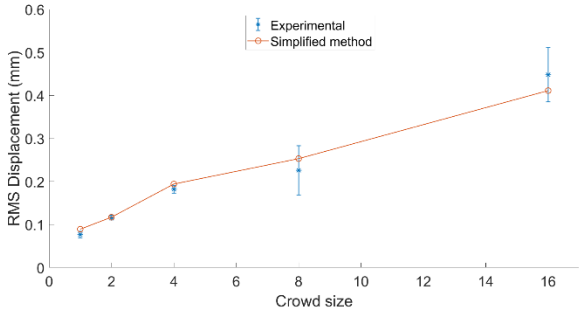


(c) Skipping 1

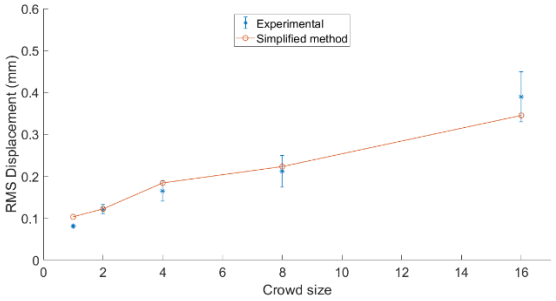


(d) Skipping 2

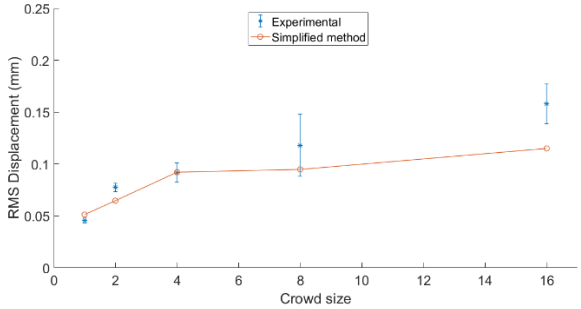
for the four investigated rhythmic activities.



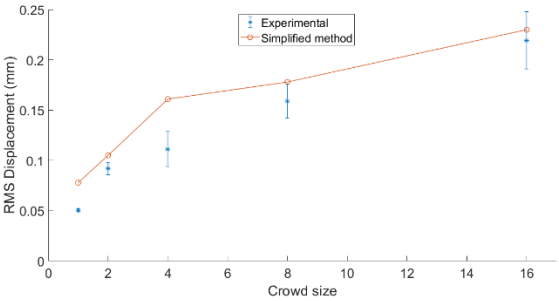
(a) Jumping 1



(b) Jumping 2



(c) Skipping 1



(d) Skipping 2

Figure 12: Comparison between experimental RMS displacements (mean in asterisk marks, standard deviation in error bars) and simplified method RMS displacements (circles) against crowd size

RMS displacements obtained by the simplified method are greater than the experimental ones for smaller crowd sizes (up to 8) whereas the inverse trend is observed for larger crowd sizes.

"Jumping 1" and "skipping 2" activities (having mean excitation frequencies of 2.36Hz and 2.62Hz, respectively) have a third load harmonic close to the fundamental mode in terms of frequency (having natural frequency of 7.65Hz). More randomness in frequency occur for large groups of individuals which could have caused resonance at that harmonic during experiments. On the other hand, "jumping 2" and "skipping 1" activities have smaller coordination factors compared to the other corresponding activity in the same category (see Table 6), which leads to lower numerical RMS displacements for such non-resonant cases. RMS response differences are also more important for skipping than for jumping since more randomness in motion and excitation positions was observed for the first category compared to the second.

Overall, the mean relative difference between the numerical and mean experimental RMS displacements ranges between 8.8% and 26.1% for the investigated rhythmic activities. These differences could be attributed to variabilities noticed while performing movements during vibration experiments as well as the potential irregularities resulting from the estimation of modal shapes (because of a coarse mesh). The inevitable residue resulting from the load identification process could also play a major role in these discrepancies. The identified load model along with the simplified method are thus verified for the tested floor specimen.

The proposed response evaluation method could be used as part of a quick assessment of regular floor structures subjected to crowd-rhythmic activities against human discomfort. This requires two principal elements to be agreed by the stakeholders: (i) design values for rhythmic load parameters (based on Table 5) and (ii) vibration acceptability criteria with regards to the comfort of occupants.

5 Conclusions

A spectral modelling approach for crowd-rhythmic activities has been proposed in this paper, comprising a PSD load model for a single person to which coordination factors are added to take into account the lack of synchronization between individuals. The load model parameters were identified using least-squares techniques based on force measurements on a floor specimen. Two particular types of rhythmic activity (jumping and skipping) were investigated with crowd sizes ranging between 1 and 16. Participants were asked to move following audible and visual stimuli to be as close as possible to real life situations.

The determined PSD load parameters for a single person differ for each activity according to their particular motion style. The obtained variation range (mean ± standard deviation) allows the application of the model for situations similar to experimental conditions. For a group of people, the coordination factors exhibit a hyperbolic decrease by crowd size for jumping activities against an exponential decrease for skipping activities. Individuals performing skipping on feet soles were found to better synchronize their motion compared to those skipping on feet toes.

Based on an equivalent load model derived from the above-established spectral model, a simplified random vibration method was proposed for the response evaluation of floors subjected to crowd-rhythmic activities. The method was based on a closed-form expression of the exponential function describing the rhythmic load. The accuracy of this method was then

assessed on the floor specimen by computing displacements resulting from the investigated rhythmic activities. The experimental and numerical responses were then compared. The obtained differences could be attributed to variabilities observed during experiments on one hand and the inevitable residue from the least-squares load identification procedure on the other. The simplified method can be used for a straightforward assessment of floors against human discomfort resulting from such activities.

As a perspective, the identified spectral load models for the four investigated activities could be further verified on full-scale floor structures with various usages (gymnasiums, sports halls, fitness centres, etc.). In that case, larger numbers of participants could be accommodated compared with the maximum crowd size of 16 imposed by the limited surface of the floor specimen. Individuals more experienced to coordinate their motion in a group might be involved in such tests. Human-structure interaction may also be analysed with different excitation frequencies and densities of occupation. This effect could then be included in the spectral load model with higher accuracy using a separated term depending on the flexibility of the structure.

Acknowledgments

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