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# Continuous Rectangular Phoenix Cells Mapping for Direct Copolar and Crosspolar Optimization of Quasi-Periodic Reflective Surfaces

Andrea Guarriello\*, Renaud Loison\*, Daniele Bresciani<sup>†</sup>, Hervé Legay<sup>†</sup> George Goussetis<sup>‡</sup>

\*IETR, National Institute of Applied Sciences, Rennes, France, Andrea.Guarriello@insa-rennes.fr

<sup>†</sup>Direction of Research, Thales Alenia Space, Toulouse, France

<sup>‡</sup>Institute of Sensors, Signals and Systems, Heriot-Watt University, Edinburgh, United Kingdom

**Abstract**—This paper proposes a general methodology to enhance the efficiency of the direct optimization of the copolar and crosspolar radiation of large aperture quasi-periodic reflective surfaces (QPRS) antennas by ensuring the layout continuity without abrupt geometrical variation. This is done through the exploitation of the Phoenix cells (PC) properties, by efficiently parametrizing and interpolating PC lookup tables. In particular, the proposed parametrization allows the description of the QPRS layout through continuous functions without losing precision. Indeed, the description of the QPRS through continuous functions, namely B-spline functions, allows for to reduction of the large-scale optimization problem dimensions and naturally ensures the layout smoothness. A use case of a large faceted contour beam reflectarray (RA) is presented, showing a remarkable improvement in both the copolar and the crosspolar discrimination levels compared to classical synthesis techniques and the RA layout smoothness is practically demonstrated.

**Index Terms**—Quasi-Periodic Reflective Surfaces, Optimization, Reflectarrays, B-Spline Projection, Periodic Lookup Table Mapping.

## I. INTRODUCTION

Contoured beam reflectarrays (RAs) design and optimization for high gain antenna applications is a rather complex process involving a large number of design parameters. Indeed, RAs are spatially fed quasi-periodic reflective surfaces (QPRS) composed of hundreds or thousands of unit cells, each one described by a set of geometrical parameters, defining the degrees of freedom of the unit cell design.

The layout distribution across the surface can be conceived by selecting the unit cells one by one, which are able of reflecting the local incident field according to a target local aperture field, previously computed. These approaches, traditionally known as phase-only synthesis, consider the cell as an ideal phase shifter, without information about cell amplitude or cross components [1].

On the other hand, the cell distribution can be conceived at a global level, where all the unit cells are simultaneously optimized by taking into account the full reflection matrix in phase and amplitude, with an objective on the scattered far-field, that usually defines the ultimate target of the design [2], [3], [4]. Although the first approach is computationally efficient, it presents the inconvenience of leading to sub-optimal designs. The second approach leads to a more optimal design

in terms of far-field performances [5], but it is computationally more expensive since at each iteration the cells composing the layout should be re-characterized or interpolated.

The optimization procedures of large QPRS involve in general a large number of degrees of freedom (d.o.f.). Some hypotheses can be done to handle the problem dimension and complexity. First of all, the local periodicity hypothesis is done to consider each cell as part of a periodic environment, by avoiding expensive simulations of the entire layout. This implies that at a local level the cell must be surrounded by similar cells in terms of topology. The layout continuity and smoothness are therefore design constraints that should be respected [6], [7], [8].

A technique to considerably reduce the optimization problem dimensions and better handle the optimization variables, is to use continuous analytical functions, depending on fewer d.o.f., to describe the geometries of the unit cells composing the array. In [9], [10] the geometrical variation of square patch elements is represented through continuous function, e.g. splines. However, the considered unit cell implied sharp geometrical variation after a complete geometrical cycle, which is not well represented by the spline projection of the layout. As a consequence and, as claimed by the authors, there is a deterioration of the performance obtained using continuous functions to represent the elements' geometries compared to that of a design where the array elements are directly optimized. This issue is solved by representing complex field quantities (aperture field) through continuous functions instead of the elements' geometry. The drawback is that the process is led back to the case in which each cell is chosen individually to match the optimized field quantities and abrupt layout variations may occur.

Phoenix cells (PC), originally presented in [11], have the interesting features of rebirth capability, i.e. the cell's geometry loops to its initial configuration after a complete phase cycle without abrupt geometry variation, with the possibility of considering mixed order/type [12] to design the PC cycle. This property is efficiently exploited in [2], [13] for the direct copolar component optimization using square PCs. In this work, the RA layout is projected on a spline functions basis to reduce the optimization problem complexity. The layout continuity is ensured by the first and second-order square PCs

spherical mapping allowing an efficient spline projection of the cell geometries composing the layout.

This work is an extension of the technique presented in [2] to a double circular polarization case. Indeed, square PCs are not suitable to reduce the crosspolar components. The aim is to present a general and efficient lookup table parametrization and interpolation to exploit the rectangular PCs topology to enable the dual polarization mode, notably by directly optimizing the copolar and crosspolar components of the far-field patterns by preserving the layout smoothness. The optimization problem dimensions are reduced and the layout smoothness is ensured by projecting the RA layout on B-spline functions.

## II. ELEMENTARY CELLS LOOK-UP TABLE PARAMETRIZATION

### A. Periodic feature of PC and PC cycle

The selection of the cells composing the QPRS layout must satisfy at least two main constraints: realize a suitable phase delay of the local reflected field, therefore being able to supply a complete (or almost)  $360^\circ$  phase shift by varying the cell geometry and have a topology very similar to those of the surrounding cells. This section aims to present how to efficiently construct a lookup table that can suitably be employed in the QPRS design process by facilitating and improving the fast access to the scattering properties of the cell during the optimization process and simultaneously reducing the number of degrees of freedom, through the parametrized layout projection on B-splines.

By combining different PC types and order one can design a periodic cycle where the geometry after a complete phase cycle comes back to its initial geometry. A basic square PC cycle comprises first-order inductive and capacitive cells whose aperture and patch, respectively, grow from zero until a maximum size [14]. As reported in this work, the advantage of using a first-order cell's PC cycle resides in the fact that the geometric variation is very smooth, and the rebirth capability of the cell is highlighted. Nevertheless, it presents the main disadvantage: the phase shift range presents a gap of more than  $50^\circ$ , that is non-negligible in the lookup table conception, i.e., the cells are not able to cover the whole  $[-180^\circ; 180^\circ]$  reflection phase range. To make the realizable phase shift cycle larger, we should consider the patch/slots' maximum dimensions very close to the lattice dimensions. This is, in practice, not feasible, since it would make the cell unstable (small geometry variations are accompanied by step reflection matrix variation). Another solution could be to change the lattice size, but in this and in previous works in which PCs are employed, the lattice size is always fixed to  $d_0 = \frac{\lambda}{3}$ , since this value allows the avoidance of grating lobes apparition and the avoidance of Wood anomalies [15]. Moreover, such a small lattice allows a homogenization impedance of the QPRS [8] with reduction of the quantization effects due to abrupt contiguous cells geometry variation. Solutions to overcome this inconvenience have been proposed in [14], where meandered inductive grids have been introduced to complete the phase

cycle. However, to exploit the asymmetry of the cell for double circular polarization purposes [16], square PCs, to be deformed into rectangles, would be preferable.

Therefore, it is necessary to consider higher-order cells to conceive an adapted lookup table conveniently. The first and second-order inductive PCs are considered to overcome this issue. The square loop/slot periodic cycle on the bottom of figure 1 is considered. Figure 1 shows the behavior of the cells as a function of the auxiliary periodic variable  $\xi$  defined as follows:

$$\xi(d_1, d_2) = \begin{cases} -\pi + \pi \frac{d_1}{d_{1,max}} & , \text{if } d_2 = 0 \\ \pi \frac{d_2}{d_{2,max}} & , \text{if } d_1 = d_{1,max} \end{cases} \quad (1)$$

The inverse relation is easily derivable. Figure 1 shows the phase of the reflection evolution of the proposed PCs cycle at central frequency  $f_0 = 2.33$  GHz, with dielectric substrate  $\epsilon_r = 1.07 - j3.5 \cdot 10^{-3}$  and normal incidence. The lattice size is set to  $d_0 = \frac{\lambda}{3} \approx 43$  mm, while the substrate height is set to  $\tau = \frac{\lambda}{4} \approx 30$  mm. The maximum slot/patch size is set to  $d_{1,max} = 41$  mm. The geometrical parameters  $d_1$  and  $d_2$  are shown in the inset of figure 1, while the maximum values are set to avoid geometries that would not be realizable. The phase gap present in the first-order PC cycle has been reduced to  $27^\circ$ . Since we are dealing with direct optimization, the effect of such a small phase gap would not affect the QPRS design process's effectiveness. In general, we can mix higher-order cells to obtain new PC cycles when lower profile cells are required [17].

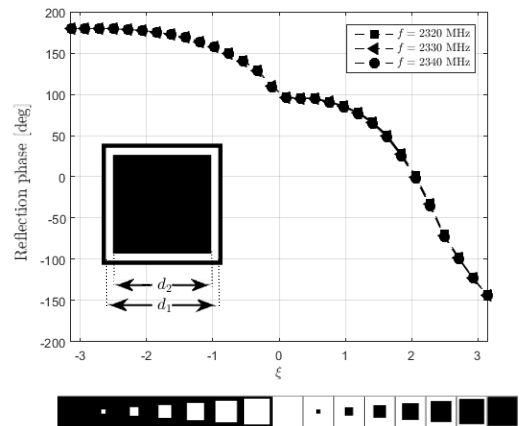


Fig. 1: The phase of the reflection coefficient of the PC cycle proposed (on the bottom) as a function of the periodic parameter  $\xi$  describing the geometry.

To have more flexibility in the QPRS design when dealing with double circular polarization, we introduce non-symmetric cells, in which the degrees of freedom are the horizontal and vertical size of the patch and slots. The unit cells can exhibit therefore  $d_{i_x} \neq d_{i_y}$ . This allows to control of the impinging wave delay and compensate for depolarization [16].

The cells contained in the lookup table described in the previous figure 1 are now parametrized through two auxiliary variables  $(\xi, h)$  that allow the description of the rectangular PC geometries through continuous mapping. A similar concept has been proposed in [2]. The lookup table 2D mapping of first and second-order capacitive/inductive PCs was performed aiming at the enlargement of the RA bandwidth.

Being  $\mathbf{d} = [d_{1_x} \ d_{1_y} \ d_{2_x} \ d_{2_y}]$  the patch and slot dimensions along the vertical and the horizontal axis, the lookup table linear parametrization is the following:

$$\xi(\mathbf{d}) = \begin{cases} -\pi + \pi \frac{d_{1_x}}{d_{1_{max}}}, & \text{if } d_2 = 0, d_{1_x} > d_{1_y} \\ -\pi + \pi \frac{d_{1_y}}{d_{1_{max}}}, & \text{if } d_2 = 0, d_{1_x} < d_{1_y} \\ \pi \frac{d_{2_x}}{d_{2_{max}}}, & \text{if } d_1 = d_{1_{max}}, d_{2_x} > d_{2_y} \\ \pi \frac{d_{2_y}}{d_{2_{max}}}, & \text{if } d_1 = d_{1_{max}}, d_{2_x} < d_{2_y} \end{cases} \quad (2)$$

$$h(\mathbf{d}) = \begin{cases} \frac{d_{1_y} - d_{1_x}}{d_{1_y} - d_{1_{max}}}, & \text{if } d_2 = 0, d_{1_x} > d_{1_y} \\ \frac{d_{1_x} - d_{1_y}}{d_{1_{max}} - d_{1_x}}, & \text{if } d_2 = 0, d_{1_x} < d_{1_y} \\ \frac{d_{2_y} - d_{2_x}}{d_{2_y} - d_{2_{max}}}, & \text{if } d_1 = d_{1_{max}}, d_{2_x} > d_{2_y} \\ \frac{d_{2_x} - d_{2_y}}{d_{2_{max}} - d_{2_x}}, & \text{if } d_1 = d_{1_{max}}, d_{2_x} < d_{2_y} \end{cases} \quad (3)$$

The inverse relations can be easily derived.

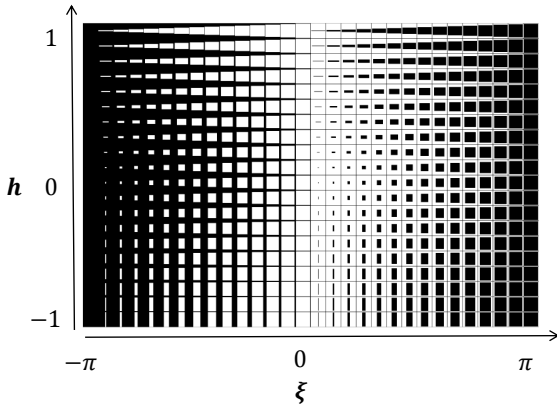


Fig. 2: Cylindrical lookup table representation.  $\xi$  represent the periodic dimension while  $h$  the cylinder height.

As shown in figure 2, this parametrization leads to a cylindrical mapping of the geometries stored in the lookup table, in which the  $\xi$  variable represents the periodic angle, varying between  $[-\pi, \pi]$ , representing the cells geometrical index, while  $h$ , non-periodic and bounded between  $[-1, 1]$ , is the axial coordinate or height that indicates the degree of cell distortion into rectangles (figure 2). This implies that

in an optimization process the continuous variable  $\xi$  is left unbounded, while  $h$  is bounded.

### B. Advantages of lookup table continuous and periodic description: B-spline projection of the QPRS parametrized layout

The continuous mapping of the lookup table has a direct link to how the cell distribution on the QPRS layout can be represented. Indeed, the QPRS can be described by two parametric surfaces  $\xi_{m,n}(x_{m,n}, y_{m,n})$  and  $h_{m,n}(x_{m,n}, y_{m,n})$ , being  $m, n$  the indices of the cells position on the surface.

In a QPRS optimization problem formulation, as general principle, the optimization variables are represented by the geometrical parameters of each cell composing the layout, i.e.  $\mathbf{d}(x_{n,m}, y_{n,m})$ . This means that being  $N \times M$  the QPRS cells grid dimensions, and  $i$  the number of geometrical parameters describing the cells  $[d_1 \ d_2 \ \dots \ d_i]$ , the total number of d.o.f. is

$$N_{d.o.f.} = N \times M \times i. \quad (4)$$

If we consider the cells contained in the lookup table described in this paper, in which the cells geometry is described through the vector  $\mathbf{d} = [d_{1_x} \ d_{1_y} \ d_{2_x} \ d_{2_y}]$ , the number of d.o.f. is equal to  $N_{d.o.f.} = N \times M \times 4$ . The cylindrical parametrization of the cells geometries through the variables  $\xi$  and  $h$ , reduces the number of d.o.f. to half of the latest  $N_{d.o.f.} = N \times M \times 2$ .

Another advantage of the cells parametrization is that it is possible to describe the QPRS layout through continuous and derivable functions, such as tensorial bi-cubic splines. To do so, the optimization variables are defined by tensors of splines coefficients  $\Delta \Xi_{k_x, k_y}$ ,  $\Delta H_{k_x, k_y}$  describing a variation with respect to an initial parameter distribution  $\xi_{initial}(x_{n,m}, y_{n,m})$  and  $h_{initial}(x_{n,m}, y_{n,m})$ . The smooth surface distribution of the parameters  $\xi$  and  $h$  defining the QPRS is then given by:

$$\xi(x_{n,m}, y_{n,m}) = \xi_{initial}(x_{n,m}, y_{n,m}) + c_{n, m_{k_x}} \Delta \Xi_{k_x, k_y} c_{n, m_{k_y}} \quad (5)$$

$$h(x_{n,m}, y_{n,m}) = h_{initial}(x_{n,m}, y_{n,m}) + c_{n, m_{k_x}} \Delta H_{k_x, k_y} c_{n, m_{k_y}} \quad (6)$$

This representation presents two main advantages: the spline description of the variation with respect to an initial layout distribution ensures the layout parametric surface continuity and derivability (i.e. layout smoothness) at each iteration, and, the number of degrees of freedom in the optimization problem is radically reduced. Being  $K_x \times K_y$  the dimension of the knots grid along the  $x, y$  directions, the number of d.o.f. depends on the number of the spline knots considered:

$$N_{d.o.f.} = (K_x + k - 2) \times (K_y + k - 2) \times 2, \quad (7)$$

where  $k$  is the knots multiplicity on the knots grid domain borders, for bi-cubic splines  $k = 4$  [18]. In general, few knots are sufficient to accurately describe a QPRS  $K_{x,y} \ll M, N$ . The presented optimization problem description allows us to continuously modulate the surface cell geometries to attain

objectives on the far field by avoiding layout discontinuities. Usually, a gradient-based multi-objective algorithm is employed where the initial guess design is the one obtained through phase-only synthesis [2], [3].

### C. Fast access to unit cell reflection matrices

The cells samples composing the lookup table are fully characterized in reflection by exploiting an industrial in-house spectral domain Method of Moments software, proven to be very fast and accurate with respect to full-wave software [19], [20]. The cells are characterized on a regular grid of incidence samples  $(\theta^{inc}, \phi^{inc})$ , selected posterior of the QPRS architecture analysis. Usually  $\theta \in [0^\circ, 40^\circ]$  with a  $5^\circ$  interval, and  $\phi \in [-180^\circ, -90^\circ]$  with a  $15^\circ$  interval (by exploiting the rectangular symmetry it is sufficient to characterize for incidences  $\phi$  comprised in just one angular quadrant). The incidence grid is therefore composed of  $9 \times 7$  incidence samples, sufficient to have a very accurate bilinear interpolation to derive the phase and the amplitude of each cell comprised in the lookup table. Being  $(m, n)$  the cell position on the surface, the lookup tables are interpolated on the actual cell incidence at  $(\theta_{m,n}^{inc}, \phi_{m,n}^{inc})$ . This means that for every cell position on the QPRS, a complete lookup table is stored.

By characterizing a limited set of samples (in this case, roughly 631 samples), it is possible to have rapid access to the scattering matrix of an arbitrary cell by local bi-cubic interpolation of the phase and amplitude of each reflection matrix component. Local cubic interpolation of scattering coefficients grants a high accuracy while maintaining a low computational effort [5]. In Figure 3 are mapped the amplitude and phases of the direct reflection matrix terms at central frequency  $f_0 = 2.33$  GHz.

### III. USE CASE: LARGE DEPLOYABLE RA OPTIMIZATION

The presented methodology is applied to the direct optimization of a very large multi-facet RA composed of 22434 cells in S-band for direct broadcast GEO satellite. Without entering the details of the RA architecture, we show here a comparison of the RA obtained with phase-only (PO) synthesis and the RA obtained with the direct copolar and crosspolar optimization (DO) for the two circular polarizations. The results are shown in terms of minimum gain on the coverage region, minimum crosspolar discrimination (XPD), and of percentage of ground stations violating the constraints on XPD. Results for North America coverage (CONUS), with a total of 1600 stations, are presented in table I. The DO leads to a notably enhanced design compared to PO. The obtained RA is shown in figure 4, highlighting the RA layout continuity and the absence of sharp transitions. Moreover, the advantages of the proposed methodology are evident when comparing the problem dimensions of a classical optimization compared to the one described in this work. With reference to the previous section II-B, the original RA optimization problem dimension is  $22434 \times 4 = 89736$  d.o.f.. With the PC parametrization and spline projection of the parametrized layout on a knots

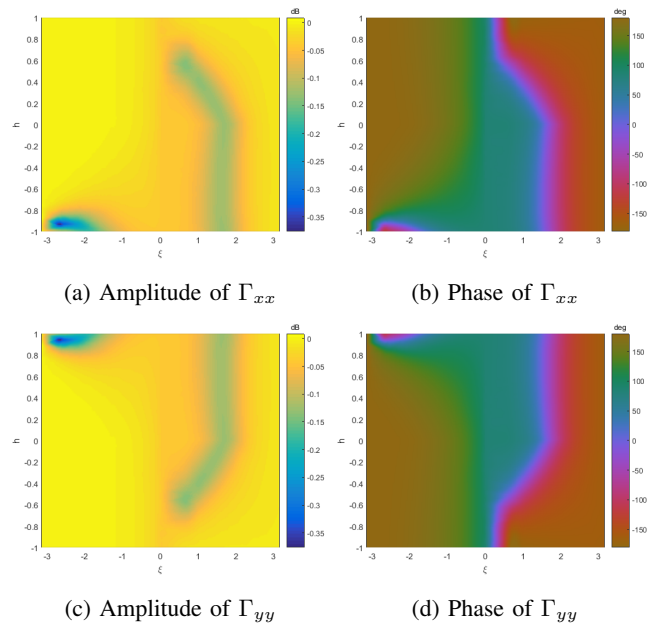


Fig. 3: Mapping over the cylinder of the principal scattering matrices terms  $\Gamma_{xx}$  and  $\Gamma_{yy}$  at normal incidences calculated by local cubic interpolation.

grid  $20 \times 20$ , the problem dimension has been reduced to  $22 \times 22 \times 2 = 968$  d.o.f..

TABLE I: RAs performances comparison

	$G_{CO_{min}}$	$XPD_{min}$	$\% < XPD_{min}$
requirements	25.00 dB	25.00 dB	< 5%
PO (LHCP)	25.05 dB	22.16 dB	10.5 %
DO (LHCP)	27.18 dB	27.00 dB	0%
PO (RHCP)	26.13 dB	21.27 dB	13.5 %
DO (RHCP)	27.35 dB	27.02 dB	0%

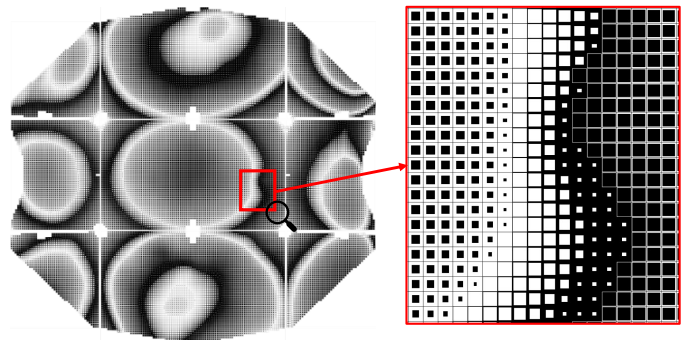


Fig. 4: Layout issued from direct optimization.

### IV. CONCLUSION

This paper presents an innovative lookup table mapping and interpolation scheme for efficient QPRS direct optimization. The methodology presented is namely an extension of the

spherical mapping presented in [2] to the double circular polarization case. It exploits the rebirth capability of the PC to map the lookup table periodically and the parametrization proposed allows to map of rectangular PCs to have more flexibility in the QPRS far-field polarization control. The methodology is applied to a concrete case of a large-scale optimization problem of a large multifaceted RA for direct broadcast satellite showing remarkable improvements in the performances with respect to classical synthesis techniques. The presented methodology allows obtaining RA layout without any sharp geometrical transition, thus respecting the local periodicity of the RA layout. Future works would include the generalization of the procedure to different periodic geometric cycles for RA cells.

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