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# Frequency changes during the propagation of a light beam

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**Abstract** – The energy and therefore the frequency of electromagnetic fields are quantities which are generally considered to be conserved in free space propagation. However, we observe here a frequency varying optical wave in free space propagation. The light comes from an Arago spot. Its frequency changes due to the rotational Doppler effect. More precisely, the light diffracted by a two-dimensional asymmetric object carries orbital angular momentum that varies with distance. As the object rotates, the frequency of the diffracted light experiences different rotational Doppler shifts along propagation. It varies in a discrete quantized way as it propagates. It can be adjusted, without violation of the conservation of energy. Since this phenomenon is deeply rooted in the diffraction process, it shines some new light on this still open issue.

**Introduction.** – The Arago spot is a bright spot that appears at the center of a circular object's shadow. It belongs to the Bessel beams category [1]. They are non-diffracting beams that are solutions of the equation of propagation [2]. Such Bessel beams are considered as self-accelerating beams [3–7] and can be designed to deviate from straight-line propagation. They can follow any trajectory [8]. Moreover, Bessel beams can carry Orbital Angular Momentum (OAM) [9]. When the Arago spot originates from an asymmetric object, it carries OAM that varies along propagation [10, 11]. On the other hand, beams carrying OAM experience so-called rotational Doppler shifts when the emitter and the receiver rotate with respect to each other [12–14]. Although the light frequency is usually assumed to be constant in free space propagation [1, 15, 16], the question arises whether such Arago spots could experience frequency variations along the direction of propagation, when the diffracting asymmetric object is rotating. The aim of this letter is thus to investigate the frequency changes of the Arago spot behind a rotating asymmetric object.

**Experimental set-up.** – The experimental set-up is displayed in Fig. 1. The eye-safe telecommunication-wavelengths light from a laser source (CEFL, Keopsys,  $\lambda = 1550$  nm,  $P = 1$  W), connected to an optical fiber (SMF 28) and a self-focusing lens (self-foc) is diffracted by an asymmetric object. The laser polarization is linear

and it is oriented towards the vertical axis. The outgoing beam is slightly diverging. A spherical lens ( $f_1 = 50$  cm) makes the beam parallel. The beam waist is 2 mm. This diffracted light interferes constructively or destructively in the shadow of the object, on the beam axis. When the object is a regular disk, the diffraction pattern is a small bright spot called the Arago spot [1]. In the case of an asymmetric static object, such as a snail-shaped object (cam), diffraction leads to a tiny dark spot in the middle of a small bright spot, close to the object and to a small bright spot with outer rings far from the object. These spots carry OAM [10].

For OAM beams in general, the equiphase surface is a spiral sheet with a pitch equal to  $\ell$  times the wavelength. The electromagnetic fields have thus a  $\exp(i\ell\phi)$  azimuthal dependence, where  $\ell$  is known as the “topological charge”. The object is printed on a transparency and fixed on a homemade hollow shaft. Special care is taken to align the center of the object (center of mass) with the rotation axis of the shaft. At a given distance  $d$  from the object, the OAM nature of the beam and the value of its topological charge (including its sign) are investigated by the double slit experiment [17] or either by the diffraction triangle method [18]. We image the diffracted spot with an infrared InGaAs camera (Hamamatsu C12741-03) for various distances from the object. Alternatively, we use a laser viewing card (Thorlabs VRC2).

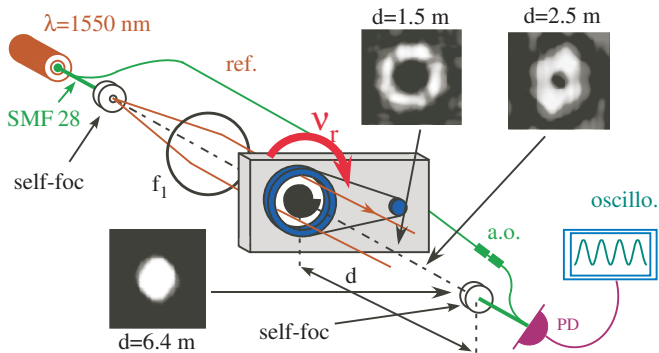
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Fig. 1: Experimental set-up. The collimated light originating from a laser ( $\lambda = 1.55 \mu\text{m}$ ) is diffracted by a rotating cam (rotating frequency  $\nu_r$ ) printed on a transparency glued on a rotating hollow shaft. It interferes on the optical axis (see inserts showing the light intensity at different positions). This axis also corresponds to the rotational axis. The beat frequency against a reference beam (ref.) is recorded on a photodiode (PD) and on an oscilloscope (oscillo.). Self-foc and  $f_1$ : collimating lenses. a.o.: acousto-optic modulators.  $d$ : distance between the rotating object and the detection. The spot is visualized with a laser-viewing card.

**Theoretical considerations.** – We evaluate the amplitude and the phase of the diffracted electric field by the cam at different positions. We first calculate the propagation of the electric field from the laser up to the diffracting object. We then use the Huygens Fresnel formula [1,16,19] to obtain the diffracted field. This formula states that every point on the area surrounding a diffracting object is itself the source of spherical wavelets having the same frequency as the incident wave which also propagate. These secondary wavelets mutually interfere at a given position. The sum of these spherical wavelets generates a spot that propagates behind the object. We then compute the amplitude and the phase of the spot in a plane perpendicular to the direction of propagation, for various distances from the object.

The results of these calculations, for the cam of Fig. 1, are shown in Fig. 2. Obviously, the phase varies with the azimuthal angle. The diffracted field is then a vortex beam that carries OAM. We evaluate the topological charge carried by the central spot. The light is decomposed into modes with integer topological charges. These calculations confirm the results of Ref. [10]. In regards to the distribution of the amplitude of the light spot, it is in qualitative agreement with the inserts of the pictures of Fig. 1, that correspond to the experimental intensity distribution of the fields at various given distances, in the shadow of the cam. From the numerical simulations, it seems that the size of the spot scales as  $d\lambda/R$ ,  $R$  being the mean radius of the cam, as one changes the size of the occulting object. This same kind of dependence can be found in every interference phenomena.

**Results.** –

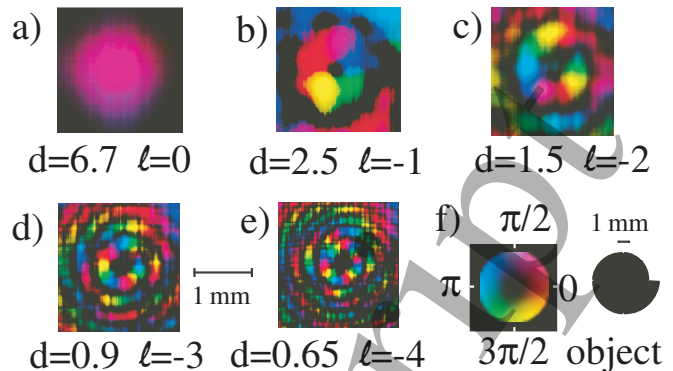


Fig. 2: Calculated phase distribution and deduced topological charge of the diffracted beam at a distance  $d$  from the object a)  $d = 6.7 \text{ m}$ , the phase is constant:  $\ell = 0$ , b)  $d = 2.5 \text{ m}$ , the phase varies from  $0$  to  $-2\pi$ :  $\ell = -1$ , c)  $d = 1.5 \text{ m}$ ,  $\ell = -2$ , d)  $d = 0.9 \text{ m}$ ,  $\ell = -3$ , e)  $d = 0.65 \text{ m}$ ,  $\ell = -4$ , respectively. f) Phase color code and shape of the absorbing object corresponding to the experimental object.

**Static object.** We have experimentally investigated the topological charge of the central spot of the diffracted beam. Please note that the charge is evaluated on a limited size area corresponding only to the spot. The results are displayed in Table 1, for the distances shown in Fig. 2. They confirm that the diffracted beams indeed carry OAM. They are in good agreement with the calculated phase variations in the previous section. For distances  $d$  between  $0.9$  and  $1.5 \text{ m}$ , the topological charge cannot be precisely determined experimentally. It is predominantly equal to  $\ell = -3$  close to  $d=0.9 \text{ m}$  and to  $\ell = -2$  close to  $d=1.5 \text{ m}$ , respectively. Inbetween, from the calculations of the previous section, the spot is actually a superposition of modes having a topological charge equal to  $\ell = -3$  and  $\ell = -2$  (see Fig. 3a for the OAM spectrum decomposition).

For  $1.5 \text{ m} \leq d \leq 2.5 \text{ m}$ , and  $2.5 \text{ m} \leq d \leq 6.7 \text{ m}$ , the spot is a linear superposition of  $\ell = -2$  and  $\ell = -1$ , and  $\ell = -1$  and  $\ell = 0$ , respectively. As the diffracting object is reversed, the sign of the topological charge changes, both experimentally and theoretically. For distances  $d$  below  $0.9 \text{ m}$ , the intensity of the spot is too low to perform any accurate experimental determination of the topological charge of the beam. Nevertheless, from our calculations, the absolute value of the topological charge increases further closer to the cam. We have changed the laser polarization (linear, vertical or horizontal, or circular) and we haven't noticed any change in the experimental results. They seem to be independent of the polarization.

**Rotating cam.** Let us consider a distance where the topological charge is unambiguously known, for example  $d = 2.5 \text{ m}$ , where  $\ell = -1$ . The estimated phase variation is displayed on Fig. 2b. Let us now rotate the diffracting object by a given angle  $\theta$ . Then, the phase distribution of Fig. 2b rotates accordingly. The azimuthal dependence

Table 1: Experimental values of the topological charge  $\ell$  and of the rotational Doppler shift  $\Delta\nu$  for different distances  $d$  between the occulting disk and the detection system. They correspond to the calculations of Fig. 2, and to a rotation frequency of the object of  $\nu_r = 9.0 \pm 0.1$  Hz. The main uncertainty of the results comes from the accurate determination of the rotation frequency of the object. The results are in agreement with the formula  $\Delta\nu = \ell\nu_r$ .

$d(\text{m})$	0.9	1.5	2.5	6.7
$\ell$	-3	-2	-1	0
$\Delta\nu(\text{Hz})$	$27.1 \pm 0.3$	$18.0 \pm 0.2$	$9.0 \pm 0.2$	$0.0 \pm 0.1$

now simply writes  $\exp(i\ell(\phi + \theta))$ . It is nothing but a shift of the azimuthal angle origin. As the object is rotated at a constant angular velocity  $2\pi\nu_r$ , at a given time  $t$ , the rotation angle equals  $\theta = 2\pi\nu_r t$ . Thus, the electromagnetic field has an azimuthal dependence  $\exp(i\ell(\phi + 2\pi\nu_r t))$ . Let us now separate this formula into the product of the azimuthal variation  $\exp(i\ell\phi)$  due to the OAM nature of the beam, and a time-dependent term  $\exp(i\ell 2\pi\nu_r t)$

$$\exp(i\ell(\phi + 2\pi\nu_r t)) = \exp(i\ell\phi) \exp(i\ell 2\pi\nu_r t) \quad (1)$$

This last term could be included in the time variation term of the electromagnetic field  $\exp(-i\omega t)$ , leading to a time oscillation  $\exp(-i(\omega t - \ell 2\pi\nu_r t))$ . This is nothing but a shift of the light frequency

$$\Delta\nu = -\ell\nu_r \quad (2)$$

which is the usual rotational Doppler shift formula [14]. Since the integer number  $\ell$  varies along propagation, as previously discussed, the frequency of the spot must vary accordingly, in a discrete way, depending on the  $\ell$  value.

Could it be verified experimentally? To this purpose, we collect part of the diffracted spot with a self-focusing lens and inject it into an optical fibre. We take advantages of existing elements such as couplers, attenuators and acousto-optic shifters at telecom wavelengths to perform a heterodyne measurement of the frequency shift. More precisely, the collected light is coupled to a reference light coming directly from the laser source thanks to a 50/50 coupler. Its intensity could be attenuated to match the collected light intensity. Besides, this reference beam is frequency shifted by two acousto-optics modulators by a fixed 70 Hz quantity. Thus, the beat frequency should equal to the frequency shift of the diffracted beam plus 70 Hz. Then, the mixed light is sent to a photodiode and an oscilloscope, looking for a beat frequency. We then perform a Fourier transform of the beat signal to isolate a single frequency (see Fig. 3b). We finally subtract the 70 Hz offset to find the rotational Doppler frequency shift (see Table 1), including its sign.

The results are in good agreement with the expected value. By blocking the reference beam, we check on the detected intensity, that for  $d > 6.5$  m,  $d = 2.5$  m,  $d = 1.5$  m and  $d = 0.9$  m, the residual intensity modulation is

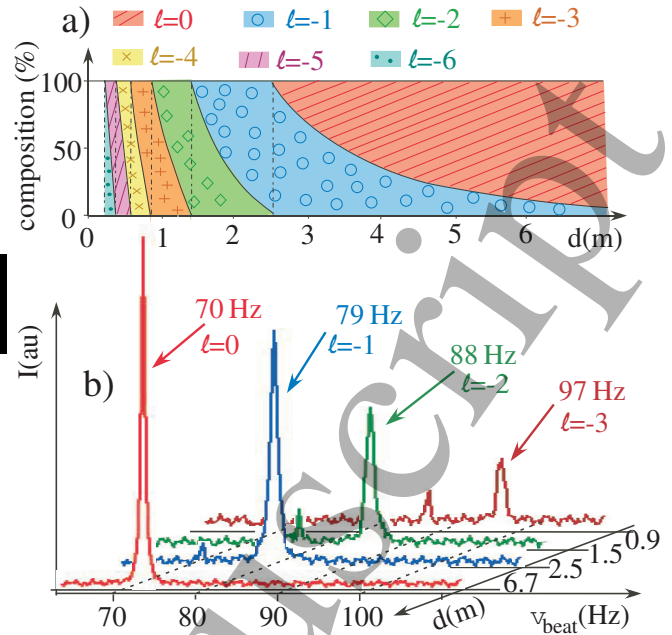


Fig. 3: Decomposition of the diffracted beam. a) Calculated distribution of the amplitude of the different modes in the decomposition of the diffracted beam, versus the distance  $d$  from the object. The dotted vertical lines correspond to positions where  $\ell$  is well-defined. The experimental measurements of Table 1 have been performed at these positions. b) Fourier transform of the experimental beat signal between the spot light and a reference beam at different distances from the rotating disk. Rotating frequency of the object:  $9.0 \pm 0.1$  Hz; fixed frequency shift of the reference beam due to the acousto-optics: 70 Hz.

negligible. Thus, the beat signal cannot be attributed to an intensity modulation of the collected light. Obviously, the frequency of the spot changes along propagation in a discrete way. As the rotation frequency  $\nu_r$  is changed, the shift varies linearly according to Eq. 2 (see also Fig. 4). Note that the picture of the object corresponds to the object used in the experiment. Such behavior can be found as soon as the diffracting object is asymmetric and generates fields carrying OAM [11]. As the object is rotated clockwise (in the other direction), the light frequency decreases. The sign of the shifts is changed as the object is reversed.

**Discussion.** – Light that changes frequency along propagation may have several applications. It may be adapted to any wavelength, including radio waves, and both the size and the rotation velocity of the object can be modified to match any situation. Using very narrow filters, this kind of frequency varying-beam would enable to address light in very specific and limited places. In telecommunications, addressing a signal to a given area, would drastically reduce the energy density that may be detrimental to human electromagnetic compatibility [20]. Thus, it could significantly reduce electromagnetic pollution. It may also be a valuable tool in material processing

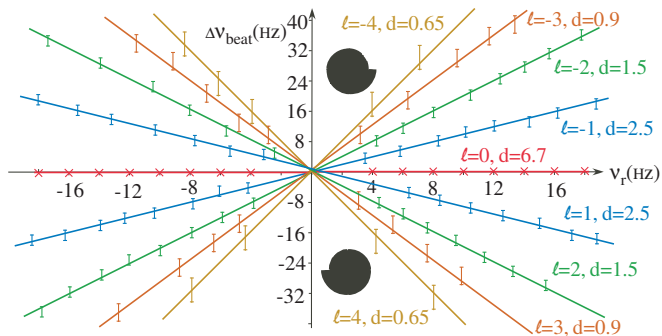


Fig. 4: Variation of the measured frequency shift versus the rotation frequency of the object for different distances between the detector and the object. A negative rotation frequency corresponds to a rotation in the other direction (clockwise). As the object is reversed, the sign of the topological charge of the diffracted beam is changed, as well as the Doppler shift.

or biomedical imaging to determine a well-defined zone and to avoid any residual absorption in sensitive, remote or hidden places [21, 22].

Cases of frequency-change during propagation are rare. To the best of our knowledge, the gravitational red shift is the only other one [23]. Nevertheless, this frequency variation may seem paradoxical here, since, at first sight, it violates the energy conservation law. The energy per photon equals  $h\nu$ ,  $h$  being the so-called Plank constant and  $\nu$  being the light frequency. This frequency increases or decreases along propagation, depending on the sense of rotation and on the orientation of the asymmetry of the cam.

However, regarded to the energy conservation in the half space after the cam, one must consider the whole electromagnetic field in a plane perpendicular to the direction of propagation. The central spot is only part of the total electromagnetic field in the shadow of the cam. For example, close to it, apart from the central spot, there are other components that form much less intense rings around this spot (see Fig. 2c, d and e). Part of these components interfere constructively on the axis at a longer distance from the cam and form the spot further on. Their frequencies are of course different from the one of the spot close to the cam. Analogously, at longer distances from the cam, there are also outer rings. They are formed with several components that interfere constructively close to the cam. They also have different frequencies from the one of the spot. Actually, all the frequency components are present in a plane perpendicular to the direction of propagation at a given distance. These components remain present whatever the distance, but with a different repartition distribution. Thus there is no violation of the energy conservation law. There is neither a violation of the OAM conservation law for exactly the same reason. All the components that lead to a given topological charge at a given position (and in a limited detection area), are present in the outer rings of the central beam.

Nevertheless, since the light frequency has changed during the interaction with the diffracting object compared with the incident one, there must have been energy exchanges with the diffracting object, due to the total energy conservation [24]. Actually, during the diffraction process, depending on the orientation of the cam and its sense of rotation, the cam should gain energy and the light should lose energy or vice versa. The components of the beam carrying OAM interact with the rotating diffracting object, leading to a work of the torque of the light beam [26]. Depending of the relative sign of the topological charge and the sense of rotation, the work could be positive or negative, independently of the laser polarization. This leads to a small variation of the rotational kinetic energy of the object. In the experimental situation depicted in Fig. 1, for a counter-clockwise rotation, the object should have lost energy. However, this energy lost is very small and is thus difficult to directly evidence experimentally. In the case where the object is actuated by light [11], it gains energy whereas the light loses energy.

**Concluding remarks.** – There is however a kind of paradox in the experiment described above. According to the theory of diffraction, in the frame rotating with the object, every point on the area surrounding a diffracting object is itself a source of spherical wavelets [16, 25]. Since wavelets are by definition spherical waves, they must have the same frequency as the incident beam, in the frame rotating with the object. In the laboratory frame, there may be some usual linear Doppler effect. However, since the rotation axis, the centre of the object and the optical axis are superimposed, there is no linear Doppler effect for any spherical wavelet detected on the optical axis. And yet, as the wavelets interfere on the optical axis forming a spot, the phase distribution of this spot is rotating as the object rotates. Besides, we have shown that the beat frequency with a reference beam varies in a *discrete* way along propagation, reflecting a *discrete* change of frequency of the propagating spot, which is obviously in contradiction with the wavelet propagation. It might be that the diffracted field in the rotating frame has components that carry OAM. These components experience a rotational Doppler shift in the laboratory frame. However, this intriguing observation may open some new discussions on the theory of diffraction, which is still an open and highly debated issue [27, 28], not only in optics [29, 30].

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