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Receivers placement for UAV localization in a surveillance area

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***Abstract* — Ensuring security of sensitive areas from unmanned aerial vehicles threats has become a major issue. In this context, multistatic radars present significant advantages over monostatic radars due to exploitation of spatial diversity and improved target detectability. In that perspective, sensors placement has an important impact in target detection or localization performance. In this paper, we propose an empirical method to place receivers in a multistatic radar network in order to cover an area of interest representing a sensitive site. The performance is measured with localization error estimation for different numbers of receivers and different target positions.**

***Keywords* — multistatic radar, CRLB, optimum receivers placement, MLE, UAVs localization**

I. Introduction

In recent years, Unmanned Aerial Vehicles (UAVs) have become very popular. Their proliferation leads however to the emergence of new threats. Due to their small size and low velocity flight capacity, UAVs detection and localization is very challenging to radar systems [1]. Therefore, ensuring security of sensitive areas from UAVs threats has become a major issue. Multistatic radar is a prominent choice for UAVs detection and localization for various reasons. One can cite for example its capacity to exploit targets Radar Cross Section (RCS) diversity [2], to support slow moving targets by exploiting Doppler estimates from multiple directions [3], or for the high resolution target localization it features [4].

One issue in multistatic radar is the sensors placement. This placement has an impact on the target detection performance and on the target localization accuracy. Improving this accuracy by the sensors placement can be addressed either by considering the output Signal to Noise Ratio (SNR) [5], the angle of arrival [6], the hyperbolic localization by time difference of arrival measurements [7], the elliptical localization by time of arrival measurements [8], or the variance minimization of range and Doppler shift estimates [9]. The placement of multiple receivers is optimized depending on the target position and velocity [9], or depending on the target trajectory [10]. However, as far as we know, no study addresses the placement problem in case of an entire area to be covered.

The contribution of this paper consists of receiver placements to cover a given surveillance area. We propose a method to define how many receivers are needed and where to deploy them in a multistatic radar network in order to meet the localization performance requirement. This method is based on the Cramér-Rao Lower Bound (CRLB) of the target position and velocity estimator. This paper is organized as follows. Section II provides the model description and the theoretical tools on which the method is based. Then, the receivers deployment method is described in Section III. Simulations and results are presented in Section IV. Finally, conclusion and perspectives are discussed in Section V.

II. Model description

The receiver placement is derived with a geometrical configuration of the radar network, consisting in the possible positions of the transmitter, the targets and the receivers in the considered area. This placement is also based on a given waveform leading to particular CRLB of the target parameters, used to define optimality of the receivers placement. In this article, the 3D case is not addressed.

A. Geometrical configuration

A 2D active multistatic radar network consists of one transmitter and N omnidirectional receivers. The receivers are assumed to be all constrained at a same fixed distance R_r from the center of the site under surveillance, as illustrated in Fig. 1. The possible receivers locus is then a circle. The position of the nth receiver is defined by the angle ϕ_n with respect to the x-axis and the center of the site, with ϕ_n sorted such that $\phi_1 \leq \phi_2 \leq ... \leq \phi_N$. The surveillance area is modeled as a disk centered at the site center, with a fixed radius R_a such that $R_a < R_r$. The target position is any point in this disk. Let's introduce $\boldsymbol{\theta}$ as the target parameters vector defined by $\boldsymbol{\theta} = (p_x\, p_y\, v_x\, v_y)^T$, with p_x and p_y the Cartesian coordinates of the position, v_x and v_y the velocity coordinates in the 2D-plane. The distance of the center of the site from the transmitter is $\sqrt{\Delta x^2 + \Delta y^2}$.

B. Signal model

The studied scenario is assumed to be multipath free. Each receiver of the network receives a backscattered signal which is a noisy, scaled, time delayed and Doppler shifted version of the transmitted signal with complex envelope $u(t)$. The backscattered signal to the nth receiver is expressed in baseband by:

$$s_n(t) = Au(t - \tau_n)e^{2\pi i f_n t} + w_n(t) \tag{1}$$

where A is the channel attenuation including the target RCS, τ_n and f_n respectively its delay and Doppler shift, and $w_n(t)$

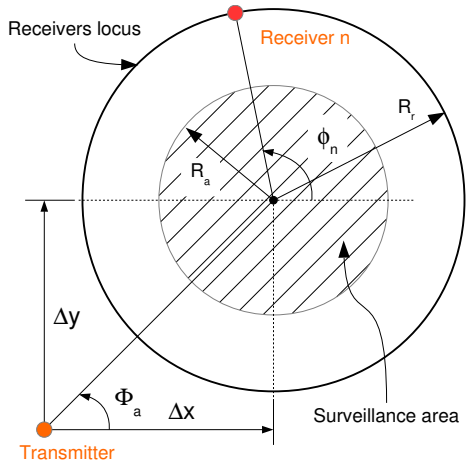


Fig. 1. Multistatic configuration.

is a complex zero mean additive white Gaussian noise. In our study, the target RCS behaviour is considered as isotropic.

Linear frequency modulation (LFM) pulses are commonly used in radar due to the great bandwidth they offer while keeping the pulse duration short and the envelope constant. The LFM is the signal waveform used in this article. This waveform is characterized by its four conventional parameters: T the pulse length, T_{PRI} the pulse repetition interval, M the number of pulses and B the bandwidth.

C. Delay-Doppler estimation for receiver placement

The ambiguity function of the LFM $u(t)$ is used to estimate the delay-Doppler that characterizes the target. With one receiver n identified by its position ϕ_n, the minimum variance unbiased estimations of τ_n and f_n are given by the CRLB of the joint estimation [9] [11]:

$$\sigma_r^2 = \frac{c^2}{d}(T^2 + T_{\text{PRI}}^2(M^2-1)),\ \sigma_f^2 = \frac{B^2}{d},\ \sigma_{rf} = \frac{cBT}{d} \quad (2)$$

with $d = \frac{1}{3}\pi^2 B^2 T_{\text{PRI}}^2(M^2-1)$ and c the light velocity. Assuming Gaussian error, the corresponding measurement model achieved by the MLE at high SNR regime is [9]:

$$\hat{\tau}_n = \tau_n(\boldsymbol{\theta}) + e_{\tau_n} \text{ and } \hat{f}_n = f_n(\boldsymbol{\theta}) + e_{f_n} \quad (3)$$

where the covariance matrix of the error is:

$$E\left[(e_{\tau_n} e_{f_n})^T (e_{\tau_n} e_{f_n})\right] = \frac{1}{\text{SNR}_n}\begin{pmatrix} \sigma_r^2 & \sigma_{rf} \\ \sigma_{fr} & \sigma_f^2 \end{pmatrix} \quad (4)$$

and SNR_n is the SNR at the receiver n.

With one receiver, the CRLB depends on θ, ϕ_n and on the LFM parameters. It depends also on the geometry of the configuration through the bistatic delay, or range, and Doppler equations [12]. With N receivers, the measurement model combines N delay and Doppler estimations. The CRLB is then derived from the general Gaussian case [13], and depends on the N receiver positions $(\phi_1, \cdots, \phi_N)$. This CRLB depends on the given positions of the transmitter, the target and the receivers.

Many criteria can be used to optimize the receivers placement [14]. In this study as in [9], the trace of the CRLB is adopted as the criterion to define receivers placement when a single target configuration is considered, that is for a given position and velocity of the target. The trace of the CRLB, as a function of the receiver positions, is then minimized, leading to the optimal receivers positions, according to the criterion used. However, these optimal positions depend on the target parameter $\boldsymbol{\theta}$ and do not ensure optimality for the whole site under surveillance. Instead of the complex N-dimension optimization problem, the proposed algorithm that solves the placement of N receivers is presented in the following section.

III. Receivers placement method

The presented method aims at placing N receivers in order to cover the localization of any target present in the surveillance area. The receivers placement procedure is synthetically described in Algorithm 1 and is organized in 3 parts:

1) lines 1 to 6, the distribution of the optimal positions of $N = 2$ receivers is first evaluated;
2) lines 7 to 11, the quantile regression is applied to set the positions of $N \geq 2$ receivers;
3) lines 12 to 17, the RMSE of the asymptotic MLE of $\boldsymbol{\theta}$ is computed to evaluate the performance of the proposed algorithm.

Algorithm 1 Receivers placement and validation algorithm.

1: Set $N = 2$, $K \in \mathbb{N}$
2: **for** $j = 1$ **to** K **do**
3: Generate the parameter $(\boldsymbol{\theta})_j$ of the target
4: $\{(\phi_1)_j, (\phi_2)_j\} \leftarrow \arg\min \text{trace}(\text{CRLB}(\boldsymbol{\theta})_j)$
5: **end for**
6: Compute PDF_i and CDF_i of ϕ_i, $i \in \{1, 2\}$
7: Set $N \in 2\mathbb{N}$
8: **for** $i = 1$ **to** $N/2$ **do**
9: $\phi_i \leftarrow \text{CDF}_1^{-1}\left(\frac{i}{N/2+1}\right)$
10: $\phi_{i+N/2} \leftarrow \text{CDF}_2^{-1}\left(\frac{i}{N/2+1}\right)$
11: **end for**
12: **for** $j = 1$ **to** K **do**
13: Generate the parameter $(\boldsymbol{\theta})_j$ of the target
14: $(\hat{\boldsymbol{\theta}})_j \leftarrow \text{MLE}\big((\phi_1)_j, \cdots, (\phi_N)_j\big)$
15: $\varepsilon_j^2 \leftarrow \|(\boldsymbol{\theta})_j - (\hat{\boldsymbol{\theta}})_j\|^2$
16: **end for**
17: Output the error and the RMSE

The first part of the procedure aims at providing empirical Probability Density Function (PDF) and Cumulative Distributive Function (CDF) of optimal positions of $N = 2$ receivers, for a set of K targets mapping the surveillance area. The algorithm starts with determining optimal locations of $N = 2$ receivers for each given target located in the surveillance area, with optimality understood as the minimization of the trace of CRLB, as introduced in

Section II-C. The process of determining the 2 receivers placement is then repeated for $K = 1575$ targets uniformly distributed in the surveillance area. Thus, each processed target yields pairs of receivers positions ϕ_1 and ϕ_2 that are collected to produce the PDF and CDF of each of the receivers positions, denoted by PDF_i and CDF_i for the ith receiver, $i \in \{1, 2\}$. Note that because of the symmetry of the geometrical configuration with respect to the transmitter-site axis, PDF_1 and PDF_2 distributions are symmetrically arranged with respect to the angle Φ_a defined by the transmitter-site axis.

In the second part of the algorithm, the placement of $N \geq 2$ receivers is based on the PDF and CDF obtained with $N = 2$. Yet, due to the PDF_1 and PDF_2 symmetry, the case of N receivers placement is considered only for N even. In each of the two PDF_i distributions, $N/2$ angular positions are selected. The $(N/2 + 1)$-quantiles is the used criterion to select the $N/2$ receivers angular positions in each CDF_i distribution. For $N = 4$, these are 3-quantiles (or tercile) that are considered to determine the 2 receivers placement in each PDF distribution, PDF_1 and PDF_2. The sets of N optimal receivers positions are thus defined. Note that the optimality of the receivers position applied to a covered area is defined here in the sense of quantile regression and should not be confused with the optimality as defined in Section II-C when considering a single target configuration.

It is worth noting that target localization with one transmitter usually requires at least 3 receivers, as in the localization by elliptical intersection. However, when additional assumptions are available, one may be able to disambiguate incorrect candidate intersections with 2 receivers only. As in the configuration, such conditions are met when the distances from the target to each of the 2 transmitters are of the same order. Due to the limitation of space, the demonstration of this result is not presented in this paper.

Finally, in the third part of the algorithm, the performance of the N receivers positions obtained in the second part or the algorithm is then evaluated with the error and the RMSE of the estimated parameters $\hat{\boldsymbol{\theta}}$. To this end, the surveillance area is mapped as in the first part by a set of K targets uniformly distributed within this area. The asymptotic MLE is performed for each target configuration. This estimator is based on the asymptotic parameters estimation given by (2) for each receiver.

IV. Results

In order to materialize the targets in the surveillance area, Monte Carlo simulations are done with target positions uniformly distributed in the area. The targets velocities are also generated with a uniform distribution from 0 to a fixed maximum velocity set at 5.6 m/s as an example and with a uniform distribution of the direction from 0 to 2π. The transmitter power is tuned such that the SNR, as the reference value, is set to 20 dB at a distance of 1000 m from a target placed at the center of the surveillance site. For other target placement, the SNR is evaluated regarding this reference, the target position, and the receiver position. Table 1 provides numerical values of the multistatic network configuration. The Nelder-Mead simplex method is used both for the minimization of the trace of the CRLB, and for the asymptotic MLE.

Table 1. Model Parameters.

Parameter	Value
Area center position, (Δ_x, Δ_y)	(2000 m, 3000 m)
Transmitter-area angle, (Φ_a)	236°
Surveillance area radius, R_a	150 m
Maximum target velocity	5.6 m/s
Receiver to area center distance, R_r	1000 m
Bandwidth, B	1 MHz
Pulse Repetition Interval, T_{PRI}	100 µs
Number of Pulses, M	16
Pulse Length, T	50 µs
Wavelength, λ	0.21 m

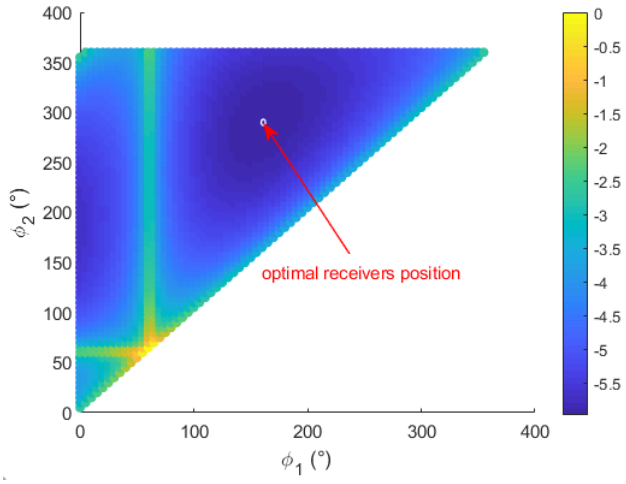


Fig. 2. Normalized trace of CRLB (dB) for one given target configuration.

According to Algorithm 1, the first step is dedicated to calculate the positions $(\phi_1)_j$ and $(\phi_2)_j$ regarding the jth target position. As an example, Fig. 2 displays the trace of CRLB for all combinations of ϕ_1 and ϕ_2 with $\phi_1 \leq \phi_2$, for a given target configuration $\boldsymbol{\theta} = (1894 \text{ m}, 3000 \text{ m}, 1 \text{ m/s}, 2 \text{ m/s})^T$. This target configuration yields $(165°, 295°)$ as optimal positions $(\phi_1)_j$ and $(\phi_2)_j$, obtained with the lower trace of the CRLB. These optimal positions and the ones obtained for each random target configuration are used to get the empirical PDF_1 and PDF_2 presented in Fig. 3. The optimal receivers positions are not scattered everywhere from 0 to 2π, but are structured as two clustered PDF distributions. As expected, PDF_1 and PDF_2 are symmetrically arranged with respect to $\Phi_a = 236°$. This cluster shape of the distribution can be explained: recall that in the case of a single target, the receivers placement would lead to a single position of each receiver. Even though, there are multiple targets velocities and positions mapping the surveillance area, when the receivers distance to the site is far greater than the surveillance area dimension, this latter is then viewed by the receivers as a single point. This can be seen as the extreme case of a clustered pattern.

Table 2 presents the sets of receivers positions obtained

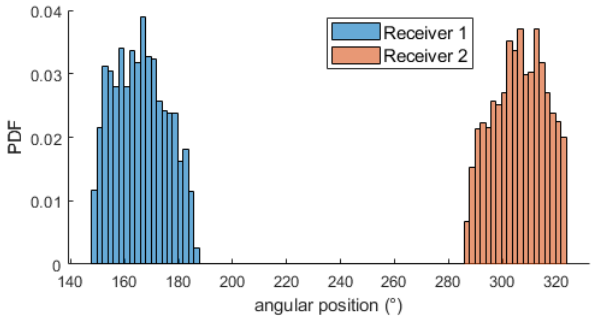


Fig. 3. PDF_1 and PDF_2 of 2 receiver positions.

Table 2. Optimal angular positions and RMSE of N receivers.

	Receivers positions						RMSE	
N	1	2	3	4	5	6	$\|\|p\|\|$ (m)	$\|\|v\|\|$ (m/s)
2	166°	306°					12.2	5.4
4	160°	170°	302°	312°			8.8	3.8
6	158°	166°	173°	299°	306°	314°	7.0	3.0

using quantile regression on CDF_1 and CDF_2. This table shows that the RMSE decreases with the number of receivers used to cover the surveillance area, leading to an increase in accuracy of the target parameters estimation. Fig. 4 plots the set of the error ε_j of each target parameter, for each set of $N \in \{2, 4, 6\}$ receivers positions. It exhibits the fact that the accuracy of the position and velocity estimations also increases with the number of receivers. The maximal value of the position errors can be compared to the range resolution of the LFM, which is $\frac{c}{2B} = 150$ m: the range resolution is then increased for all the target positions within the surveillance area.

The method can be generalized for higher number N of receivers. According to the minimum estimation accuracy fixed by an acceptable RMSE for the target parameters, N can be deduced, hence fixing one of the sets of N receivers positions. The question of how many receivers to deploy and where to deploy them is thus addressed.

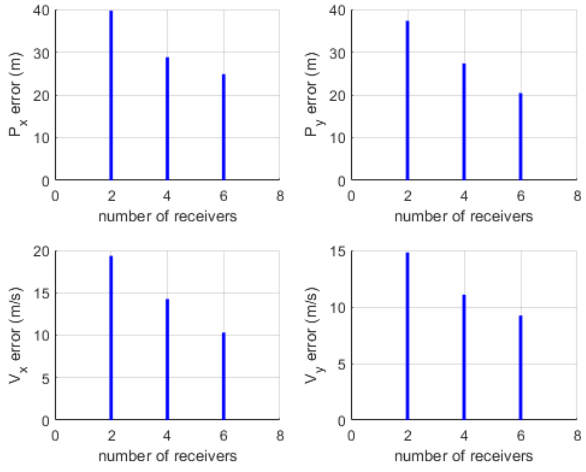


Fig. 4. Error of target parameters estimation.

V. Conclusion

In this article, we propose to study a multistatic radar system for the purpose of detecting UAVs inside a sensitive area. The receivers placement issue is addressed through an empirical method considering a fixed transmitter and several receivers around the area to be covered. We present a method to select the number of receivers and to place them according to the maximum admissible RMSE on target parameters estimation. This method exploits the CRLB of the position and velocity estimation of the target, and defines the optimal receivers position according to a quantile regression.

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