Reduced-Order Models for Fast Antenna Characterization

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***Abstract*—A reduced order model (ROM) for antenna characterization problems is proposed. It exploits the outer dimensions of the antenna under test (AUT) and the geometry of the measurement surface scan and leads to a significant reduction of the required number of field sampling points and therefore time to measure antenna radiation patterns. The inputs of the ROM are the equivalent currents enclosing the AUT and the outputs are their corresponding radiated fields on the measurement surface. By performing the singular value decomposition (SVD) of the radiating operator, we derive the minimal order model of the system and thereby numerically construct the basis of the fields radiated by the AUT. The evaluation of the so-reduced model is expedited by using a discrete empirical interpolation method (DEIM) that returns the sampling positions of the radiated field. The approach is tailored to the antenna characterization problem and specifically the antenna shape and the measurement surface scan. The proposed methodology is general, it can be easily adapted to any type of radiating structures and shape of the field measurement scans. Two experimental results of complex radiating structures measured in near and far field demonstrate the interest and potentialities of the approach.**

Keywords: antenna measurements, antenna radiation patterns, reduced order systems, non uniform sampling.

I. Introduction

The aim of model order reduction is to lower the computational complexity of mathematical models describing real-life processes. Reduced-order models (ROMs) are typically used to replace complex systems, e.g. dynamical or control systems, by a simpler one while preserving the input-output relationship. In electromagnetics, ROMs have been recently successfully applied for scattering problems to quickly analyze the interaction of a scatterer with other scatterers or antennas [1]–[3]. The idea is to model a scatterer by precomputing (offline) the response to a set of excitations. The response to an arbitrary excitation boils down to a weighted summation of these precomputed responses. For electromagnetic scattering problems, the ROM can be seen as a generalization of the spherical waves used for spherical scatterers [4] and shares some similarities with the characteristic modes approach for arbitrarily shaped scatterers [5]. The complete methodology to construct and evaluate ROMs for scattering problems is described in [1]. The ROM is built by computing the randomized singular value decomposition (RSVD) of the interaction matrix between the radiator and the scatterer. Its evaluation is then expedited using the discrete interpolation method (DEIM).

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In this paper, we apply the ROM scheme proposed in [1] and adapt it to antenna characterization problems and more specifically antenna radiation pattern measurements. The investigated scenario is represented in Fig. 1. The inputs and outputs of the ROM are, respectively, the equivalent currents ($\mathbf{J}_{\mathrm{eq}}$ and/or $\mathbf{M}_{\mathrm{eq}}$) surrounding the antenna under test (AUT) on a surface S' and the radiated near or far field ($\mathbf{E}^{\mathrm{NF}}$ or $\mathbf{E}^{\mathrm{FF}}$) on a surface S. They are tailored to the antenna measurement setup at hand since S' surrounds the AUT and S corresponds to the measurement surface scan. From this geometry, we build the matrix that maps any current distribution on S' to the radiated field on S. We perform the singular value decomposition (SVD) of this matrix in order to extract a low rank approximation of the radiation operator and get the ROM of the antenna characterization problem. The discrete empirical interpolation method (DEIM) is then applied to identify the dominant equivalent currents from a reduced number of measurement points.

The proposed ROM approach enables a fast characterization of antennas by interpolating their radiated (near or far) field from a small number of measurement points. It exploits only readily available data, the outer dimensions of the AUT and the geometry of the measurement surface scan. This approach differs radically from previously proposed fast antenna characterization strategies. In [6] the insertion of a-priori information on the AUT combined to the suitable use of advanced numerical modeling tools enables to achieve an important under-sampling and therefore speed up the measurement time. In [7]–[9], the sparse representation of the electromagnetic field on general purpose basis functions, spherical harmonics, leads to a significant reduction of the required sampling points and a reduction of measurement time.

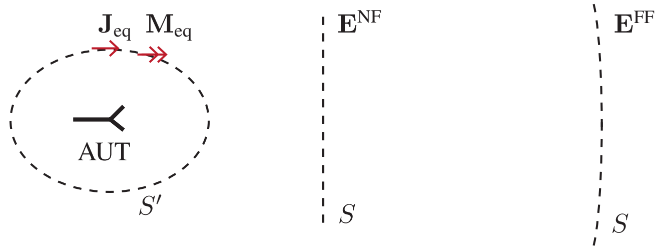

Fig. 1: Illustration of the antenna characterization problem. Equivalent electric and/or magnetic currents ($\mathbf{J}_{\mathrm{eq}}$ and/or $\mathbf{M}_{\mathrm{eq}}$) are placed on a surface S' enclosing the antenna under test (AUT). The near or far field ($\mathbf{E}^{\mathrm{NF}}$ or $\mathbf{E}^{\mathrm{FF}}$) is measured on a surface S.

The paper is organized as follows. The antenna characterization problem is formulated in Section II. The computation of the ROM and its evaluation are detailed in Section III and IV. The proposed ROM is applied in Section V to both near and far field experimental data in order to show its potentialities and interests.

II. Problem Formulation

The electric field radiated by the AUT can be expressed as a function of electric and/or magnetic equivalent currents (denoted $\mathbf{J}_{\text{eq}}$ and $\mathbf{M}_{\text{eq}}$ respectively) according to the surface equivalence theorem [10]. Later, a formulation of the equivalence theorem based on the use of only electric (magnetic) equivalent currents has been proposed [11]. The electric field, at a position $\mathbf{r}$ on the measurement surface S, is given from only the equivalent electric currents on S' by:

$$\mathbf{E}(\mathbf{r}) = \int_{S'} \overline{\mathbf{G}}_{\text{EJ}}(\mathbf{r}|\mathbf{r}') \cdot \mathbf{J}_{\text{eq}}(\mathbf{r}') dS' \tag{1}$$

where $\overline{\mathbf{G}}_{\text{EJ}}$ is the electric field free space dyadic Green function for electric sources. The expression of this dyadic can be found in many textbooks, for instance in [10]. Note that these equivalent electric currents $\mathbf{J}_{\text{eq}}$ are tangential to the surface S'.

After a proper discretization of the surfaces S' and S, the electric field can be expressed in the following matrix-vector form:

$$\mathbf{e} = \mathbf{G}\,\mathbf{j}_{\text{eq}} \tag{2}$$

where the vector $\mathbf{e}$ is the discretized electric field tangential to S, the vector $\mathbf{j}_{\text{eq}}$ gathers the discretized equivalent electric currents tangential to S' and $\mathbf{G}$ is the radiation matrix.

III. ROM Computation - Numerical Basis Generation

The input and output regions of our antenna characterization problem, S' and S respectively, are readily available. The equivalent current surface S' surrounds the AUT whereas S is the surface where the field is measured, see Fig. 1. These two surfaces are distinct which implies that the radiating operator $\mathbf{G}$ in (2) is compact. As a consequence, it is possible to compute the SVD of $\mathbf{G}$ [12]. This matrix can be approximated with a controlled accuracy by keeping only the singular vectors associated to the largest singular values and dropping those below a predefined error tolerance. The radiation matrix is, in general, not full rank since there are several sets of current distributions $\mathbf{j}_{\text{eq}}$ on S' can generate the same field $\mathbf{e}$ on S. The truncation of the SVD factorization enables to build a compressed representation, aka. ROM, of our antenna characterization problem:

$$\mathbf{G} \approx \mathbf{U}\mathbf{\Sigma}\mathbf{V}^{\dagger} \tag{3}$$

where only the R largest singular values σ_r are kept and $\mathbf{V}^{\dagger}$ is the Hermitian matrix of $\mathbf{V}$. The corresponding singular vectors are the R first columns of $\mathbf{U}$ and $\mathbf{V}$, denoted $\mathbf{U}_r$ and $\mathbf{V}_r$, they form the orthonormal basis of the equivalent current surface S' and the measurement surface S respectively. These singular vectors depends on the shape of the surfaces S' and S, they can be seen as the characteristic modes radiated by the apertures defined by these two surfaces. The singular values σ_r quantify the amount of power that is coupled from one characteristic mode of S' to S. This procedure enables to generate numerically the basis associated to the characterization problem geometry.

The application of the SVD factorization can be computationally expensive for large matrices. The radiation matrix $\mathbf{G}$ is large when the surfaces and S and/or S' are electrically large. To overcome this limitation and avoid having to form the full matrix $\mathbf{G}$ explicitely, the randomized SVD (RSVD) algorithm can be used [13]. It enables to obtain a predefined order approximation of the SVD factorization by taking (only) a collection of random input vectors (here current distributions) $\{\Omega_1, \Omega_2, \cdots\}$ to estimate the range of $\mathbf{G}$, ie. to span the range of fields radiated by the AUT. By increasing the number of random input vectors, the approach is proven to converge to the SVD [13]. The RSVD algorithm is used to efficiently compute the leading singular vectors of $\mathbf{G}$ and thereby numerically build the basis vectors of our problem. Details about the RSVD algorithm are provided in [13].

IV. ROM Evaluation - Sampling Point Determination

The goal of fast antenna characterization is to approximate the field radiated by the AUT on S from a small number of field samples. We know that the field $\mathbf{e}$ is a weighted combination of the previously constructed orthonormal basis formed by R vectors, ie. the columns the matrix $\mathbf{U}_r$:

$$\mathbf{e} \approx \mathbf{U}_r \boldsymbol{\alpha} \tag{4}$$

where $\boldsymbol{\alpha}$ is a vector of complex coefficients ($\boldsymbol{\alpha} \in \mathbb{C}^R$). The best approximation of α in the least square sense is $\alpha = \mathbf{U}_r^{\dagger}\mathbf{e}$. The length of $\mathbf{e}$, denoted M, is the number of measurement points that we want as small as possible so as to reduce the characterization time.

One way to reduce the number of field sampling points from M to R is to apply the discrete empirical interpolation method (DEIM) [1], [2], [14], [15]. This algorithm provides, from the basis $\mathbf{U}_r \in \mathbb{C}^{M\times R}$, a set of R interpolation points $\mathbf{e}_r \in \mathbb{C}^R$ to determine an approximation of $\boldsymbol{\alpha}$. The DEIM is explained and described in [1], it provides a matrix $\mathbf{Q}$ that selects R (among M) field samples to be measured: $\mathbf{e}_r = \mathbf{Q}^T\mathbf{e}$. The coefficients $\boldsymbol{\alpha}$ are then:

$$\boldsymbol{\alpha} = \left(\mathbf{Q}^T\mathbf{U}_r\right)^{-1}\mathbf{e}_r \tag{5}$$

Note that the DEIM is guaranteed to yield an invertible $\mathbf{Q}^T\mathbf{U}_r$ as shown in [15]. Finally, the field radiated by the AUT can be approximated from a small number R of sampling points as follows:

$$\mathbf{e} \approx \mathbf{U}_r\left(\mathbf{Q}^T\mathbf{U}_r\right)^{-1}\mathbf{e}_r. \tag{6}$$

The steps of the DEIM are given in [15].

V. Experimental Validations

The ROM procedure has been experimentally validated to speed up the characterization in both near and far fields of many antenna prototypes in various frequency bands. The examples of two radiating structures are here exposed.

A. Far Field Characterization

The AUT is a reflectarray of 193 cells (see Fig. 2(a)) of maximum dimension 8.8λ at 12 GHz that radiates a beam tilted in both planes as shown in 3(a). This antenna has been designed by Thales Alenia Space in the framework of the project R3MEMS.

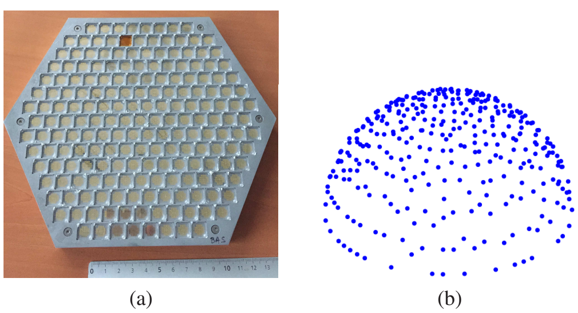

Fig. 2: (a) Picture of the reflectarray of maximum dimension 8.8λ at 12 GHz. (b) Far field sampling points (total number of 353) selected by the DEIM and distributed on the measurement surface S (half a sphere).

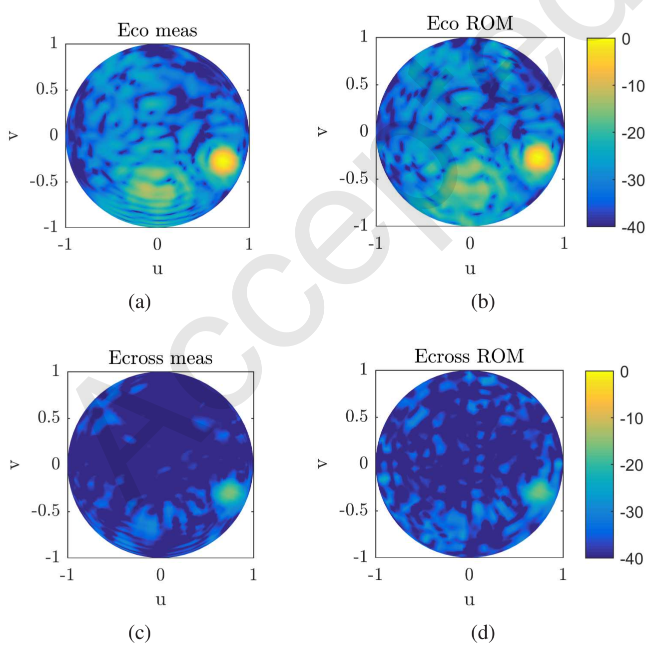

Fig. 3: 3D far field patterns (co- and cross- polarization) of the reflectarray at 12 GHz: (a,c) measured patterns and (b,d) patterns reconstructed using the ROM approach from a small number of sampling points.

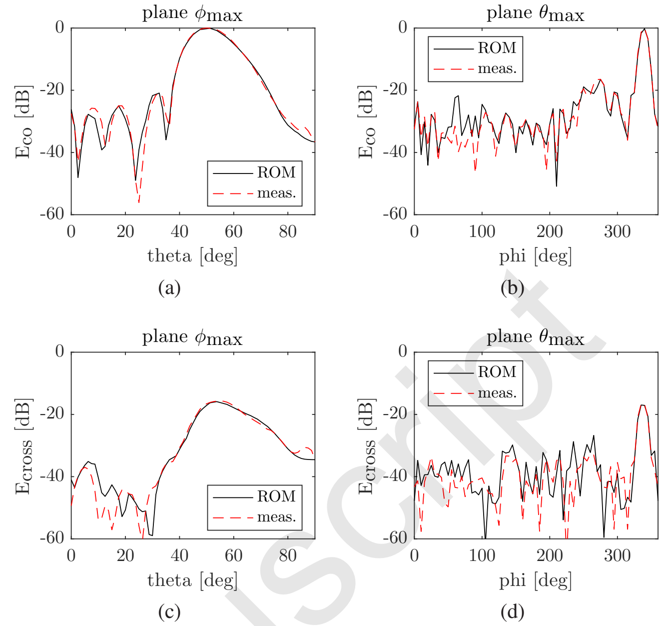

Fig. 4: 2D far field patterns (co- and cross- polarization) of the reflectarray at 12 GHz in the cutting planes corresponding to the maximum radiated field.

Equivalent electric currents are placed on a rectangle S' above the reflectarray aperture with a step size of 0.25λ. The reference radiation far field is measured with a very dense sampling (elevation and azimutal step of 0.5° and 1° respectively) on half a sphere (see Fig. 3(a,c)).
The RSVD is applied on the radiation matrix and the singular vectors associated to singular values above $\sigma_{\max}/10$, which corresponds to the "knee" in the singular value distribution, are kept to construct the numerical basis composed of 353 vectors.
The DEIM algorithm is then applied to select the 353 measured points that are plotted in Fig. 2(b). As a comparison, more than 800 measurements points were necessary to properly characterize the reflectarray radiation pattern with the sparse spherical harmonics approach proposed in [9]. This significant reduction, and therefore important gain in measurement time, is made possible because the proposed approach encompasses information about the geometry (a flat rectangular aperture) of the AUT. A more accurate description of the AUT, for instance a surface S' having the shape of an hexagon, would enable to even further reduce the number of required measurement points.
The agreement between the reference and the interpolated far field is very good despite the very small number of sampling points, as shown in Fig. 3 and 4. In terms of computation time, note that the whole procedure including the RSVD and DEIM takes less than 5s with a biprocessor 2.79 GHz-CPU 64 GB-RAM Xeon.

B. Near Field Characterization

The AUT is a metal-only metasurface that has been designed to generate a circular polarization in Ka band. All details about

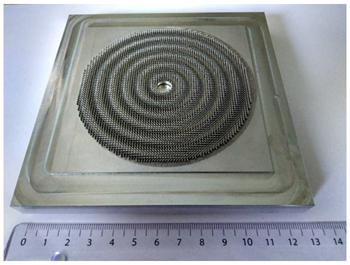

Fig. 5: Picture of the circularly polarized metasurface [16] of diameter 10λ at 32 GHz.

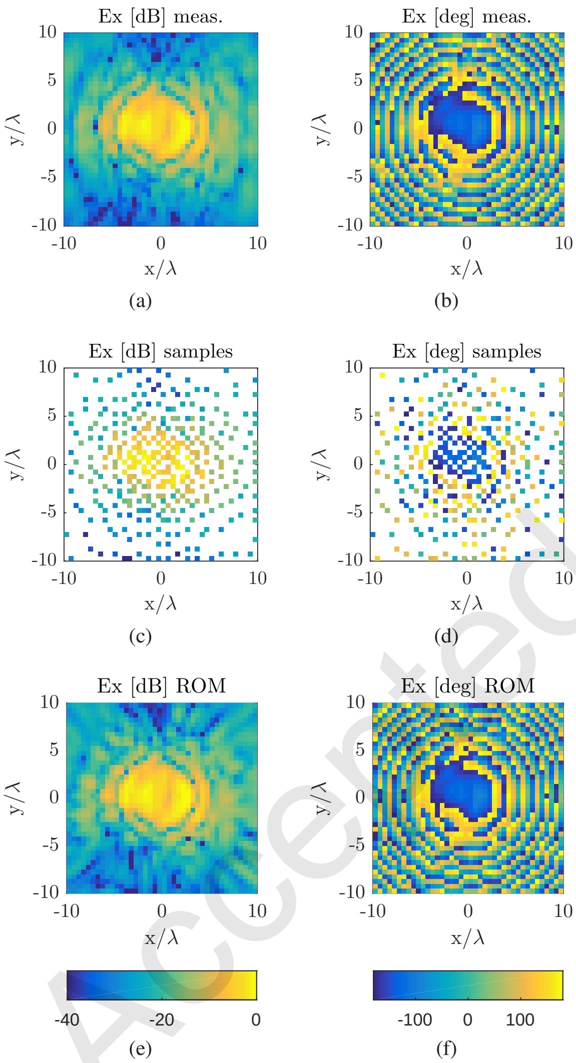

Fig. 6: Planar near field mappings (left (a,c,e) magnitude and right (b,d,f) phase) radiated by the metasurface at 32 GHz at a height of 5 λ. Near field mappings of (top (a,b)) the measured data, (middle (c,d)) the samples selected by the DEIM and (bottom (e,f)) the reconstructed field using the ROM approach.

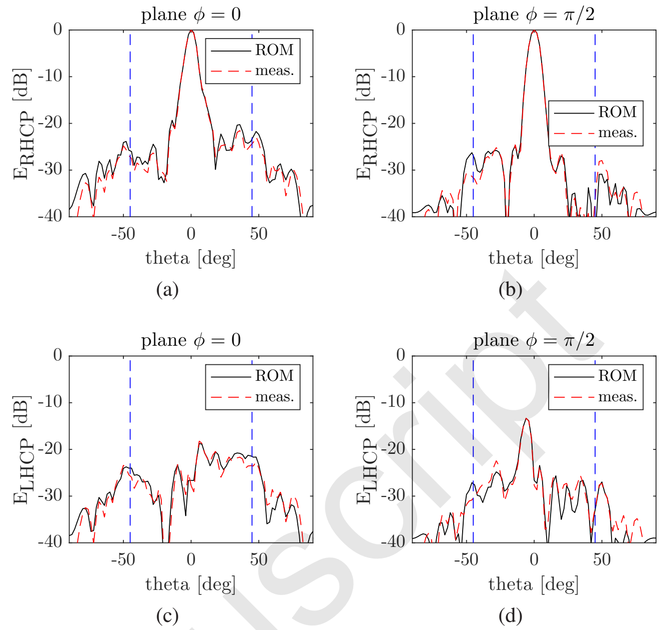

Fig. 7: 2D far field patterns (RHCP and LHCP) of the metasurface at 32 GHz derived from the measured near field and the near field reconstructed using the ROM approach. (a,b) show the RHCP component for the planes $\phi = 0$ and $\phi = \pi/2$ respectively. (c,d) show the LHCP component for the planes $\phi = 0$ and $\phi = \pi/2$ respectively. The vertical dashed lines represent the limits of validity of the far field reconstruction.

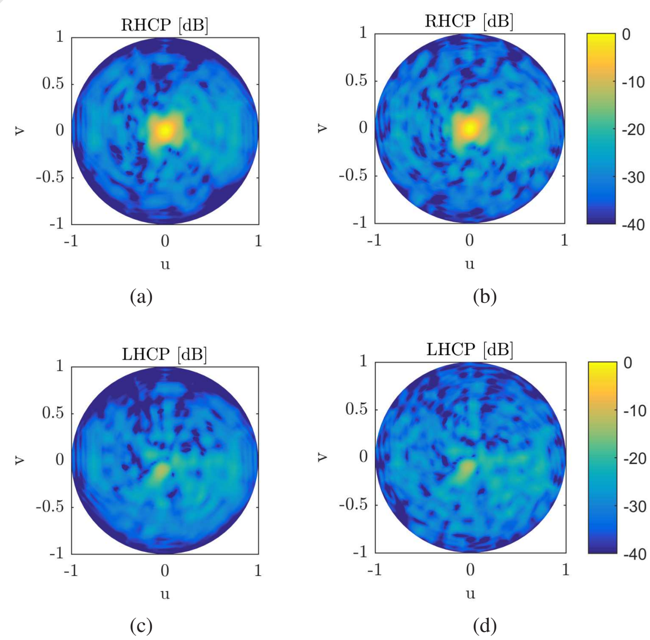

Fig. 8: 3D far field patterns (RHCP and LHCP) of the metasurface at 32 GHz derived from the measured near field (a,c) and the reconstructed near field (b,d).

the design and manufacturing of this radiating structure, shown in Fig. 5, are provided in [16].

Equivalent electric currents are placed on a square surface S' of side 10λ above the metasurface with a step size of 0.25λ. The metasurface is measured each 0.5λ at 32 GHz on a square surface S of side 20λ located at a height of 5λ. Considering the geometry of S and S', the far field patterns can be derived accurately from the near field measurements for angles between $\pm 45^\circ$.

The RSVD is applied and the singular values above $\sigma_{\max}/10^3$ are kept to construct the numerical basis that is composed of 358 vectors. The DEIM algorithm is applied to select only 358 measurement points, see Fig. 6(c,d). The number of near field points then drops from 1681 (with a standard 0.5λ step) to 358. Considering a regular distribution over the near field surface, it means that the average distance between the field samples is now of only 1.1λ. With our near field scanner, it takes 1h02min to measure both components of the field with a 0.5λ sampling step (Fig. 7(a,b)) whereas only 24min are necessary to collect the 358 selected measurement points (Fig. 7(c,d)). It means that a reduction of 60% of the field acquisition time is achieved in this case. The computation time of the whole procedure, RSVD and DEIM, takes less than 8s.

To assess the proposed ROM procedure, we reconstruct the near field radiated by the metasurface from selected measurements points and compare it to the measured complex near field. The mappings of the near field, magnitude and phase, are shown in Fig. 6(a,d) and (c,f) respectively. They are in very good agreement. To better estimate the quality of the field reconstruction from the ROM procedure, we derive the far field and, more specifically, right and left hand circular polarizations, denoted RHCP and LHCP respectively. An excellent agreement is obtained between the measured and reconstructed far field as shown in Fig. 7 and 8 despite the coarse near field sampling which validates the proposed ROM procedure.

VI. Conclusion

A reduced order model (ROM) is proposed for antenna characterization problems enabling a significant acceleration of antenna radiation pattern measurements. The AUT is enclosed by equivalent currents and the operator that maps these currents to the radiated field on the measurement surface is built. By computing the RSVD of this operator and keeping only the leading singular vectors, the ROM and specifically the orthormal basis tailored to our antenna characterization problem is constructed numerically. Note that the left and right singular vectors can be seen as the characteristics modes of the AUT and the measurement surface, respectively, whereas the singular values represent the coupling between these modes. The DEIM is then applied to select a small number of field sampling points from which the field radiated by the AUT can be properly interpolated. Two complex radiating structures have been characterized to demonstrate the efficiency and potentialities of the proposed approach. It has been found that the required number of sampling points, both for near and far field measurement configurations, can be significantly reduced as compared to previously proposed antenna characterization strategy. The ROM concept exploits only readily available information about the antenna characterization, the outer dimensions of the AUT and the measurement surface geometry. It is directly applicable to any kind of radiating structures and geometry of measurement scans and works both for near and far field characterization. Our procedure paves the way to a more efficient use of antenna near and far field measurement facilities.

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