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Recent Advances in Modeling and Simulation of Nanofluid Flows-Part II: Applications

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Abstract

Modeling and simulation of nanofluid flows is crucial for applications ranging from the cooling of electronic devices to solar water heating systems, particularly when compared to the high expense of experimental studies. Accurate simulation of a thermal-fluid system requires a deep understanding of the underlying physical phenomena occurring in the system. In the case of a complex nanofluid-based system, suitable simplifying approximations must be chosen to strike a balance between the nano-scale and macro-scale phenomena. Based on these choices, the computational approach-or set of approaches-to solve the mathematical model can be identified, implemented and validated. In part I of this review [1], we presented the details of various approaches that are used for modeling nanofluid flows, which can be classified into single-phase and two-phase approaches. Now, in part II, the main computational methods for solving the transport equations associated with nanofluid flow are briefly summarized, including the finite difference, finite volume, finite element, lattice Boltzmann methods, and Lagrangian methods (such as dissipative particle dynamics and molecular dynamics). Next, the latest studies on 3D simulation of nanofluid flow in various regimes and configurations are reviewed. The numerical studies in the literature mostly focus on various forms of heat exchangers, such as solar collectors (flat plate and parabolic solar collectors), microchannels, car radiators, and blast furnace stove coolers along with a few other important nanofluid flow applications. Attention is given to the difference between 2D and 3D simulation, the effect of using different computational approaches on the flow and thermal performance predictions, and the influence of the selected physical model on the computational results. Finally, the knowledge gaps in this field are discussed in detail, along with some suggestions for the next steps in this field. The present review, prepared in two parts, is intended to be a comprehensive reference for researchers and practitioners interested in nanofluids and in the many applications of nanofluid flows.

Keywords: Nanofluids, physical models, 3D modeling, CFD techniques

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1- Introduction

Although the study of thermal systems through experimental physics is always a valuable approach, it has some limitations and disadvantages. One of the drawbacks might be the difficulty in determining the contribution of various physical phenomena to the system efficacy. Moreover, high costs of experiments are often a barrier for doing in-depth experimental studies. Modeling and simulation of the desired system increasingly offer a suitable alternative to experiments. As mentioned in Part I of this review [1], experiments on nanofluids are sometimes prohibitive due to the high expense of nanofluid preparation, and information gained from such experiments is limited by difficulties in imaging at the nanoscale in flowing systems. These challenges motivate researchers to use computational fluid dynamics (CFD) techniques to simulate nanofluids-based systems, which can often be done at far lower expense and yield highly detailed data on system performance and its relationship to microscale physics. For modeling of nanofluids-based systems, it is critical that the dominant physical phenomena affecting the performance of the system are first determined, following which a suitable model to describe these phenomena can be selected. Part I of this review gave a comprehensive discussion of various physical phenomena responsible for heat transfer enhancement in nanofluid flows along with associated physical models. In part II of the review, the aim is to summarize the most popular computational fluid dynamics (CFD) techniques that are used for simulation of nanofluids. Following this, recent studies on 3D modeling of thermal systems employing a nanofluid as the working fluid are reviewed. Finally, gaps and challenges in modeling of nanofluid flows are presented and some proposals are given for future work.

2. CFD techniques for nanofluid flow solution

Generally, it is difficult to solve the nanofluid flow problems analytically because of the nonlinear nature of the governing equations. Therefore, to find the velocity and temperature fields, computational fluid dynamics (CFD) techniques may be used. In this part, the main CFD techniques that are used to determine the flow and heat transfer characteristics in nanofluid flows are briefly reviewed.

These techniques can be classified into three main groups as follows[2]:

- Macroscale based techniques
- Mesoscale based techniques
- Microscale based techniques

Figure 1 presents different CFD techniques for solution of nanofluid flows.

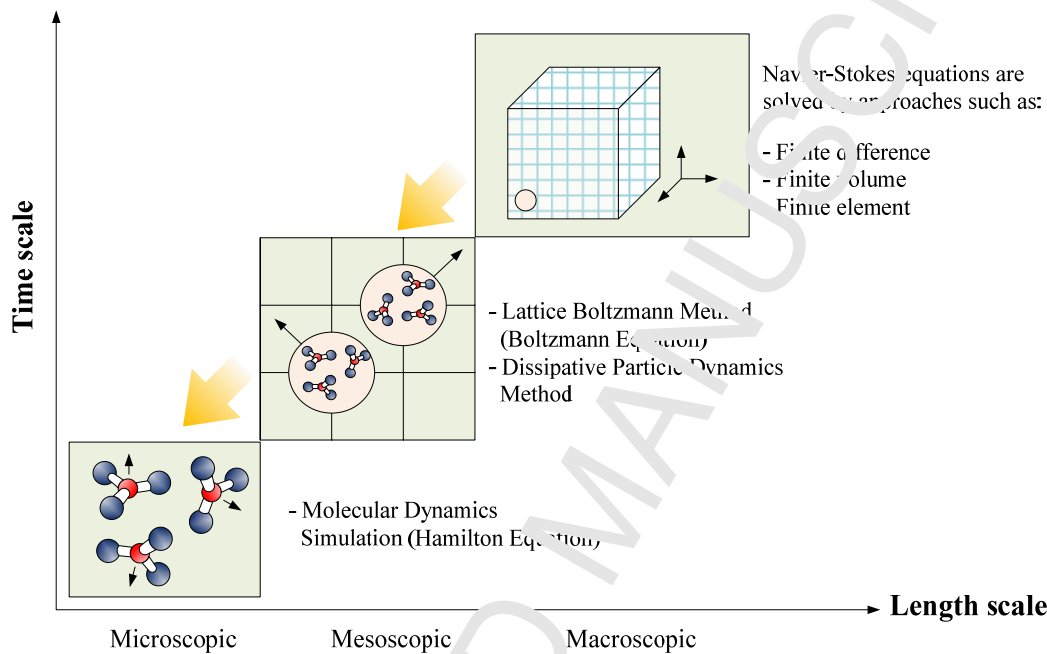


Figure 1. Different CFD techniques for solution of nanofluid flows.

Since the aim of the present work is mainly to focus on physical aspects of nanofluid flows, in the following just a brief description of each CFD technique is presented.

2.1. Macroscale based techniques

Macroscale based techniques such as finite difference, finite volume, and finite element deal with converting partial differential equations to algebraic equations through discretization. Here, the common macroscale based approaches are briefly reviewed.

2.1.1. Finite difference method

Euler was probably the first one who introduced “finite difference method” to solve differential equations in more than 200 years ago. Finite difference method (FDM) is the oldest and easiest numerical approach to solve the flow and heat transfer problems where the domain of solution is not complicated. To use this method, first, the governing equations should be converted (discretized) to algebraic form using the Taylor series (for example $(\frac{\partial u}{\partial x})_i$ becomes $\frac{u_{i+1} - u_{i-1}}{2\Delta x}$, where index of i stands for the node number in the x direction). Next, the computational domain should be divided into a series of nodes in each coordinate direction. Finally, the linear algebraic equations are solved by iterative techniques and the value of desired parameters are obtained at each node. The FDM is easy to use and effective on structured meshes. On the other hand, it is not as helpful an approach for complex flows and geometries[3,4], where it is generally required to discretize the flow field using a body-fitted grid in order to use FDM. For example, Astanina et al. [5] used FDM to simulate the mixed convection of nanofluids in a lid-driven porous cavity. Sheremet et al. [6] employed FDM to simulate natural convection of nanofluids in a lid-driven cavity where boundary conditions are as a function of temperature. Astanina et al. [7] studied MHD flow in a square cavity with variable properties of nanofluids using FDM. Using FDM, Ismael et al. [8] investigated the mixed convection of nanofluids in a square cavity where a heater was installed in one of its corners. Alsabery et al. [9] applied FDM to solve the conjugate natural convection in a square cavity where the temperature boundary conditions in top and bottom walls change as a sinusoidal function. Hassan and Harmand [10] used FDM to analysis the flow of nanofluids in a heat pipe under periodic heat load. Figure 2 displays schematic of some problems that have been solved by FDM.

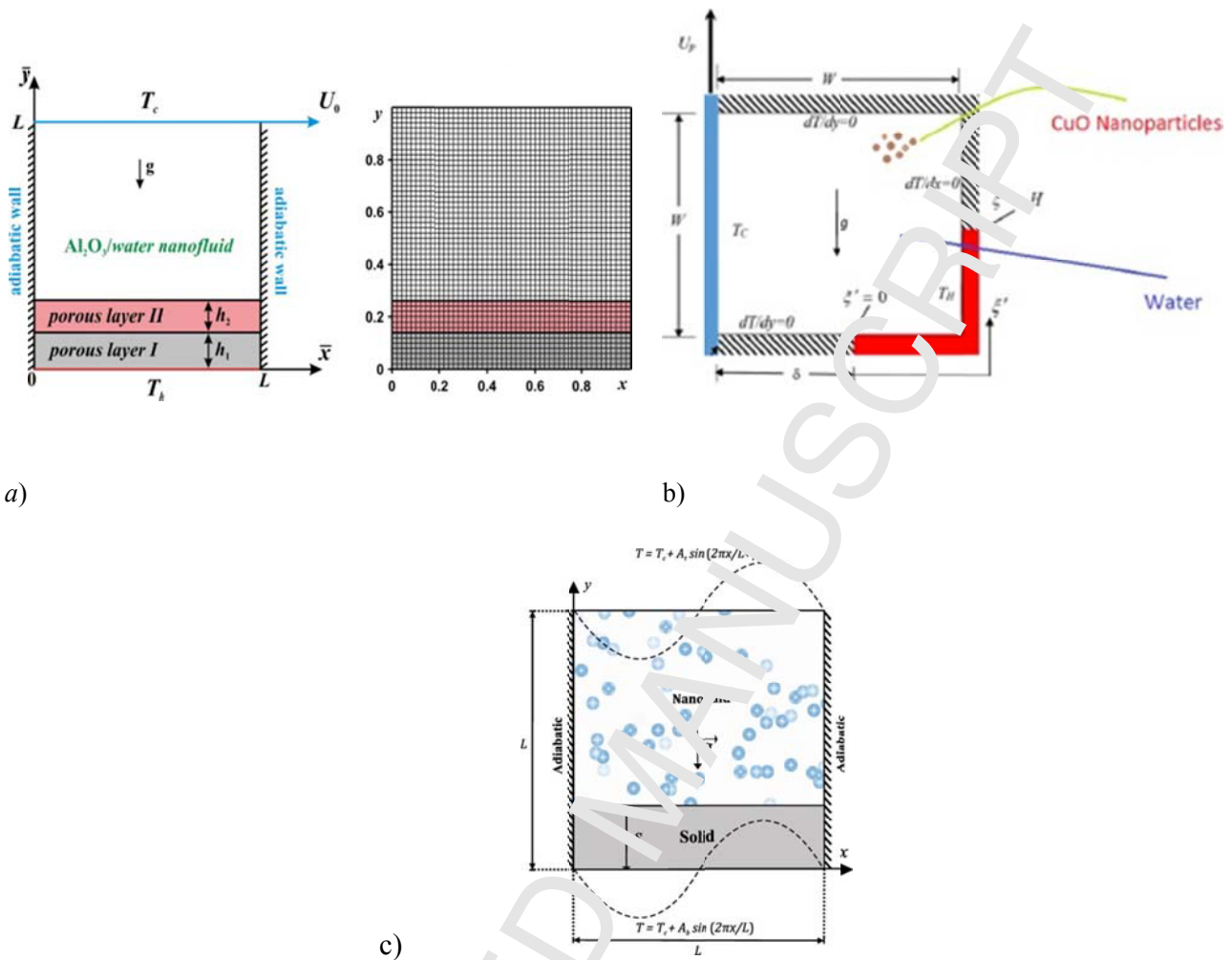


Figure 2. Schematics of some problems in which finite difference method is employed as the CFD technique (a) porous cavity [5], (b) cavity with a heater at the corner [8], (c) cavity with sinusoidal boundary conditions [9] (figures are reprinted with permission from publisher).

2.1.2. Finite volume method

The finite volume method (FVM) was introduced as an alternative to FDM to better handle complex flow domains. A review of the literature shows that FVM is the most common CFD procedure used in solving nanofluid flow problems. In this approach, first the differential equations are converted into integral form. For example, $\frac{\partial u}{\partial t} + \nabla \cdot f(u) = 0$, after integrating over any volume V , becomes

$\frac{\partial}{\partial t} \int_V u dx + \oint_{\partial V} f_i n_i ds = 0$ where n_i is the unit outward normal and s is the control surface. Next, the

domain of solution is divided into continuous control volumes, and the governing equation is applied to each of them. The values of desired parameters are calculated at the center of each control volume, then at the surfaces of control volumes the parameter values are estimated through interpolation. Finally, for each control volume, the achieved integral terms are converted to algebraic equations by using quadrature formulae. The finite volume approach is easy to understand because every term has a clear physical description [3,4]. The advantage of FVM is its flexibility for use in complex flows. On the other, one of its main disadvantage is the difficulty of extending this method for higher than 2nd order accuracy in 3D modeling.

As mentioned earlier, most numerical studies on nanofluids have used FVM, several examples of which are given in the following. Cho et al. [11] employed FVM to model the natural convection of nanofluids in a cavity with wavy walls. Ismael et al. [12] used FVM to simulate the nanofluid flow in a cavity under magnetic field by considering partial slip and heat flux on sidewalls. Mohammadian et al. [13] numerically simulated conjugate heat transfer of nanofluids in a microchannel heat exchanger under the laminar regime. They employed FVM and the single-phase model in their study. Tang et al. [14] employed FVM to model free convection of nanofluids in a cavity with double sinusoidal wavy walls where the Eulerian-Lagrangian technique was applied. Minea and El-Maghlany [15] solved the natural convection of an ionanofluid (a mixture of ionic liquids and nanoparticles) inside a square cavity under different sets of boundary conditions. They employed FVM and single-phase homogenous approach model with temperature-dependent properties in the simulation. In another work, Minea and El-Maghlany [16] employed FVM for modeling of a receiver of a parabolic trough collector. Figure 3 shows the schematics of abovementioned problems that have employed FVM.

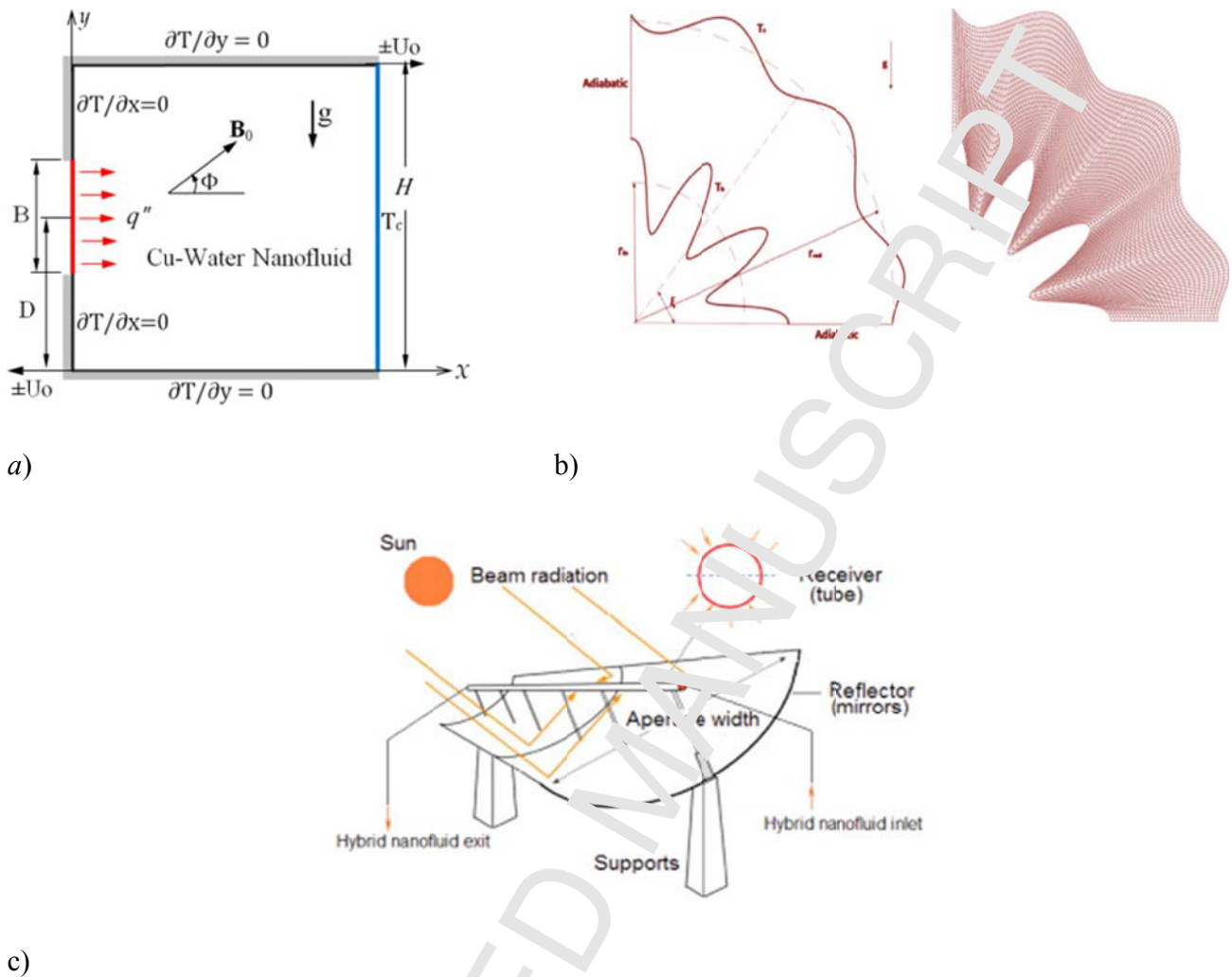


Figure 3. Schematics of some problems in which finite volume method is employed as the CFD technique (a) cavity with a heater under magnetic field [21], (b) cavity with double sinusoidal wavy walls [14] (c) parabolic trough collector [16] (figures are reprinted with permission from publisher).

2.1.2.1 Common FVM based Software

Two common FVM based software packages used for simulation of nanofluid flows are ANSYS FLUENT and ANSYS CFX. For example, Esfahani et al. [17] simulated nanofluid flow in a wavy channel using ANSYS FLUENT and the single-phase approach. Effects of various models for thermophysical properties on the forced convection turbulent flow in a tube was investigated by Minea

[18] where ANSYS FLUENT was used. Minea and Lorenzini [19] employed ANSYS FLUENT to solve natural convection of ZnO/water nanofluid in a cavity by assuming single phase homogenous model for nanofluid flow. Minea [20] utilized ANSYS FLUENT to analysis the variations of Brinkman number in a partially heated tube in which $\text{Al}_2\text{O}_3/\text{water}$ nanofluid was used as coolant. Chereches et al. [21] employed ANSYS FLUENT to simulate the flow of a nanofluid in a circular tube in both laminar and turbulent regimes while the flow was assumed to be single-phase. In another study, Kaloudis et al. [22] used ANSYS CFX to model the nanofluid flow in a parabolic trough collector by two-phase approach (Eulerian-Eulerian). Figure 4 shows schematics of the problems considered by [17,22]. Researchers are increasingly using open source software, such as OpenFOAM, which provides a C++ toolbox for solving nanofluid-based problems. The advantage of open-source software is both the reduced cost as well as the ease of adapting and modifying the software for specific problems. For example, Meng et al. [23] utilized OpenFOAM to model nanofluid flow in a square cavity using both single-phase and two-phase approaches.

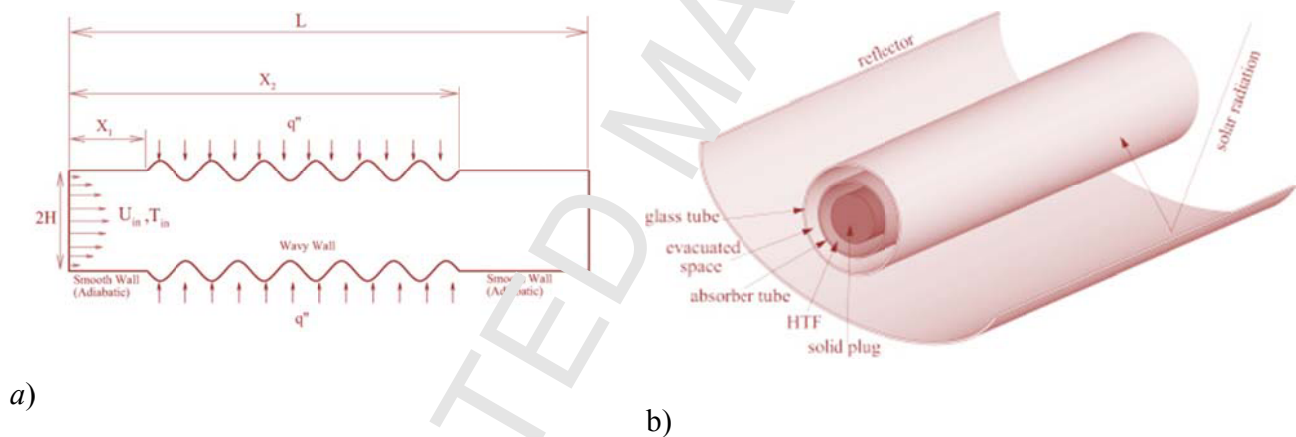


Figure 4. (a) Flow in a wavy channel solved by ANSYS FLUENT[17] (b) flow in a parabolic trough collector solved by ANSYS CFX[22] (figures are reprinted with permission from publisher).

2.1.3. Finite element method

In the finite element method (FEM), the domain of solution is divided into a series of finite elements (often unstructured meshes) and the governing equations are solved as weighted integrals over these elements. Before integrating the governing equations, each term is multiplied by a weight function, the use of which is one of the main differences between FEM and FVM. The final equations are in the

form of non-linear algebraic equations. The main advantage of FEM is its flexibility for dealing with complicated geometries[3]. One of disadvantages of FEM may be related to round off errors and error accumulation.

2.1.3.1. Common FEM based Software

Two FEM based software packages that have been used for nanofluid flow simulation are COMSOL Multiphysics and FlexPDE, although the former package is more commonly used.

For example, Nasrin et al. [24] used COMSOL to model the performance of a photovoltaic thermal (PV/T) system using nanofluids with the single-phase approach. They compared the results of the numerical simulation with experimental data and found good agreement between them. In another work, Bianco et al. [25] investigated thermal efficiency and entropy generation in a PV/T system using COMSOL where nanofluid flow was considered as a single-phase fluid. Gunjo et al. [26] investigated melting and solidification in a shell-and-tube regenerative type latent heat storage system where the working fluid was a mixture of paraffin and nanoparticles. They used COMSOL for the modeling by considering the temperature-dependent of properties (see Fig. 5 (a)).

Hatami and Jing [27,28] used FlexPDE to evaluate the performance of direct absorber solar collectors by nanofluids (see Fig. 5 (b)).

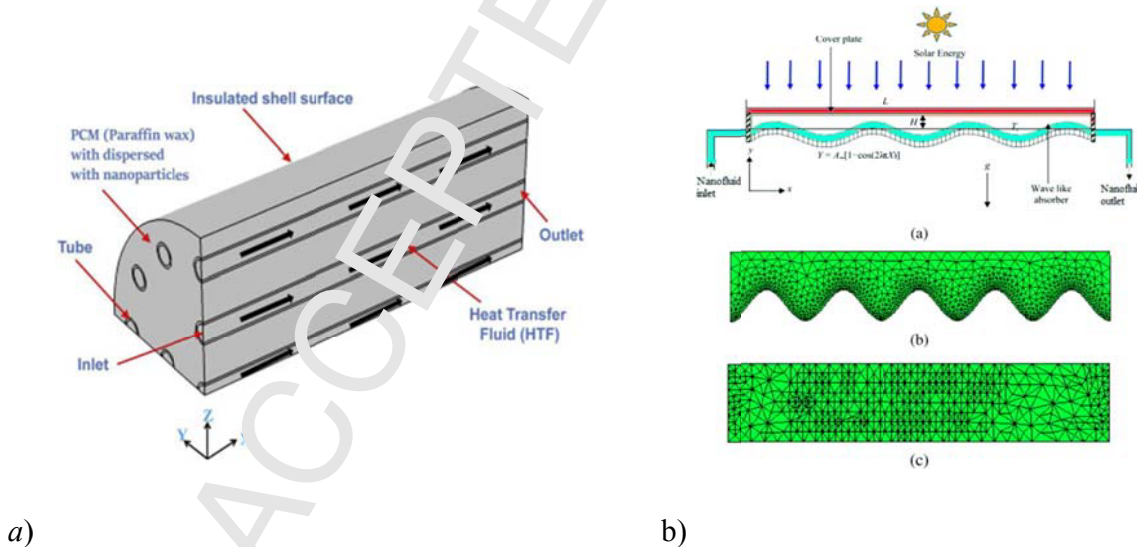


Figure 5. (a) Flow in a shell and tube regenerative type latent heat storage system solved by COMSOL Multiphysics[26] (b) analysis of a direct solar absorption collector by FlexPDE[27] (figures are reprinted with permission from publisher).

2.1.4. Other macroscale based techniques

There are some other macro scale based techniques that have been used to solve nanofluid flows which are briefly reviewed here.

2.1.4.1. Control Volume Finite Element Method

Control Volume Finite Element Method (CVFEM) is a powerful CFD technique to solve fluid flows in complex geometries. Indeed, CVFEM is a combination of FEM and FVM so that it has the benefits of these two techniques in itself [29]. Sheikholeslami is a pioneer researcher in the use of CVFEM to solve nanofluid problems (for example see [30–36]). Sheikholeslami and his colleagues have used CVFEM to solve flow and heat transfer of nanofluids in complex geometries by both the single-phase and two-phase approaches. For example, Sheikholeslami and Ganji [35] considered the geometry shown in Fig. 6(a) and investigated the effect of electric field on nanofluid flow in the presence of porous media and radiation. In another work, Sheikholeslami [37] simulated the effect of magnetic field on natural convection inside a porous curved cavity by CVFEM (schematic is shown in Fig. 6 (b)).

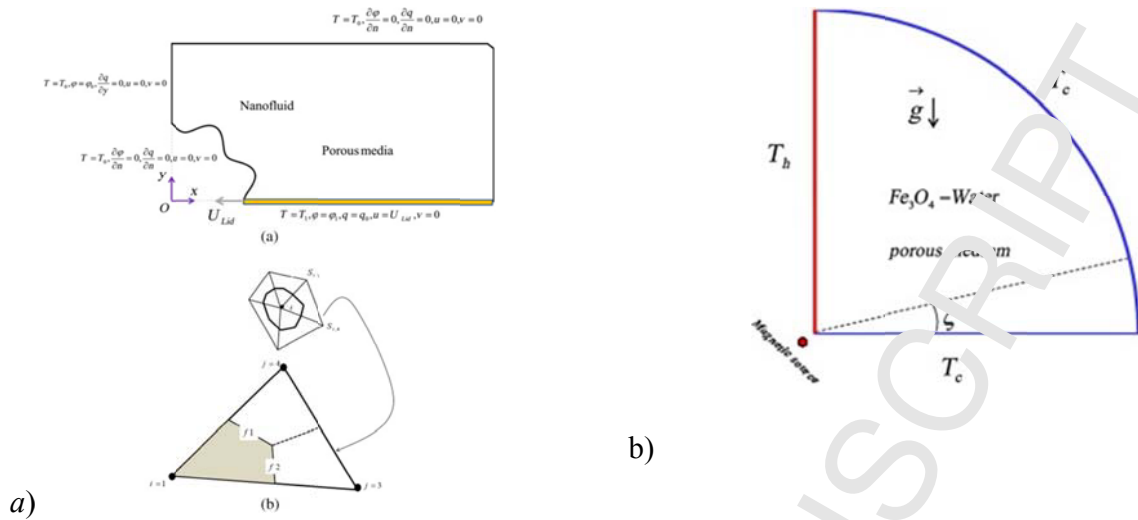


Figure 6. Schematics of problems solved by CVFEM (a) Porous rectangular enclosure[35] (b) Porous curved cavity [37] (figures are reprinted with permission from publisher).

2.1.4.2. Boundary Element Method

The term “boundary element method” (BEM) is used to describe a numerical approach that uses a Green's function formulation to convert a 3D partial differential equation to an integral equation over a surface, typically the bounding surface of a body immersed in the flow field (typically called the 'boundary integral equation' [38]. This boundary integral equation can then be discretized to obtain a matrix system. When applicable, BEM entails a very significant reduction of computation time since it is necessary only to solve for the value of the unknown variables on the bounding surface, rather than throughout the domain. For example, Oğtakkaya and Bozkaya [39] employed BEM to solve natural convection of water-based nanofluids inside an inclined semi-annulus cavity under a magnetic field by considering nanofluids as a single-phase working fluid (See Fig. 7 (a)). Applicability of BEM, however, is limited to problems for which a Greens function solution exists.

2.1.4.3. Spectral Method

The spectral method is a global approach (unlike FDM) that is based on expansion of the dependent variables in eigenfunction expansions, which must be selected to be compatible with the differential equation, the boundary conditions and the coordinate system used for numerical solution. In a spectral

method, the value of a parameter at each point is obtained by inversion of the eigenfunction expansions, and so it depends on information obtained from the whole computational domain and not only neighboring nodes. When applicable to a given problem, spectral methods are extremely fast, easy to formulate and exhibit exponential accuracy, which provides convergence faster than any other numerical method with algebraic accuracy [40]. Due to the restrictions on existence of the eigenfunction expansion, however, spectral methods are limited to very simple geometries, such as channel flow between infinite flat walls or flow in a triply periodic square. A few studies have used spectral methods to solve nanofluid problems. For example, Wakif et al. [41] utilized a spectral method to study the effect of a magnetic field on the onset of convection of nanofluids between two infinite plates where the space between the two plates was saturated with porous media (see Fig. 7 (b)). The authors have used the Buongiorno model in their study.

2.1.4.4. Meshless methods

To use common CFD techniques such as FDM, FVM, and FEM, a mesh must first be generated, which can sometimes be a challenge. To solve this problem, meshless (also called meshfree or gridless) methods have been proposed. In meshless methods, a set of nodes (without any connection) are scattered throughout the solution domain [42-43]. The most common meshless methods are [43]:

- Meshfree local Petrov-Galerkin
- Finite point methods
- Radial basis function approaches
- Meshfree boundary schemes

In comparison with a standard FEM, a meshless approach is easier to use for modeling 3D problems and can have higher accuracy. On the other hand, the computational cost of a meshless approach is often greater than a standard FEM [42].

As an alternative, Lagrangian solvers are another example of meshless methods where no mesh is used in the solution. Generally, a set number of fluid molecules are distributed randomly in the flow domain. For example, in Molecular Dynamics (MD) and Dissipative Particle Dynamics (DPD), the motion of these molecules is governed by Newton's second law of motion where a set of forces are acting on the fluid molecules [44,45]. In MD, the forces between the molecules are calculated by using the inter-particle potentials, such as the Lennard-Jones potential. In the DPD approach, the forces consist of friction forces, conservative forces, and random forces. Based on knowledge of the forces

acting on molecules, the acceleration, velocity, and position of the particles can be calculated. This allows statistical averaging to be implemented to calculate the bulk properties of the flow domain such as density, viscosity, mass diffusivity, velocity, pressure, stream function, and vorticity. For thermal transport problems, the first law of Thermodynamics is applied to the fluid molecules which can then be solved for the molecules temperature in the flow domain. Similarly, the bulk properties such as thermal conductivity, thermal diffusivity, internal energy, and enthalpy of the flow domain can be calculated using statistical averaging [46]. Another Lagrangian method is Smooth Particle Hydrodynamics (SPH) [47], which is also a meshless method. In this method, the continuum conservation laws governing the flow, i.e., Navier-Stokes equations, are transformed from partial differential form into integral using an interpolation function that calculate the kernel of the flow variables at any given point. The main difference between SPH and MD methods is that the inter-particle forces in SPH are derived from the continuum conservation laws, as opposed to using inter-particle potentials in MD [48,49]. The SPH is appropriate for studying large discontinuities in the flow and has the advantage that bulk fluid properties are an input to the solution (unlike MD, where they are outputs). However, its application to mimic meso-scale and nanoscale applications is not as well established as MD. Other meshless methods are the discrete element method [49] and the Smooth Dissipative Particle Dynamics (SDPD) [50]. The SDPD was introduced to combine the advantages of both DPD and SPH, where it retains the thermal fluctuation features found in DPD and has the flexibility in discretizing the Navier–Stokes (N–S) equation governing of the system using the SPH approximations [50].

In the literature, there are a limited number of papers on nanofluids that have employed meshless methods. For instance, Nikfar and Mahmoodi [51] used the meshless local Petrov–Galerkin approach to solve natural convection in a square cavity filled with alumina/water nanofluid. Figure 7 (c) shows the domain, boundary conditions and nodes distribution in the problem considered by Nikfar and Mahmoodi [51]—as can be seen, there is no connection between grids. Ooi and Popov [52] studied the effect of shape of Cu nanoparticles suspended in water in terms of natural convection heat transfer in a cavity utilizing a radial basis integral equation method. Bourantas et al. [53] employed a meshless approach (combination of point collocation method with the moving least squares approximation) to solve the natural convection of nanofluids in a porous cavity.

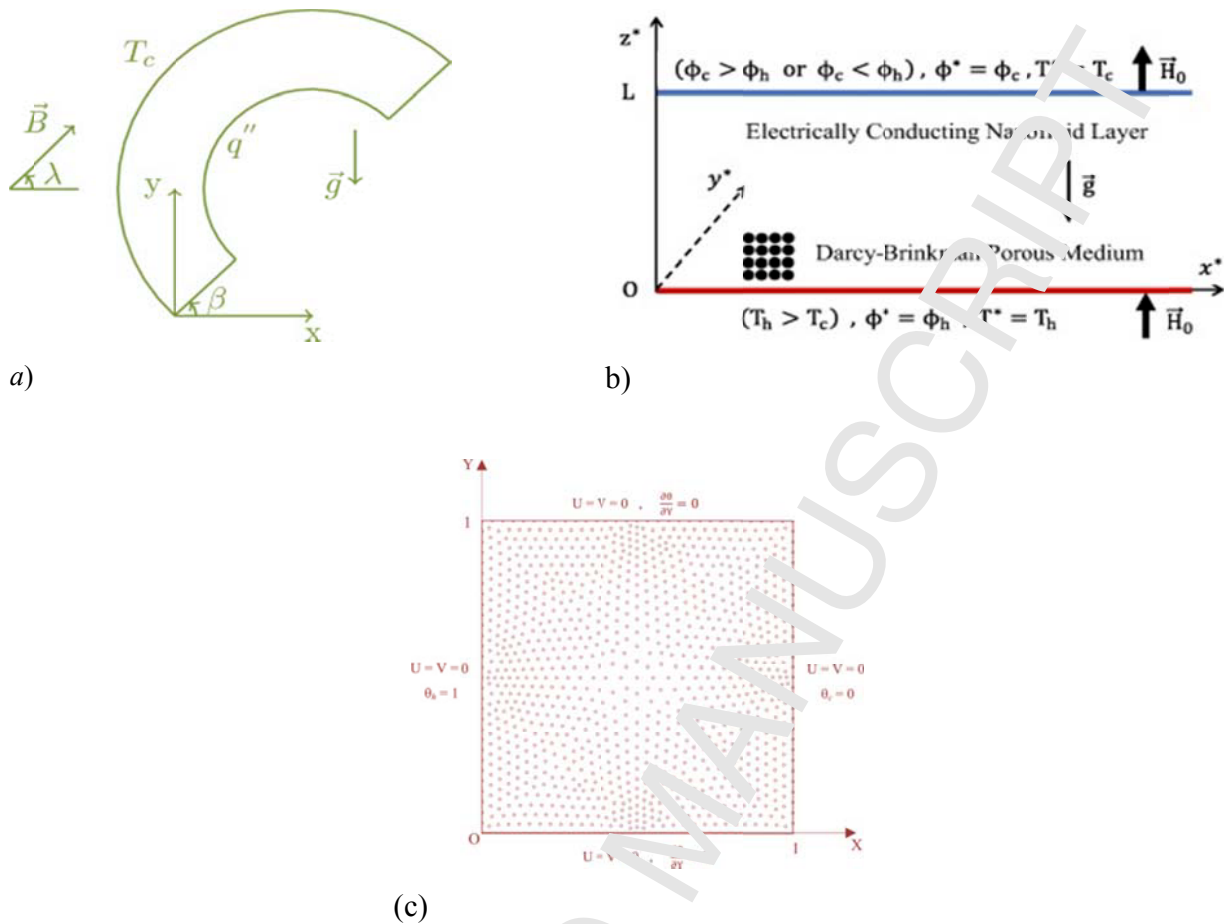


Figure 7. (a) Natural convection in an inclined semi-annulus cavity solved by boundary element method [39] (b) Flow between two infinite plates solved by spectral method[41] (c) Domain, boundary conditions, and node distribution for the problem considered by N. kfar and Mahmoodi [51] (figures are reprinted with permission from publisher).

2.2. Mesoscale based techniques

Mesoscale techniques attempt to simulate systems at a scale in-between molecular scale and the macroscopic scale. The lattice Boltzmann method and the dissipative particle dynamics method are the main mesoscale techniques that are used to simulate the nanofluid flows. Both of these methods are based on solution of a dynamical system that can be shown to converge to the Navier-Stokes equations in a certain limit.

2.2.1. Lattice Boltzmann method

The lattice Boltzmann method (LBM) was first introduced in the 1980s. The application of this method in simulating heat and mass transfer of fluids, especially in complex geometries and multicomponent flows, has increased in intervening years due to its effectiveness, flexibility and simplicity in comparison to the traditional computational fluid dynamics methods. The fundamental idea of LBM is based on the discrete movement theory of a set of artificial fluid particles' placed on lattices. The motion of these fluid particles is simulated by following the evolution of a prescribed Boltzmann equation. The most essential advantage of the LBM is the incorporation of microscopic physical interactions of the fluid particles in the numerical simulation and reveal the mesoscale mechanism of hydrodynamics. Taking the Boltzmann equation into consideration, the density distribution function $f(x, \xi, t)$ is introduced to represent a density number of the particles with the microscopic velocity ξ at the position x and the time t and the particles distribution can be expressed as $f(x, \xi, t)$ [54]. The most common models used for 3D simulations are the D3Q19 and D3Q41 lattices, providing 4th and 8th order isotropy in three dimensions respectively, which is illustrated in Fig. 8 [55]. The standard notation DnQm for m discrete velocities in n spatial dimensions is used throughout.

In order to simulate the nanofluid flow by the LBM, the dynamic parameters affecting the nanofluid flow behavior should be analyzed from the microscopic point of view due to various forces on the nanoparticles. Sidik and Razali [56] provided a concise review of research studies on the use of LBM for analyzing the convective heat transfer of nanofluids. In another study, Sidik and Mamat [57] summarized recent advances of LBM in estimating the hydrodynamic and thermal behavior of nanofluids and discuss several challenging problems that have been encountered in this issue. Kalteh and Hasani [58] studied the nanofluid free convection in an L-shaped cavity using LBM and the influences of nanoparticle concentration and diameter as well as Rayleigh number on the temperature and flow fields were evaluated. Mliki et al. [59] applied LBM to evaluate the MHD natural convection of nanofluid in an inclined C-shaped cavity and the impact of relevant parameters on the hydrothermal characteristics of the nanofluid was investigated. Zhou et al. [60] developed a 3D LBM model to analyze the mixed convection heat transfer and flow dynamics of alumina nanofluid in a cubic cavity under magnetic field. Mohebbi et al. [61] employed LBM to simulate the forced convection of TiO₂/water, Al₂O₃/water, and CuO/water nanofluids in an extended surfaces channel. Ghasemi and

Siavashi [62] studied the first and second laws of thermodynamics for natural convection of nanofluids (Cu/water) in a porous cavity employing LBM.

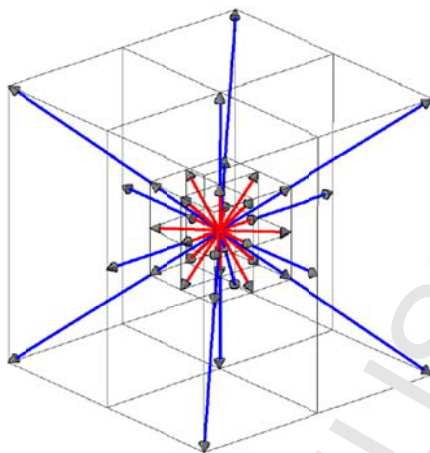


Figure 8. 4th order isotropic 19 velocities lattice (red arrows) and 6th order isotropic 41 velocities lattice (red and blue arrows) [55] (figure is reprinted with permission from publisher).

2.2.2. Dissipative particle dynamics method

The continuum Navier-Stokes equations (NSE) are generally used to study energy transport in micro/nanoscale heat transfer applications such as in nanofluids. The major concern-particularly with nanoparticle aerosols-is whether the scale of nanoscale should be simulated or if the continuum assumption is still applicable. This issue can be resolved by comparing the mean free path of the base fluid to the scale of the problem under consideration. For aerosol systems for which mean free path is of order of tens of nm (e.g. the mean free path of air is 60nm) the no-slip assumption may begin to break down, even for microscale systems. A similar breakdown of the continuum assumptions can occur in liquid nanofluid systems for problems involving length scales on the order of about 10nm or less.

The most familiar approach among discrete particle simulation methods is molecular dynamics (MD), where the process starts from tracking individual atoms of the fluid and calculating their velocities, accelerations and forces. From these microscopic details, further bulk properties such as temperature, pressure, viscosity, density, mass diffusivity, and flow rates can be recovered using statistical sampling. Such analysis gives comprehensive data about microscopic details of the simulated system. However, the primary drawback of this method is the extremely small physical and time scales, which are not appropriate for tracking larger temporal and spatial scales that occur in the

bulk heat transfer applications. In general, the MD method is computationally expensive and time consuming, even for cutting-edge supercomputers. The time and spatial scales of the heat transfer at the nanoscale are larger than MD, yet smaller than the conventional continuum NSE scales. Thus, intermediate spatial and temporal scales must be captured. These scales can be captured using mesoscopic particle-based methods, so-called “coarse-graining” methods, where each simulated particle represents a group of actual fluid molecules. These methods possess the unique capacity to model relatively bulk physical systems and efficiently capture the essential details of the pertinent interactions within fluid molecules. A relatively recent coarse-grained technique of this type is the dissipative particle dynamics (DPD) method.

The dissipative particle dynamics (DPD) method is a coarse-grained version of MD where each DPD particle represents a group or packet of actual molecules [62]. The DPD particles are randomly distributed in the flow domain, and particle interaction obeys conservation of mass, momentum and energy, as is shown Fig. 9. The figure shows a schematic for the fluid particles and wall particles where each particle represents a group of fluid molecules. To drive the flow in the domain, a body force is used and a periodic boundary condition is applied at the domain inlet and outlet to guarantee mass conservation. Further, a bounce back boundary condition is applied at the walls to produce the no-slip boundary condition. The DPD method can predict complex hydrodynamics at micro/nanoscale with a much higher computational efficiency than traditional MD [63]. During recent years the method was applied to several heat transfer applications, including conduction [64,65] and convective heat transfer applications for pure fluids and natural convection [66–68], and heat transfer with temperature-dependent properties [69].

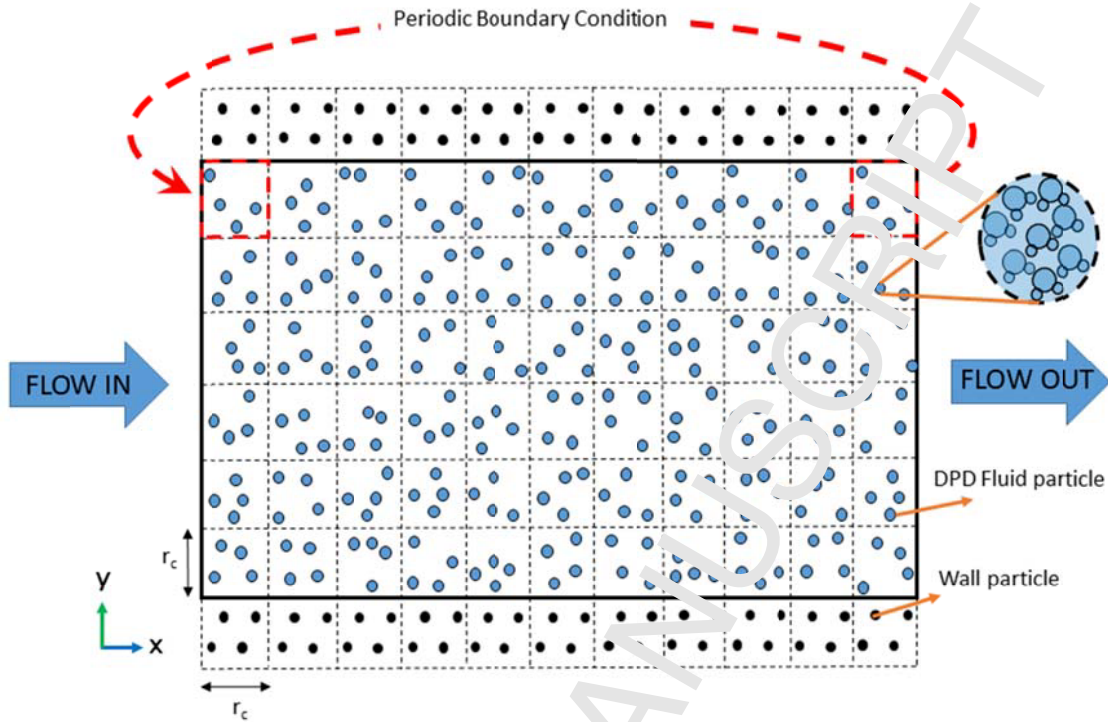


Figure 9. Schematic for DPD domain showing fluid and wall particles with appropriate boundary conditions

Recently, Abu-Nada [70,71] advanced the DPD approach to model heat transport within nanofluids. The method was able to mimic energy transfer mechanisms by studying thermal fluctuations induced by Brownian motion. Also, the method investigated the inter-particle energy transport between neighboring fluid particles [69]. The results indicated that the DPD method is well suited for nano/microscale applications, and it can be further extended to other practical heat transfer applications.

In nanofluid heat transport applications, the most influential properties are the thermal conductivity and viscosity. To understand the heat transfer mechanism that takes place microscopically within nanofluids using DPD, these two properties must be linked to the microscopic interactions between DPD particles. Abu-Nada [70,71] described the linkage procedure between nanofluid's bulk viscosity and the microscopic force interaction between DPD particles. He also described the linkage of nanofluid's bulk thermal conductivity and the microscopic heat conduction fluxes occurring between

DPD particles. In other words, he transformed the macroscopic bulk values of viscosity and thermal conductivity into microscopic particle-particle force and heat flux interactions between interacting DPD particles.

The interaction between DPD particles, shown in Fig. 9, is governed by the following equations:

$$\frac{d\vec{r}_i}{dt} = \vec{v}_i \quad (1)$$

$$\frac{d\vec{v}_i}{dt} = (\vec{f}_{ij}^C + \vec{f}_{ij}^D + \vec{f}_{ij}^R) + \vec{f}_{ext} \quad (2)$$

$$mC_v \frac{dT_i}{dt} = (q_{ij}^{visc} + q_{ij}^{cond} + q_{ij}^R) \quad (3)$$

where the conservative force \vec{f}_{ij}^C , dissipative force \vec{f}_{ij}^D and random force \vec{f}_{ij}^R are given in [70]. Also, \vec{f}_{ext} accounts for any external force applied to the DPD particles such as body force, pressure gradient, etc. The terms, q_{ij}^{cond} , q_{ij}^{visc} , q_{ij}^R are conductive, viscous and random heat fluxes, respectively, as given in [70]. The conservative interaction between the DPD particles, which regulates the potential energy between the particles, is determined from the state equation of the DPD system. Also, the strength of dissipative and random forces within the system of DPD follows the fluctuation-dissipation theorem to warranty conservation of the total momentum in system [70].

Generally, weighting functions are used in the conservative and dissipative forces where they decline monotonically with the inter-particle separation distance, becoming zero at a critical cut-off radius. The DPD conservative forces have a fine repulsive nature-one which does not account for attraction between DPD particles. The most commonly used weighting function for the conservative force is the Groot and Warren soft repulsion model [63]:

$$w(r_{ij}) = \begin{cases} \left(1 - \frac{r_{ij}}{r_c}\right)^2 & (r_{ij} < r_c) \\ 0 & (r_{ij} \geq r_c). \end{cases} \quad (4)$$

As for the dissipative and random forces weighting functions, Fan's et al. [72] generalized weighting function (which is one of the most famous models) has proven accurate for high viscosity fluids which mimic high Schmidt number mass transport applications and high Prandtl number thermal applications. Fan's model is given as [72]:

$$w^D(r_{ij}) = (w^R(r_{ij}))^2 = \begin{cases} \left(1 - \frac{r_{ij}}{r_c}\right)^S, & (r_{ij} < r_c) \\ 0, & (r_{ij} > r_c) \end{cases} \quad (5)$$

This weighting function can mimic greater DPD viscosity by either raising the exponent, S , or by modifying the value of r_c as cut-off radius. This weighting function plays a key role in mimicking thermal transport in water where $S = 1/2$ and $r_c = 1.1$ [73].

The incorporation of experimental data for the bulk nanofluid viscosity in the DPD model can be done by updating the dissipative force as proposed by Abu-Nada [70,71]. Abu-Nada's technique improves Fan's et al. [72] weighting function to account for the nanofluid's viscosity variation with the domain's concentration and temperature. Abu-Nada proposed the following analogy [70,71]:

$$\frac{\Omega^D(T, \phi, r_{ij})}{w^D(r_{ij})} = \frac{\mu(T, \phi)}{\mu_c} \quad (6)$$

Where $\mu(T, \phi)$ is the experimentally measured viscosity as a function of nanoparticles concentration and the local temperature of the nanofluid. Note that the parameter, $\Omega^D(T, r_{ij})$, is a modified weighting function and the parameter, $w^D(r_{ij})$, is weighting function [72], while μ_c is the pure fluid viscosity. Consequently, dissipative and random forces can be obtained by:

$$\vec{f}_{ij}^D(T, \phi) = \sum_{j \neq i} -\sigma_{ij} \Omega^D(T, \phi, r_{ij}) (\vec{e}_{ij} \cdot \vec{v}_{ij}) \vec{e}_{ij} \quad (7)$$

$$\vec{f}_{ij}^R(T, \phi) = \sum_{j \neq i} \sigma_{ij} \Omega^R(T, \phi, r_{ij}) \zeta_{ij} \Delta t^{-1/2} \vec{e}_{ij} \quad (8)$$

The incorporation of the bulk thermal conductivity of experimentally collected data for a given nanofluid can be included by adapting the conductive heat fluxes. Thus, the typical DPD conductive heat flux model can be described as [73]:

$$q_{ij}^{cond} = \sum_{j \neq i} \kappa_{ij} w^2(r_{ij}) \left(\frac{1}{T_i} - \frac{1}{T_j} \right) \quad (9)$$

The revised value of total heat flux is achieved by alteration of the κ_{ij} parameter, given in the former relation. This parameter is calculated as [70,71]:

$$\kappa_{ij} = \frac{C_v^2 \mathbf{K}(T, \varphi) (T_i + T_j)^2}{4k_B} \quad (10)$$

in which $\mathbf{K}(T, \varphi)$ is a modified heat friction function. In the typical DPD constant properties formulation, a constant value (k_o), called heat fraction, is used instead of $\mathbf{K}(T, \varphi)$. The amount of heat fraction (k_o) is usually estimated by tuning until it matches the value of thermal diffusivity. Here it is assumed that the function $\mathbf{K}(T, \varphi)$ changes with domain's temperature and nanofluid concentration and, consequently, a relationship was suggested as:

$$\mathbf{K}(T, \varphi) = k_o k(T, \varphi) \quad (11)$$

where $k(T, \varphi)$ is experimental measured bulk thermal conductivity of nanofluid. Using this function, the final set of equation for the DPD heat fluxes are given by the following:

$$q_{ij}^{\text{cond}} = \sum_{j \neq i} \kappa_{ij} w^2(r_{ij}) \left(\frac{1}{T} - \frac{1}{T_i} \right) \quad (12)$$

$$q_{ij}^{\text{visc}}(T) = \sum_{j \neq i} \frac{1}{2C_v} \left(\Omega^D(T, \varphi, r_{ij}) \left[\gamma_{ij} (\vec{e}_{ij} \cdot \vec{v}_{ij}) - \frac{c_{ij}^2}{m} \right] - \sigma_{ij} \Omega^R(T, \varphi, r_{ij}) (\vec{e}_{ij} \cdot \vec{v}_{ij}) \zeta_{ij} \right) \quad (13)$$

$$q_{ij}^R(T) = \sum_{j \neq i} \alpha_{ij}(T, \varphi) \Omega^R(T, \varphi, r_{ij}) \Delta t^{-1/2} \zeta_{ij}^e \quad (14)$$

The relation between parameters γ_{ij} and σ_{ij} is as [73]:

$$\gamma_{ij} = \frac{\sigma_{ij}^2 (T_i + T_j)}{4k_B T_i T_j} \quad (15)$$

Finally, the factor in the random heat flux can be expressed as:

$$\alpha_{ij} = \sqrt{2k_B \kappa_{ij}} \quad (16)$$

where k_B is the Boltzmann constant.

2.3. Microscale based techniques (Molecular dynamic simulation)

The fluid flow is usually solved for nanofluids using the continuum theory, which involves the hypothesis of continuum medium. As mentioned in the previous section, the continuum approximation

can begin to break down for problems at the smaller end of the nanometer size range (e.g. flow around carbon nanotubes, etc.) due to comparable characteristic dimensions of the fluid confined space and the molecular free path. In this regard, methods to directly solve for the fluid motion at the molecular scale were introduced. The molecular dynamic simulation (MDS) method was first introduced by Alder and Wainwright at the end of 1950. In this method, the molecules and atoms are regarded as 'fluid particles' whose dynamic and interactions are computed over a time period. The period of time for the calculation is defined according to the time needed to reach a dynamic evolution between system components. MDS solves a system of interacting particles using the Newton's equations of motion and specifies the trajectories of molecules and atoms. In addition, molecular mechanics or interatomic potential force fields are used to calculate forces between the particles and their potential energies.

Recent advance and progress in nanofluidics has made MDS a promising simulation approach in this field for problems on appropriately small length and time scales. A schematic of a water-based nanofluid containing carbon nanotubes is given in Fig. 10 [74]. The MDS method is known as a viable method to investigate the flow physics in nanoscale problems since it is able to properly determine the molecular structure of the fluid and surfaces, and to predict the atomistic interactions. Jabbari et al. [75] and Lee et al. [76] quantitatively reviewed recent advances of MDS in nanofluidic systems, including enhancements in measurement of nanofluid thermophysical properties, thermal wall modeling, the cut-off radius, and initial conditions, etc. These review papers provided very critical discoveries and proposed important areas needed to be taken into consideration in future research, and thus the reader is recommended to refer to these articles for further detailed information on this matter. The MDS method was hereto used in various investigations on nanofluids. It is used for measuring the viscosity and thermal conductivity of nanofluid at different temperatures and using different particle sizes [77], investigating heat transfer characteristics of nanofluid [78] and their surface nanostructures [79], Poiseuille flow of a nanofluid [80], chaotic movements of nanoparticles [81], etc. According to Wei et al. [77], molecular motion without temperature gradient are found to be responsible for heat transfer in the area of low viscous heat using a non-equilibrium MDS. Toghraie et al. [80] used MDS to simulate the Poiseuille flow of nanofluids containing platinum and copper particles and were able to successfully employ the Large-scale Atomic/Molecular Massively Parallel Simulator (LAMMPS) code for this purpose. In this method, the Lennard-Jones potential governs the atomic interaction. Table 1 summarizes some advantages and limitations of different CFD approaches.

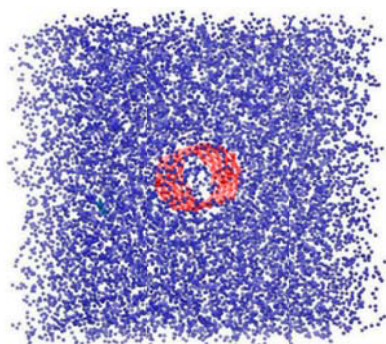


Figure 10. Nanofluid molecular dynamic model: Carbon nanotube nanoparticle (red-colored) and surrounding water (blue-colored) [74] (figure is reprinted with permission from publisher).

Table 1 advantages and limitations of different CFD approaches

Method	Advantages	Limitations
Finite Difference	<ul style="list-style-type: none"> - Simplicity. - Relatively small convergence time - Numerical evaluation of the difference equation is simple - Simple code structures - Intuitive and easy to implement for simple problems 	<ul style="list-style-type: none"> - In large scale problems small time steps are needed [82]. - Compared to FEM needs high number of grids to give good output. - Difficult to employ for complicated structures
Finite Volume	<ul style="list-style-type: none"> - Complex geometries can be considered [83] - Neumann B.C can be easily established. - Suitable for Unstructured Grids - Governing equations are written as integral form 	<ul style="list-style-type: none"> - Irregular geometries require far more effort than FEM
Finite Element	<ul style="list-style-type: none"> - Very general method and used for solving a variety of problems - The stiffness matrix and the load vector can be computed for each element. - Allowing attention to be confined to a single typical element [82]. - Spatial variation of properties can be handled with relative ease. - Irregular regions can be modelled with greater accuracy. - Suitability for non-linear equations. - High flexibility in changing element sizes - Mixed-boundary-value problems are easier to handle[84]. - Any shaped geometry can be modelled with the same effort. 	<ul style="list-style-type: none"> - Equations are much more mathematically complex. - Numerically intensive and can take more CPU time and memory storage space [84]. - More mathematics involved - less physical significance - Complex code structures
Boundary Element	<ul style="list-style-type: none"> - Less data is required to run a program efficiently [85]. - Small system of equations. discretization takes place only on the surface of body - Well-suited to problems with open domains [85]. - Good agreement between BEM and FEM - More efficient in modelling due to the reduction of dimensions. - Advantageous in the case of problems with infinite or semi-infinite domains - Less data preparation time 	<ul style="list-style-type: none"> - Programming complexity - BEM is not successfully applied to non-linear fluid flow problems. - Matrices are asymmetric, dense and not easy to solve[85]. - Solution time is long and memory size is large. - Poor for thin structures - In non-linear problems, the interior must be modelled

Lattice Boltzmann	<ul style="list-style-type: none"> - The method is scalable, stable and suitable for parallel computing - Suitable for complicated geometries. - Suitable for multiphase flows calculations - Suitable for mesoscale applications such as colloids and flow in small pores encountered in porous media. - Suitable for applying microscopic interactions between fluid molecules. 	<ul style="list-style-type: none"> - Modeling variable thermo-physical properties (such as viscosity and thermal conductivity) is not straightforward. - The method is limited to uniform grid where the modeling of very fine gradients encountered at boundaries requires fine treatment which makes the method computationally demanding, examples in shear layers and in boundary layers. - Bulk viscosity is not a direct input parameter to the method, but it is related to the relaxation time of the method - Axisymmetric flows implementations are not straightforward. - Multiphase flows with large variation in density is problematic
Dissipative Particle Dynamics	<ul style="list-style-type: none"> - Grid less method with a Lagrangian nature - Brownian motion is intrinsic in the formulation of the method - Suitable for complicated geometries - Suitable for microfluidics applications as it contains thermal fluctuations - A coarse-grained particle method suitable for spatial and temporal scales that are larger than the molecular dynamic scales and much smaller than the continuum based models (i.e., Navier-Stokes) - Suitable for suspension flows and colloids - Suitable for multiphase flows - Straightforward extension from two-dimensions to three dimensions - Suitable for inter-particle interactions at microscopic level by including attraction and repulsion forces between fluid molecules and the surrounding media. 	<ul style="list-style-type: none"> - There is no direct connection between DPD parameters and the physical parameters of the system under simulation (i.e., thermophysical properties are not input parameters) - Computationally demanding when compared with finite difference and finite volumes.

3. Three dimensional computational modeling of nanofluid flows

The following review of 3D numerical studies of nanofluid flow and heat transfer is divided into three main sections. The first section deals with simulation of natural convection flow of nanofluids where the buoyancy effect is considered. In the second part, forced convection studies are reviewed. In the third, part mixed convection flows of nanofluids are discussed.

3.1. Natural convection of nanofluid flow

For natural convection problems, the buoyancy force is added to the momentum equation which couples the momentum and energy equations. Many studies have been done to analyze natural convection in enclosures because of its importance in solar collectors, thermal energy storage systems, electronics equipment, and buildings [86]. The physical parameters affecting the 2D and 3D modeling results as well as the influence of different CFD techniques on the accuracy of results are compared. Next, the effects of using different nanofluids, boundary conditions, and even some external fields are reviewed from the literature on nanofluid natural convection.

3.1.1. Two dimensional versus three dimensional modeling

Obviously, it is tempting to find cases for which 2D simulation can provide accurate results since these are less computationally expensive than full 3D simulations (for the same resolution). Even for 3D simulations, several simplifications are typically made (including limiting the domain) to obtain a tractable model of reasonable cost. Along these lines, several studies have been done to investigate the agreement between 2D and 3D simulations. These types of studies show that the results of 2D and 3D modeling can be quite different depending on the range of physical parameters employed, such as Rayleigh number, nanoparticle volume fraction, and size of the thermal system (aspect ratio). For example, Boutra et al. [87] applied LBM coupled with FDM to solve the three dimensional natural convection flow of Ag/water nanofluids (with volume fractions up to 10%) in an inclined cube cavity with differentially heated side walls. In this simulation, Maxwell and Brinkman models were implemented to calculate the thermal conductivity and viscosity of nanofluids, respectively. Their findings revealed that at low Rayleigh numbers (i.e. 10^3) the average Nusselt number can be estimated

via 2D modeling with high accuracy; however, with increasing Rayleigh number, the 2D model overestimated the average Nusselt number compared to a 3D simulation. Moreover, the difference between 2D and 3D results increased with the increment of nanofluid concentration (See Fig. 11).

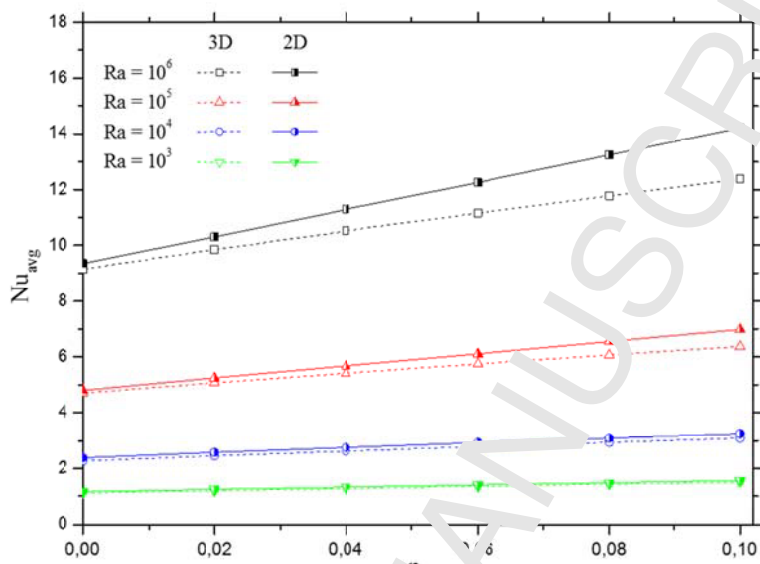


Figure 11. Comparison between 2D and 3D modeling results for average Nusselt number at various Rayleigh numbers and nanofluid concentrations [87] (figure is reprinted with permission from publisher).

In another study, Ravnik and Škerget [88] solved natural convection of nanofluids in a cube cavity where a hot cylindrical body was installed in its center while front and back surfaces of cavity were insulated, and other sides were kept at low temperature (i.e. cold walls). The cross section of this body varied from circular to elliptical. Three types of water-based nanofluids containing Al_2O_3 , Cu, and TiO_2 nanoparticles with volume fractions of 10 and 20% were considered in the analysis where the Rayleigh number varies between 10^3 and 10^6 . They used a boundary element method to simulate the flow of nanofluids. In addition, the Maxwell and Brinkman models were implemented to estimate the thermal conductivity and viscosity of nanofluids, respectively. Comparison between 2D and 3D simulations reveals that with increasing the Rayleigh number, the difference between 2D and 3D results increased, and in general the estimated heat transfer rate by 3D modeling is lower than that of 2D. However, it was concluded that the difference between 2D and 3D results is not significant, so apply 2D modeling was suggested to be suitable for similar problems.

In another study, Sheremet et al. [89] used FDM to simulate free convection of nanofluids in a porous cavity with partially heated sidewalls. They implemented the Buongiorno model (discussed above) in which Brownian motion and the effect of thermophoresis have been included. The results were presented for various parameters including Rayleigh and Lewis numbers as well as buoyancy-ratio, Brownian motion, thermophoresis, and aspect ratio parameters. They concluded that as the aspect ratio increases the difference between 2D and 3D results for average Nusselt number on a hot wall decreases. In the limit, the difference tends to zero for aspect ratios ≥ 2 (See Fig. 12). They also emphasized that considering the non-homogenous distribution of nanoparticles leads to more accurate predictions of flow and heat transfer characteristics in the cavity.

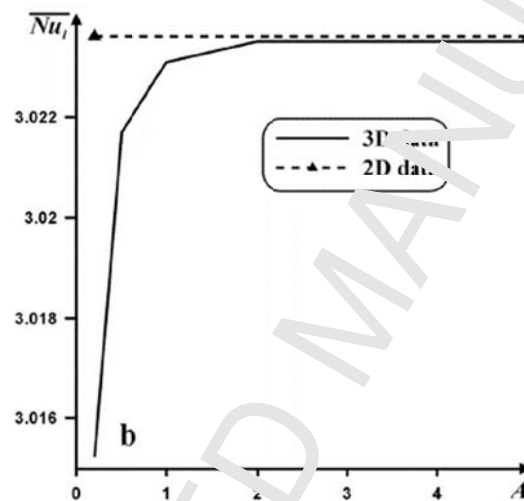


Figure 12. Comparison between 2D and 3D results for variation of average Nusselt number with aspect ratio for $Le = 1$, $Nr = Nb = Nt = 0.1$ [89] (figure is reprinted with permission from publisher).

3.1.2. What role does the CFD technique play on the modeling results?

There are not enough comparative studies that investigate the effect of both the CFD technique and 3D simulation on the accuracy of results. To date, only one paper was published in this regard in the literature. Saghir et al. [90] investigated the natural convection of Al_2O_3 /water nanofluids (with volume fractions up to 3%) in a cubic cavity where the sidewalls were differentially heated. They used the Ho et al. [51] correlations for thermal conductivity and viscosity. For the simulations, they implemented three well-known approaches including FDM, FEM, and LBM. The results of simulations were compared against experimental data for volume fraction of 3% at $Ra = 5.6 \times 10^7$

where the temperature difference between sidewalls was 12 °C. As is shown in Fig. 13, in the case of 2D simulation, FEM was more accurate than FDM and LBM. Moreover, the comparison of 2D and 3D results for finite element technique reveals that the results of 2D modeling are closer to experimental data.

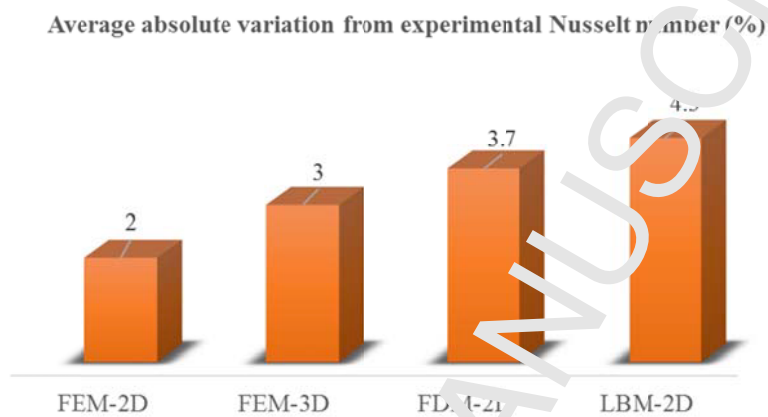


Figure 13. Comparison between the results of different numerical models for volume fraction of 3% at $Ra = 5.0 \times 10^4$ [90].

From just one study it is hard to conclude which CFD technique is better, so more studies should be done to discover this issue.

3.1.3. What is the effect of nanofluid type on the heat transfer rate?

The effective thermal conductivity and viscosity of nanofluids strongly depend on the type of nanoparticles and base fluid. Using nanoparticles with higher thermal conductivity increases the effective thermal conductivity of nanofluids. To date, two studies have dealt with the effect of nanofluid type on natural convection. In both studies the Brinkman model [92] was used to calculate the effective viscosity, so changing the type of nanoparticle has no effect on the effective viscosity. Of course, other properties including density, thermal expansion coefficient, and viscosity vary by changing the kind of nanoparticles.

In this regard, Purusothaman et al. [93] solved the natural convection of nanofluids in a cavity, which its schematic has been shown in Fig. 14, using single-phase approach and FVM. They presented the results for Rayleigh number in the range 10^5 - 10^7 , cavity aspect ratio between 1 and 7.5, nanoparticle volume fractions up to 10%, and for two different nanofluids including Cu/water and Al_2O_3 /water. They concluded that Cu/water nanofluid is more effective than Al_2O_3 /water for cooling of hot elements installed on the enclosure wall; one reason behind the better cooling performance of Cu/water nanofluid could be its higher effective thermal conductivity. They utilized the Maxwell and Brinkman models to calculate the thermal conductivity and viscosity of nanofluids, respectively.

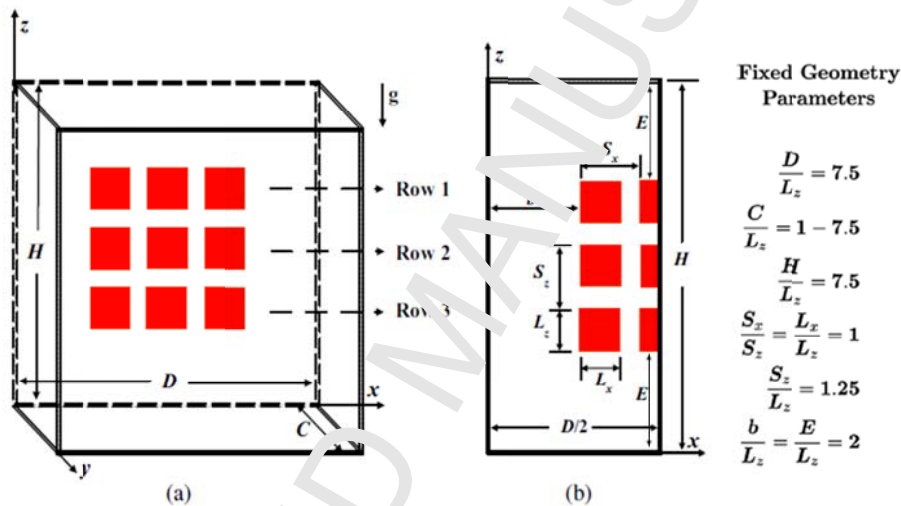


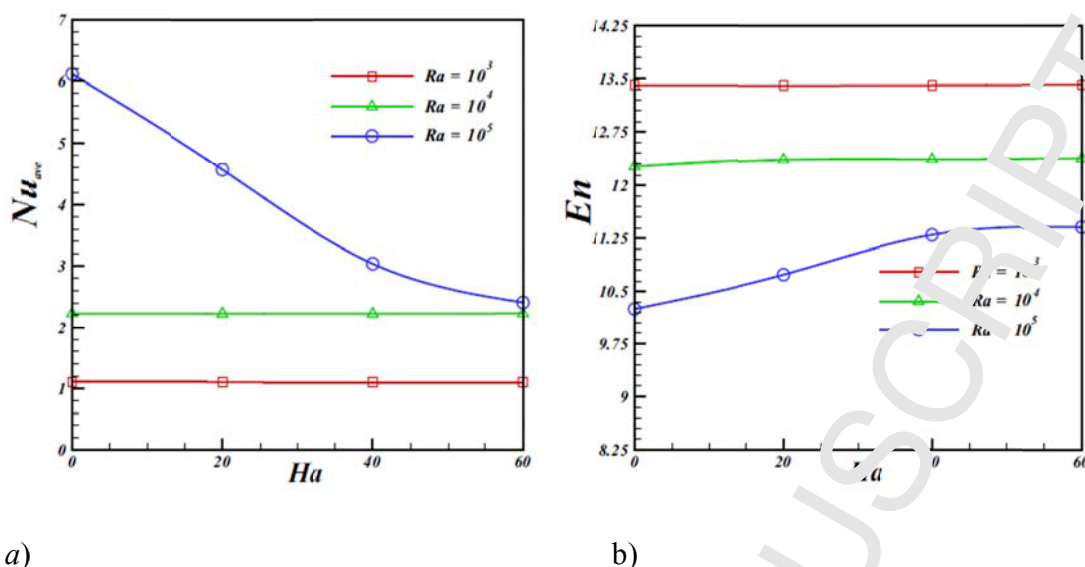
Figure 14. Physical configuration and geometric parameters (a) full view of the enclosure (b) half view of sources mounted on wall [93] (figure is reprinted with permission from publisher)..

Snoussi et al. [94] studied the steady state free convection of two water-based nanofluids containing Al_2O_3 and Ag nanoparticles (with maximum volume fraction up to 8%) in a cube cavity with differentially heated sidewalls. They used FLUENT software (based on FVM) to simulate the natural convection flow while Maxwell and Brinkman equations were utilized to estimate the thermal conductivity and viscosity of nanofluids, respectively. They found that particle loading diminishes the heat transfer rate and the reduction was highlighted when the convection mode is the dominant regime of heat transfer. In addition, the results revealed that Ag nanoparticles provide higher heat transfer rates.

3.1.4. What is the effect of a magnetic field applied to natural convection nanofluid flow?

Magnetic fields are utilized in many engineering applications (e.g. micropumps, electronic devices, and nuclear reactors). Therefore, nanofluids should also be studied with respect to MHD flow, to see if there is any additional effect on heat transfer characteristics of thermal systems [95–99]. From the 3D studies on MHD flow effect on natural convection of nanofluids, it seen that with increasing Hartmann number (a dimensionless number that indicates the ratio of magnetic to viscous forces), the average Nusselt number reduces. This was attributed to the decrease of fluid velocity induced when MHD flow field acts as a barrier to heat transfer enhancement.

In another study, Sheikholeslami and Ellahi [100] investigated the natural convection flow of $\text{Al}_2\text{O}_3/\text{water}$ nanofluids in a cube cavity under MHD flow where the temperature of bottom surface was higher than that of top surface and other surfaces were adiabatic. They used LBM to simulate the problem. To calculate the thermal conductivity and viscosity, Koo–Kleinstreuer–Li (KKL) models were utilized. The results were presented for different volume fractions (up to 4%), Hartmann numbers, and Rayleigh numbers. They concluded that applying magnetic field reduces the magnitude of velocities in the cavity, and, consequently, the Nusselt number decreases. On the other hand, Nusselt number would be enhanced through particle loading and boosting the buoyancy forces. Figure 15 presents the effects of Rayleigh and Hartmann numbers on average Nusselt number and heat transfer enhancement in the cavity. Heat transfer enhancement parameter is the ratio of heat transfer rate in the case of nanofluid to the case of base fluid. As seen heat transfer enhancement index raises with increasing the magnetic field strength especially at high Rayleigh numbers.



a) b)

Figure 15. Effects of Rayleigh number and Hartmann number on (a) average Nusselt number ($\phi = 4\%$) and (b) heat transfer enhancement [100] (figure is reprinted with permission from publisher).

Al-Rashed et al. [101] modeled the effect of the angle in which an inclined magnetic force applies to a partially heated cavity filled with CNT/water nanofluids. The magnetic source angle with respect to horizontal was measured by α ($0 \leq \alpha \leq 90^\circ$). The modeling was done for Hartmann numbers of 0, 50, and 100, volume fractions up to 5%, and Rayleigh numbers between 10^3 and 10^5 . The maximum and minimum values of Nu number were obtained for different conditions as shown in Fig. 16. They used Brinkman model for calculating viscosity and Yue model [102] for estimating the thermal conductivity of CNT/water nanofluids.

In a continuation of the work of [101], Al-Rashed et al. [103] analyzed the second law of thermodynamics for the same problem. They found that the effect of magnetic field inclination on entropy generation is more visible at $Ra = 10^5$.

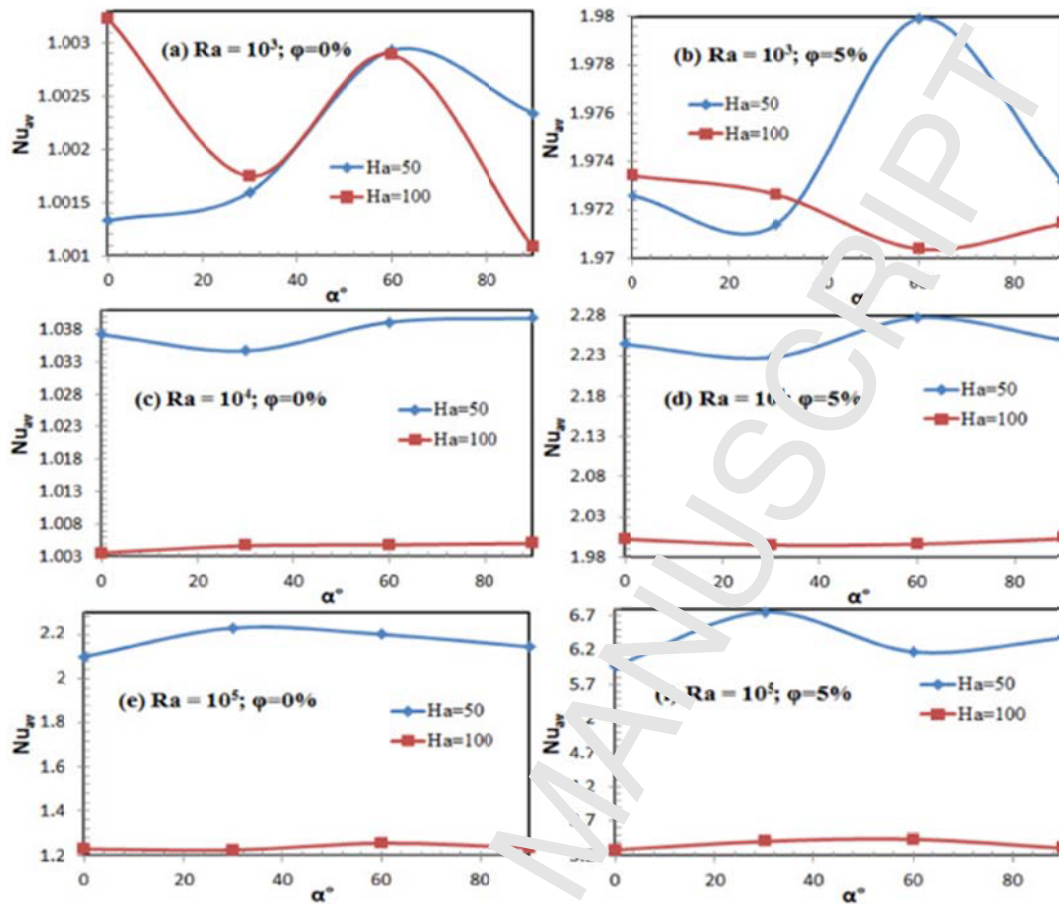


Figure 16. Effect of the angle of magnetic force applied to a partially heated cavity on Nu number [101] (figure is reprinted with permission from publisher).

Kolsi et al. [104] studied the effects of magnetic field, and an inclined fin on natural convection in an open cavity filled with CNTs/water nanofluids. The results were presented for different values of Rayleigh number, nanoparticle concentrations up to 5%, Hartmann numbers between 0 and 100, and inclination angles of fin ranging 0 to 360 degrees. It was found that heat transfer rate reduces linearly with increasing the magnetic field intensity. It was also concluded that by adding nanoparticles the heat transfer augments linearly, moreover when the fin is at an angle of 180° , the heat transfer is maximized. The problem was solved with the FVM and a single phase approach.

3.1.5. What is the effect of cavity configuration on natural convection?

Changing the slope of cavities and heating area as well as using internal fins (or blocks) can affect the heat transfer due to natural convection in cavities. In the following, the effects of these parameters are discussed.

Kolsi et al. [105] simulated heat transfer and entropy generation due to natural convection of Al_2O_3 /water nanofluids with volume concentrations up to 15% in a cube cavity by implementing the FVM and single-phase homogenous approach. Solid panels with three different positions were installed at corners of the cavity. They used Maxwell and Brinkman model to calculate the thermal conductivity and viscosity of nanofluids, respectively. It was found that with increasing nanofluid concentration, the heat transfer rate enhances. Moreover, particle loading into the base fluid is not advantageous from the second law viewpoint. By using the solid panels one can adjust the rate of heat transfer in the cavity.

Kolsi et al. [106] studied transient natural convection heat transfer and entropy generation due to Al_2O_3 /water nanofluid flow (with volume fractions up to 20%) in a cube cavity with differentially heated walls. The problem was solved via the FVM where the Maxwell and Brinkman relations were used to estimate the thermal conductivity and viscosity of nanofluids, respectively. They concluded that with increases in Rayleigh number and nanofluid concentration both average Nusselt number and entropy generation increase.

Meng and Li [107] simulated the free convection of Al_2O_3 /water nanofluids with two volume concentrations of 1 and 4% in a horizontal cylinder where the sizes of length and diameter are the same. The ends of circular cylinder had a finite temperature difference to create free convection in the cavity. The problem was solved by OpenFOAM (FVM). They found that at a given Rayleigh number, the Nusselt number decreases with the increase of nanofluid concentration. They also observed that when a temperature-dependent solver was used for simulation, the results of water, and nanofluid with a concentration of 1% are closer to experimental data. On the other hand, a temperature-independent solver should be utilized to simulate the results at the concentration of 4% to minimize the deviation from experimental data. Ternik and Rudolf [108] solved the natural convection of non-Newtonian nanofluids (Au nanoparticles with volume fractions up to 10% dispersed in carboxy methyl cellulose) in a cube cavity with differentially heated walls for Rayleigh numbers between 10 and 10^6 . The problem was again solved with the FVM and assuming power law behavior for nanofluids. They found that by loading nanoparticles to the base liquid the onset of natural convection prolongs. In

addition, after the onset of free convection in the cavity, the average Nusselt number reduces with increases in nanofluid concentration independent of Rayleigh number magnitude.

Kolsi et al. [109] examined the natural convection and entropy generation of $\text{Al}_2\text{O}_3/\text{water}$ nanofluids with volume fractions up to 15% in a cube cavity where two adiabatic blocks were installed inside it, as shown in Fig. 17. The modeling was conducted by using finite volume method and results were presented for Rayleigh numbers between 10^4 and 10^6 . They found that Nusselt number increases with increases in Rayleigh number and with the nanofluid concentration. On the other hand, when blocks with a smaller size were mounted in the cavity, the average Nusselt number ameliorates.

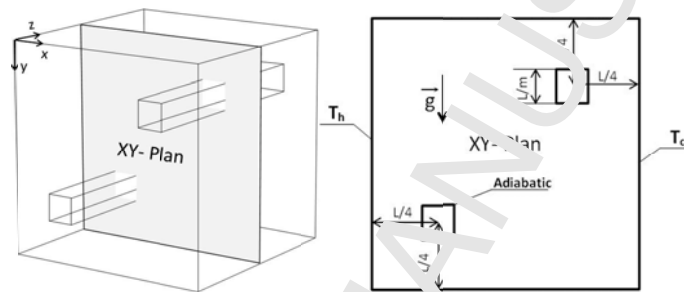


Figure 17. Schematic of the problem considered by Kolsi et al. [109] (figure is reprinted with permission from publisher).

Salari et al. [110] analyzed 3D flow, heat transfer and entropy generation in an inclined cavity that is partially filled with MWCNTs/water nanofluid and the rest space on the top of nanofluid was occupied with air. A single-phase homogeneous model was used to simulate the problem. The results were presented for volume fractions of 0.2, 0.4, and 1%, Rayleigh numbers between 10^3 and 10^6 , and cavity slopes between 0 and 60 degrees. The main results of this study can be summarized as:

- When the cavity is in vertical position (0°), two secondary eddies are formed in the vicinity of bottom side of cavity. These two eddies are converted to one eddy with an increase in the slope angle (see Fig. 18).
- With increasing volume fraction of nanoparticles the heat transfer rate increases while entropy generation reduces.

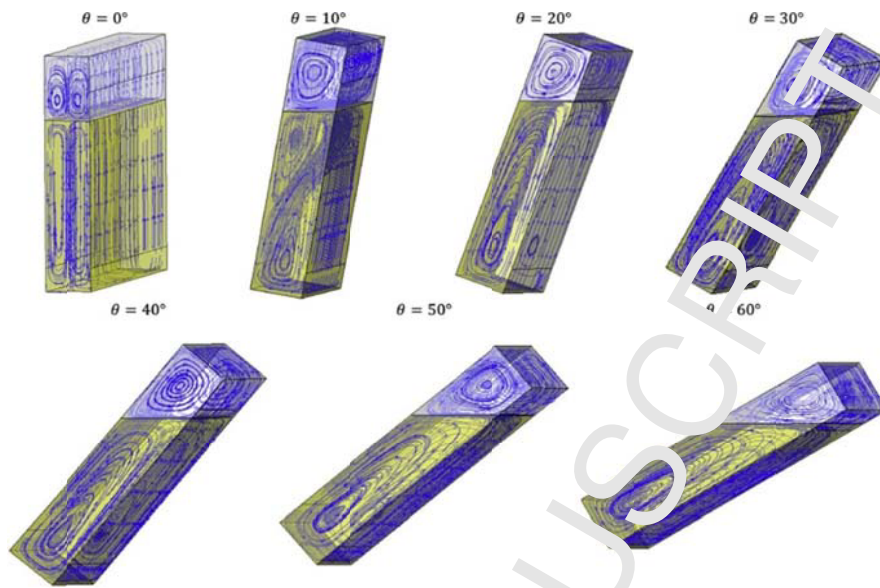


Figure 18. Three-dimensional flow pattern for enclosures with different tilt angles at $Ra = 10^5$ and $\phi = 1\%$ [110] (figure is reprinted with permission from publisher).

Al-Rashed et al. [111] simulated the natural convection of CNTs/water nanofluids in an inclined cube cavity using finite volume method where Ahmed body is installed inside it (See Fig. 19). It should be mentioned that an Ahmed body is a simplified shape of car that is used for simulation purposes in automotive industry. The simulation was performed for volume fractions up to 5%, Rayleigh numbers between 10^3 and 10^6 , inclination angles between 0 and 180 degrees, and thermal conductivity ratio between 0.01 and 100. The authors concluded that the maximum average Nusselt number was achieved in inclination angles of 30 and 150 degrees whereas the average Nusselt number was minimized at 90 degrees. Thermal conductivity ratio has no significant effect on the heat transfer rate compared to other parameters. Moreover, particle loading increased heat transfer rate inside the cavity.

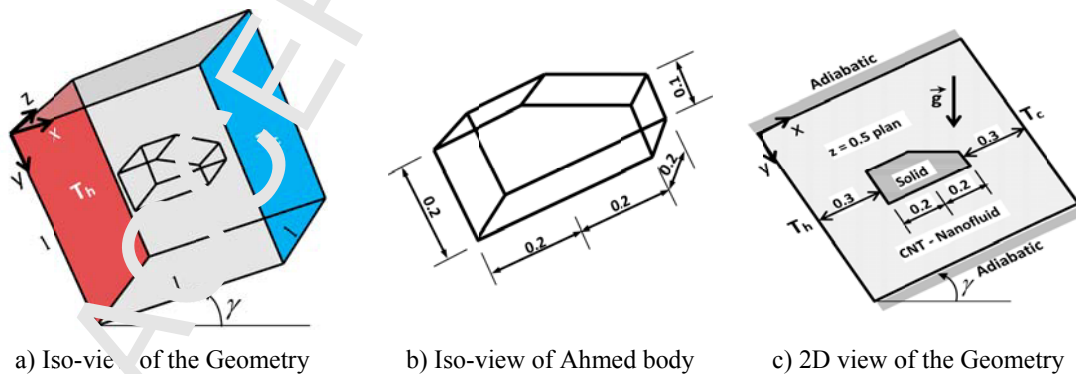


Figure 19. Geometrical representation of the problem considered by Al-Rashed et al. [111].

In another study with the same geometry and nanofluid given in [111], Al-Rashed et al. [112] analyzed entropy generation in the cavity. They found entropy generation increases with increases in volume fraction of nanoparticles.

Kolsi et al. [113] applied the FVM to study the natural convection and entropy generation of Al_2O_3 /water nanofluids with volume fractions up to 15% inside an open cube cavity where a diamond shaped body is installed in the middle of cavity. Maxwell and Brinkman equations were employed to estimate the thermal conductivity and viscosity of nanofluids, respectively. They concluded that from the first law of thermodynamics viewpoint, adding nanoparticles to the base fluid is advantageous, however, from the second law of thermodynamics viewpoint particle loading is not beneficial since the entropy generation increases in the cavity.

Kolsi et al. [114] solved the natural convection and entropy generation of Al_2O_3 /water nanofluids with volume fractions up to 20% in a cube cavity in the presence of Marangoni effect (or called thermo-capillary force). Maxwell and Brinkman equations were used to estimate the thermal conductivity and viscosity of nanofluids, respectively. Two sidewalls of cavity were partially heated and the top surface of cavity was open and in contact with free stream of gas. The analysis was done by employing finite volume method for Marangoni numbers between -1000 to 1000 and a constant Rayleigh number equal to 10^5 . It should be mentioned that a positive Marangoni number implies that thermo-capillary force boosts the buoyancy force, and, consequently the convection flow intensifies, on the other hand, a negative Marangoni number means that thermo-capillary force acts against the buoyancy force. It was found that at a given value of Marangoni number, the Nusselt number increases via particle loading. The results also revealed that the increase of Nusselt number with an increase in Marangoni number and entropy generation rises with particle loading.

Al-Rashed et al. [115] investigated the MHD natural convection inside a partially heated cavity where the active zone of magnetic field was variable (the magnetic field could be applied to a part or whole of the cavity). The working fluid was CNT/water nanofluid and the method of solution was FVM. It was found that adding nanoparticles increases the heat transfer rate. Moreover, the heat transfer reduced with extension of active zone and Hartmann number.

Rahimi et al. [116] employed LBM to model natural convection and entropy production of CuO/water nanofluid (volume fractions up to 4%) in a cavity with four different sets of thermal boundary

conditions. Figure 20 shows the schematic of the problem considered by Rahimi et al. [116]. The authors used a single-phase approach to solve the problem while KKL model has been used to obtain the properties. Figure 21 shows a sample of their results for temperature field and for different cases. It was found that adding nanoparticle to the base fluid increases the heat transfer and reduces the entropy generation.

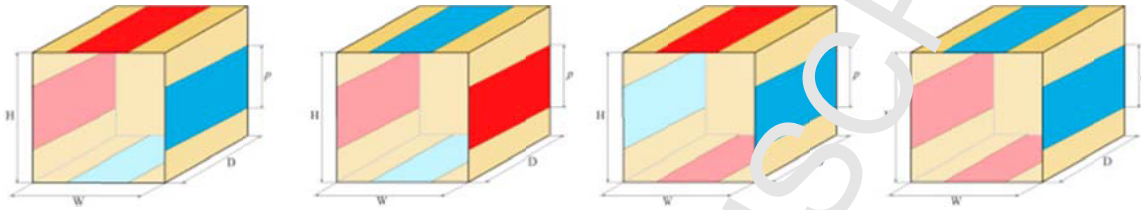


Figure 20. Schematic of the problem considered by Rahimi et al. [116] (figure is reprinted with permission from publisher).

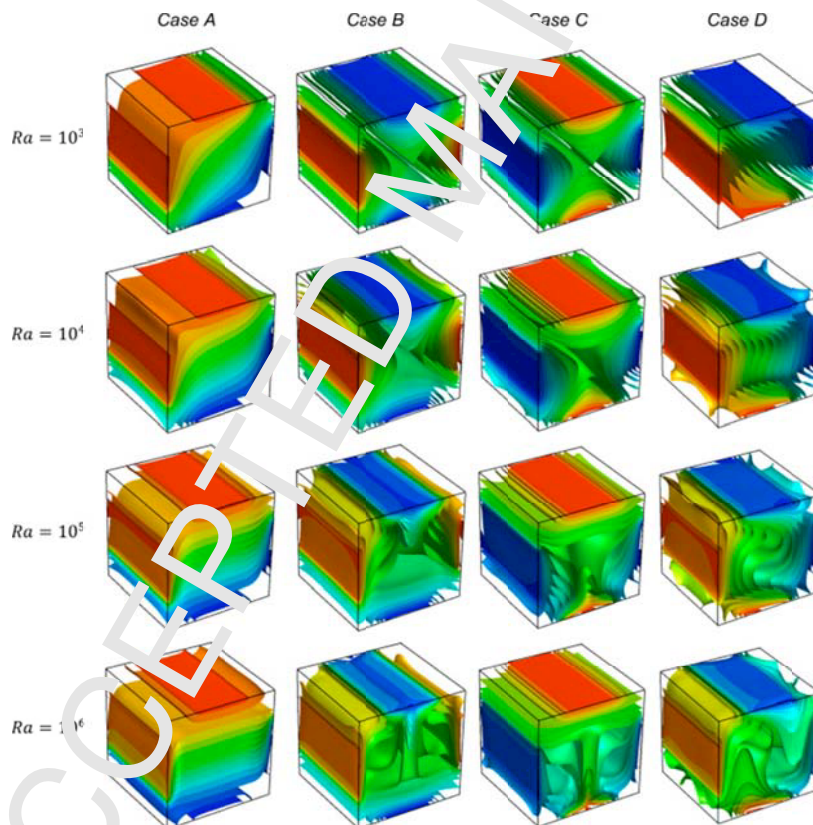
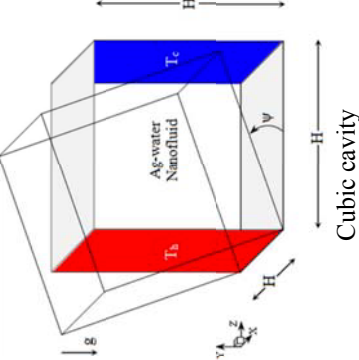
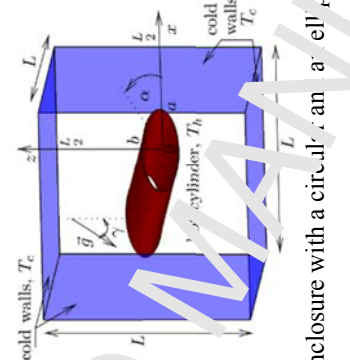
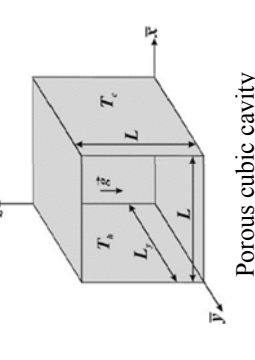


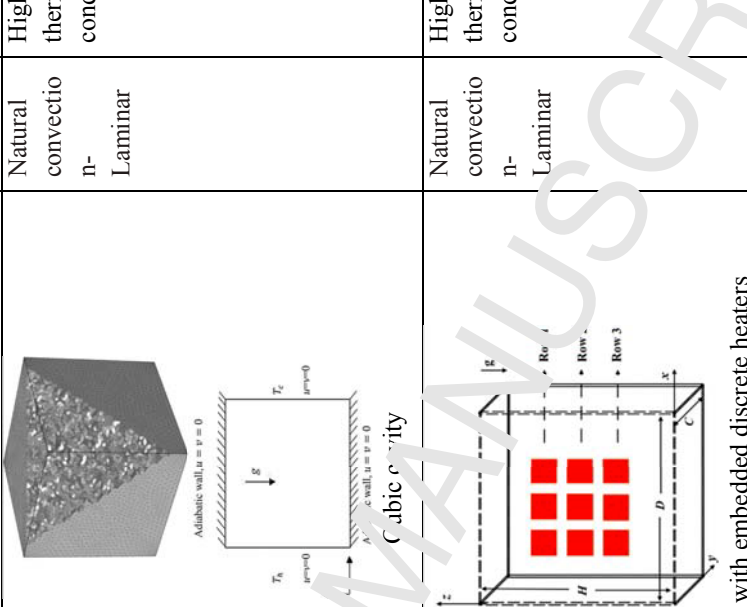
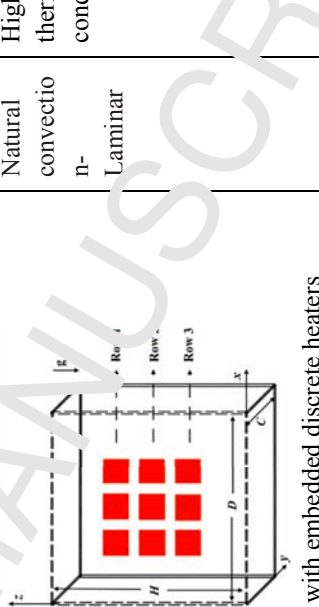
Figure 21. Three-dimensional temperature field for different arrangements of discrete active walls and Rayleigh numbers at $\phi = 4\%$ [116] (figure is reprinted with permission from publisher).

In another work, Rahimi et al.[117] conducted experiments to investigate the natural convection in a cubic cavity with partially heated walls filled by DWCNTs/water (in six different volume concentrations including 0.01, 0.02, 0.05, 0.1, 0.2, and 0.4%). They measured the properties of nanofluids experimentally. A three-dimensional code was developed based on the FVM and single-phase approach where for properties the measured data were used. They found that the heat transfer coefficient is maximized at a volume fraction of 0.05%. Sheremet et al. [118] studied the free convection of a nanofluid in a porous cubic cavity employing Tiwari and Das' model, the FDM, and a non-uniform mesh.

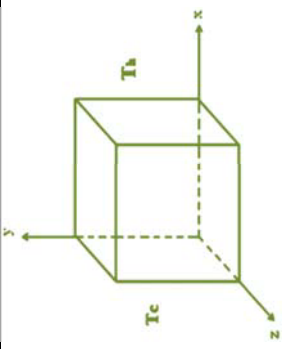
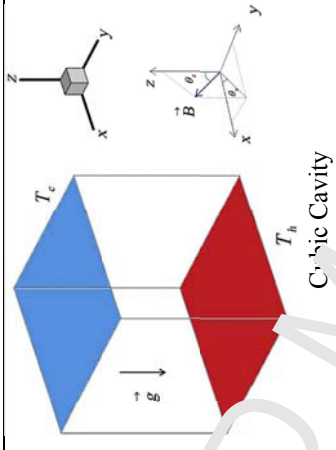
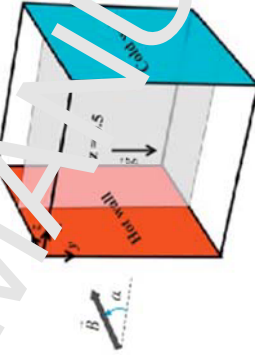
As can be appreciated at this stage, most papers model natural convection in nanofluids with single-phase approaches (mostly the homogenous temperature-independent technique). Table 2 summarizes the studies on 3D simulation of natural convection of nanofluids.

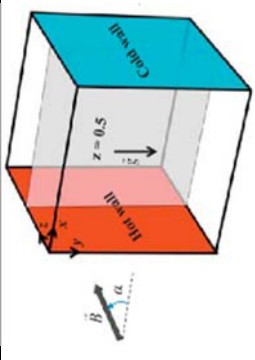
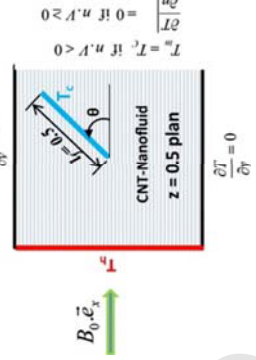
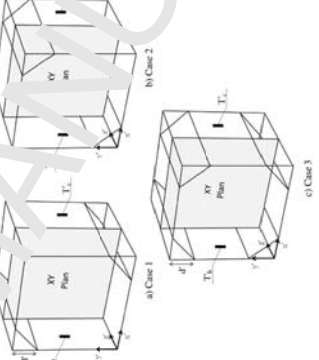
Table 2. Summary of studies on 3D simulation of natural convection of nanofluids (figures are reprinted by permission from publishers)

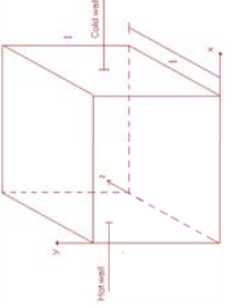
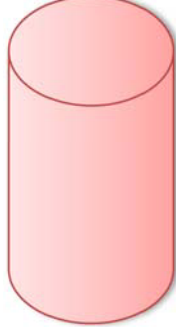
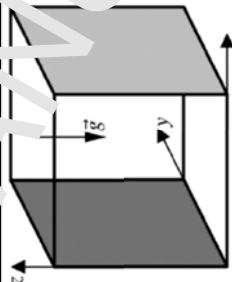
Boutra et al. [87]	LBM	-Single phase, homogenous model, Temperature independent properties	Ag/water	 <p>Cubic cavity</p>	n- Laminar	Higher thermal conductivity	With increasing Ra number, 2D modelling overestimates the amount of Nu number compared to 3D. The difference between 2D and 3D results increases with increasing nanofluid concentration
Ravnik and Škerget [88]	BEM	-Single phase, homogenous model, Temperature independent properties	Al ₂ O ₃ ; Cu and TiO ₂ nanoparticles suspended in water.	 <p>Cubic enclosure with a circular and ellipsoidal cylinder</p>	n- Laminar	Higher thermal conductivity	- With increasing the Ra number, the difference between 2D and 3D results increases - Heat transfer rate estimated by 3D modelling is lower than that of 2D -Cu/water nanofluid gives higher heat transfer enhancement because it has highest thermal conductivity.
Sheremet et al. [89]	FDM	Single phase, Buongiorno model	-	 <p>Porous cubic cavity</p>	n- Laminar	Higher thermal conductivity Brownian diffusion	With increasing the aspect ratio, the difference between 2D and 3D results for Nu number on hot wall decreases. The difference tends to zero for aspect ratios

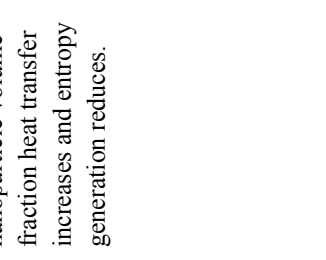
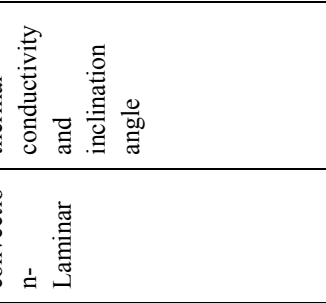
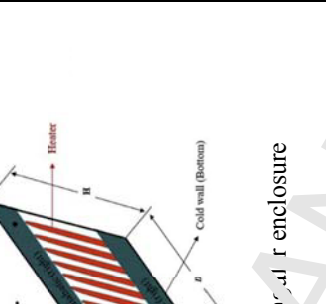
Saghir et al. [90]	FDM FEM LBM	Single phase, homogeneous model, Temperature independent properties	Al ₂ O ₃ /water	 <p>Cubic cavity</p> <p>Cavity with embedded discrete heaters</p>	Natural convection n-Laminar	Higher thermal conductivity	In the case of 2D simulation, FEM was more accurate than FDM and LBM. Comparison of 2D and 3D results for FEM unveils that the results of 2D modelling are closer to experimental data.
Purusothaman et al. [93]	FVM	-Single phase, homogeneous model, Temperature independent properties	Al ₂ O ₃ /water Cu/water	 <p>Cavity with embedded discrete heaters</p>	Natural convection n-Laminar	Higher thermal conductivity	Cu/water nanofluid gives higher heat transfer enhancement than Al ₂ O ₃ /water.

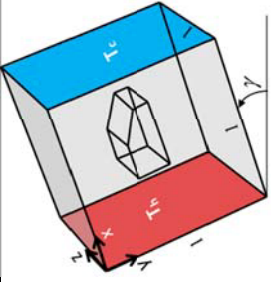
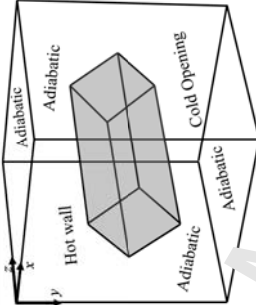
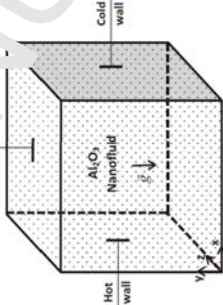
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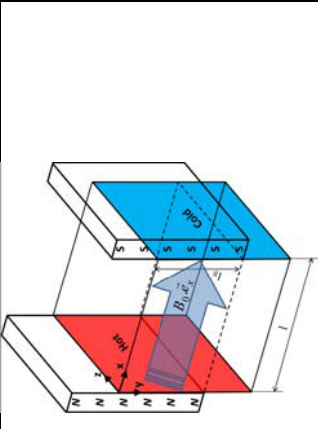
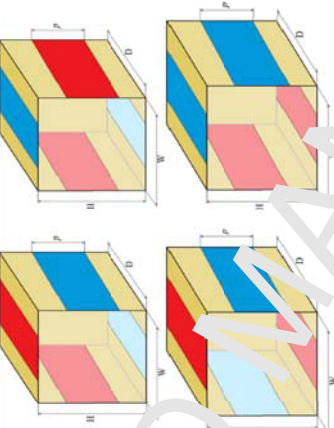
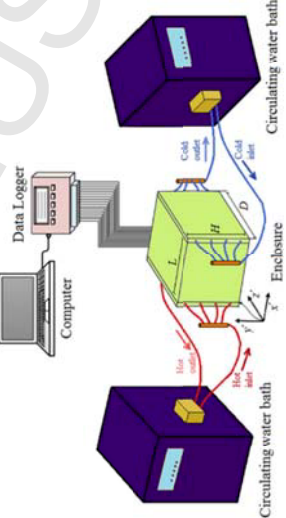
Snoussi et al. [94]	FVM/ ANSYS FLUENT	-Single phase, homogenous model, Temperature independent properties	water-based nanofluids containing Al_2O_3 and Ag nanoparticles	 <p>Cubic cavity</p>	Natural convection n-Laminar	Higher thermal conductivity	- Particle loading diminishes the heat transfer rate especially at high Ra numbers. - Silver nanoparticles provides slightly higher Nu number.
Sheikholeslami and Ellahi [100]	LBM	-Single phase homogenous model, Temperature dependent properties	Al_2O_3 /water	 <p>Cubic Cavity</p>	Natural convection n-Laminar	Higher thermal conductivity Brownian motion	Nusselt number would be enhanced through particle loading and boosting the buoyancy forces.
Al-Rashed et al [101]	FVM	-Single phase, homogenous model, Temperature independent properties	CNT/water	 <p>Cubic Cavity under magnetic field</p>	Natural convection n-Laminar	Higher thermal conductivity and greater magnetic field.	Optimum angle of magnetic force applied to the cavity to reach the maximum heat transfer rate was obtained for different concentrations and Ra numbers.

Al-Rashed et al [103]	FVM	-Single phase, homogenous model, Temperature independent properties	CNT/water	 <p>Cubic cavity under magnetic field</p>	Natural convection n-Laminar	Higher thermal conductivity and greater magnetic field.	Inclination angle is more significant at high Rayleigh numbers.
Kolsi et al. [104]	FVM	-Single phase, homogenous model, Temperature independent properties	CNT/water	 <p>Cubic cavity with a fin in middle</p>	Natural convection n-Laminar	Higher thermal conductivity and changing the fin angle.	By adding nanoparticles the heat transfer augments linearly - When the fin is at an angle of 180° the heat transfer is maximized.
Kolsi et al. [105]	FVM	-Single phase, homogenous model, Temperature independent properties	Al ₂ O ₃ /water	 <p>Cubic cavity with triangular solid insert</p>	Natural convection n-Laminar	Higher thermal conductivity	Modelling the problem as 3D is more helpful to understand the heat transfer mechanism compared 2D modelling.

Kolsi et al. [106]	FVM	-Single phase, homogenous model, Temperature independent properties	Al ₂ O ₃ /water	 <p>Cubic Cavity</p>	Natural convection n-Laminar	Higher thermal conductivity	With increases in Ra number and nanofluid concentration both average Nusselt number and entropy generation increase.
Meng and Li [107]	VVM/ OpenFOAM	Single phase, homogenous model, Both Temperature dependent and independent properties	Al ₂ O ₃ /water	 <p>Horizontal cylinder</p>	Natural convection n-Laminar	Higher thermal conductivity	- For water and $\phi = 1\%$, using temperature-dependent solver gives closer results to experimental data. - For $\phi = 4\%$, temperature-independent solver provides the minimum deviation from experimental data.
Terrik and Rudolf [108]	FVM	-Single phase, homogenous model, Temperature independent properties	Au/ carboxy methyl cellulose	 <p>Cubic cavity</p>	Natural convection n-Laminar	Higher thermal conductivity	- By loading nanoparticles, the onset of natural convection prolongs. - Nu number reduces with increases in nanofluid concentration.

Kolsi et al. [109]	FVM	-Single phase, homogenous model, Temperature independent properties	Al_2O_3 /water	 <p>Cubic cavity with twin adiabatic blocks</p>	Natural convection- Laminar	Higher thermal conductivity	Nusselt number increases with increases in Rayleigh number and nanofluid concentration
Salari et al. [110]	FVM	-Single phase, homogenous model, Temperature independent properties,	MWCNTs/water	 <p>Transient enclosure</p>	Natural convection- Laminar	-Higher thermal conductivity and inclination angle	With increasing nanoparticle volume fraction heat transfer increases and entropy generation reduces.
Al-Rashed et al. [111]	FVM	-Single phase, homogenous model, Temperature independent properties	CNT/water	 <p>Cubic cavity with incorporated Ahmed body</p>	Natural convection- Laminar	Higher thermal conductivity	Maximum average Nu number was achieved in inclination angles of 30 and 150 degrees whereas the average Nu number was minimized at 90°.

Al-Rashed et al. [112]	FVM	-Single phase, homogenous model, Temperature independent properties	CNT/water	 <p>Cubic cavity with incorporated Ahmed body</p>	Natural convection - Laminar	Higher thermal conductivity	Entropy generation increases with increases in volume fraction of nanoparticles
Kolsi et al. [113]	FVM	-Single phase, homogenous model, Temperature independent properties	Al ₂ O ₃ /water	 <p>Cubic cavity with diamond obstacle</p>	Natural convection - Laminar	Higher thermal conductivity	By adding nanoparticles both Nu and entropy generation increases.
Kolsi et al. [114]	FVM	-Single phase, homogenous model, Temperature independent properties	Al ₂ O ₃ /water	 <p>Cubic cavity with free top surface</p>	Natural convection - Laminar	Higher thermal conductivity	- Nusselt number increases with increases in Marangoni number. - Entropy generation rises with particle loading.

Al-Rashed et al. [115]	FVM	-Single phase, homogenous model, Temperature independent properties	CNT/water	 <p>3D cubic Cavity with magnetic field</p>	Natural convection n-Laminar	Higher thermal conductivity	- Adding nanoparticles increases the heat transfer rate - Heat transfer reduced with extension of active zone and Hartmann number.
Rahimi et al. [116]	LBM (based on Dual-MRT model)	-Single phase, homogenous model, Temperature dependent properties	CuO/water	 <p>Cubic cavity with different positions of heating</p>	Natural convection n-Laminar	Higher thermal conductivity Brownian motion	Adding nanoparticles increases heat transfer and decreases entropy generation.
Rahimi et al. [117]	FVM	-Single phase, homogenous model, Temperature dependent properties	DWCNTs/ water	 <p>Cubic cavity</p>	Natural convection n-Laminar	Higher thermal conductivity	- A combined experimental and numerical study was done for six concentrations of 0.01, 0.02, 0.05, 0.1, 0.2, and 0.4%. - Heat transfer coefficient is maximized at $\phi = 0.05\%$.

3.2. Forced convection of nanofluid flow

The study of forced convection flow is significant for heat exchangers. As such, researchers have simulated nanofluids in applications ranging from micro to conventional size composed and for straight and helical tubes. The hope for replacing conventional base fluids with nanofluids is that the heat transfer rate increases. However, this must be balanced with the fact that the pressure drop (pumping power) also increases due to the increase of viscosity. The ratio of heat transfer enhancement to pressure drop increment due to adding nanoparticles to base fluid is called thermal performance index (or the performance evaluation criteria). Different definitions of thermal performance index (shown by η_t) has been presented in the literature. For example, Bizhaem and Abbassi [119] used the following relationship as the thermal performance index (TPI) for the analysis of forced convection in helical tubes:

$$\eta_t = \frac{\left(\frac{h_{nf}}{h_f}\right)}{\left(\frac{\Delta p_{nf}}{\Delta p_f}\right)^{0.166}} \quad (1)$$

Hashemi and Akhavan-Behabadi [120] used the following equation as the TPI in the study of nanofluid flow in horizontal coiled tube:

$$\eta_t = \frac{\left(\frac{h_{nf}}{h_f}\right)}{\left(\frac{\Delta p_{nf}}{\Delta p_f}\right)} \quad (2)$$

Yang et al.[121] investigated the nanofluid flow in a parallel channel by assuming the same pumping power for all conditions. They used the following equation as the TPI:

$$\eta_t = \frac{\left(\frac{Nu_{nf}}{Nu_f}\right)}{\left(\frac{f_{nf}}{f_f}\right)^{1/3}} \quad (3)$$

A greater TPI implies that nanofluid efficacy increases.

Next, the studies conducted on forced convection of nanofluids are reviewed in two main sections (i.e. single-phase and two-phase studies).

6.2.1. Studies using the single-phase approach

Doshmanziari et al. [122] simulated the forced convection turbulent flow and heat transfer of Al_2O_3 /water nanofluids with volume fractions up to 1.5% in a horizontal spiral-coil tube which was kept in a water bath with a constant temperature of 60 °C. Figure 22 shows a schematic of the problem. Nanofluid enters the coil with a constant temperature of 34 °C and after heating in the bath exits. The 3D numerical results were compared against experimental data and showed a maximum 17% deviation. They concluded that using nanofluid with a volume fraction of 1.5% instead of water can enhance the heat transfer rate up to 47% for Reynolds numbers between 9,000 and 21,200. The authors assumed that there is no slip between nanoparticles and base fluid (single-phase homogenous approach was used). Moreover, the FVM was used to discretize the governing equations while RNG k- ϵ model was employed to model the turbulent flow. For thermal conductivity and viscosity, the Corcione correlations were used.

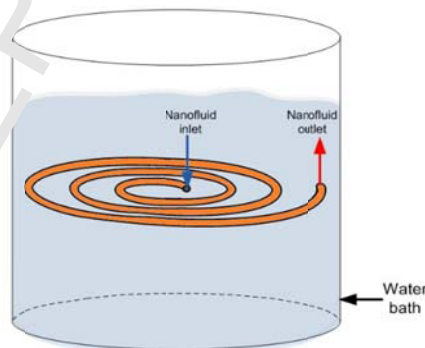
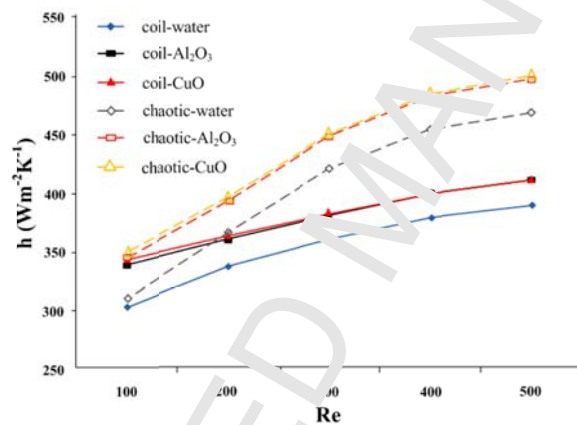
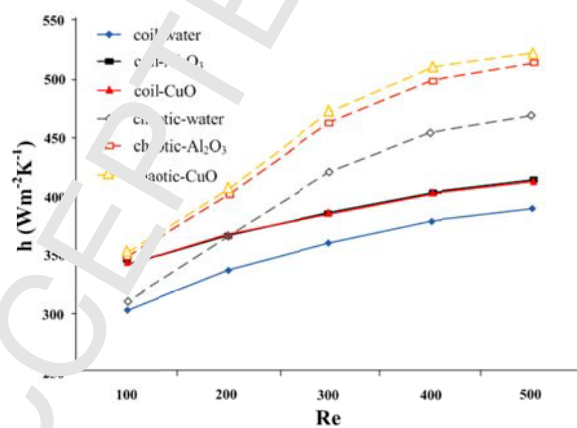


Figure 22. A schematic of the problem considered by Doshmanziari et al. [122]

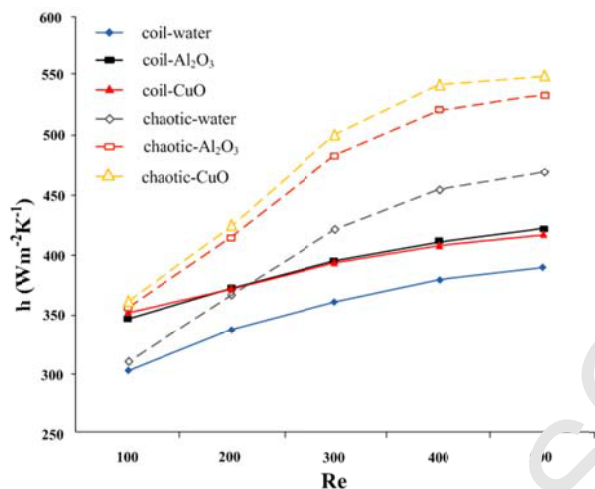
Tohidi et al. [123] applied two passive approaches for heat transfer enhancement inside a heat exchanger with helical tubes. The two techniques included the use of nanofluids instead of water as well as changing the simple structure of helical tubes to chaotic configuration. The results were presented for $\text{Al}_2\text{O}_3/\text{water}$ and CuO/water nanofluids, nanoparticle concentration up to 3%, and Reynolds numbers between 100 and 500. The single-phase governing equations were solved using FEM through the commercial software of ANSYS FLUENT. They concluded that a water-based heat exchanger with chaotic configuration provides higher heat transfer rates in comparison with a nanofluid-based heat exchanger having simple coils. It should be noted in chaotic tubes the flow turbulence is more than simple tubes that it provides higher heat transfer rate. In addition, they found that CuO/water nanofluids are more advantageous than $\text{Al}_2\text{O}_3/\text{water}$ nanofluids. Figure 23 compares the heat transfer rate for different structures and nanofluids.



(a)



(b)



(c)

Figure 23. Variations of heat transfer coefficient with Reynolds number for volume concentrations of (a) 1%, (b) 2%, and (c) 3% and different configurations [123](figure is reprinted with permission from publisher).

Togun et al. [124] solved the turbulent flow of Al₂O₃/water nanofluid with Reynolds numbers between 20000 and 50000 and volume fractions up to 2% in an annulus with sudden expansion by employing the FVM. The effects of Reynolds number, nanofluid concentration, and expansion ratio on flow and heat transfer characteristics have been investigated in the study. The authors concluded that with increasing the working fluid velocity and nanoparticle volume fraction along with a larger expansion ratio the surface heat transfer coefficient can be enhanced. In the simulation, k-ε model was used to capture the turbulence flow while the nanofluid was assumed as single-phase. Maxwell and Brinkman models were used to estimate the thermal conductivity and viscosity.

Shalchi-Tabrizi and Seyf [125] investigated numerically heat transfer and entropy generation of alumina/water nanofluids in a conical microtube heat sink using single-phase homogenous approach (temperature dependent properties). The geometry has shown in Fig. 24. The flow was assumed laminar with forced convection mode with Reynolds numbers between 50 and 200. Nanoparticles with two different sizes (i.e. 29 and 47 nm) and concentrations from 0% up to 4% were employed in the simulations. The results of this study suggested using nanoparticles with a smaller size, and higher concentration to have a higher heat transfer rate and lower entropy generation. In another work with similar geometry used in [125], Seyf et al. [126] simulated heat transfer, pressure drop and entropy generation in the microtube heat sink where nanofluids having a nano-encapsulated phase change material (NEPCM) enters the ducts of heat sink from the top. The results revealed that particle loading can enhance the heat transfer rate in the system. Moreover, increasing the Reynolds

number and the melting point of nanoparticles reduces the thermal resistance, and, consequently, the heat transfer rate ameliorates. It was shown that increasing the two main independent variables (mass concentration and Reynolds number) both have a major effect on the total entropy generation rate which means that irreversibilities in the system reduce.

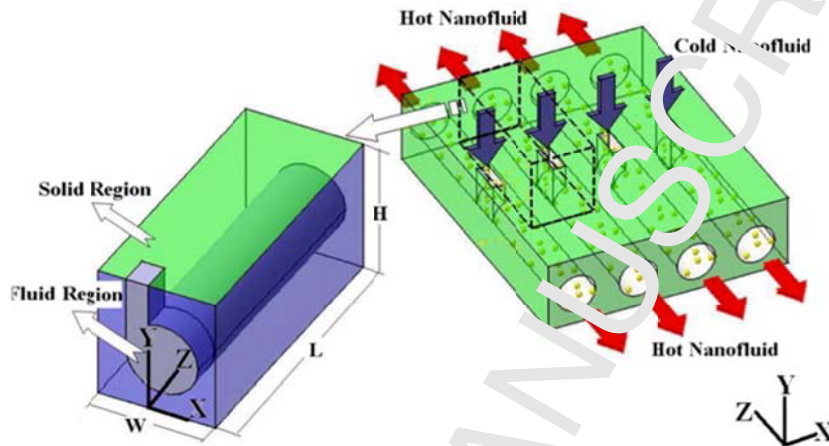


Figure 24. Schematic of microtube microchannel heat sink with tangential impingement [125] (figure is reprinted with permission from publisher)..

Yavuz and Tokgoz [127] studied single-phase flow of water, argon, Al_2O_3 /water nanofluid (with a concentration of 8%) in a tube with rectangular cross section where 8 square ribs were designed in the bottom side and 8 square ribs in the top side for disrupting the boundary layer. They concluded that using argon is more reasonable than using water and nanofluid.

Gunnasegaran et al. [128] considered the cooling of a compact heat exchanger as shown in Fig. 25. A nanofluid (coolant) with a volume fraction of 2% and under turbulent regime enters the channel with temperature of 82.5°C while the inlet temperature of air is 37.5°C . The base liquid of nanofluids was ethylene glycol and three different nanoparticles were considered in the analysis including Cu, diamond, and SiO_2 . It was found that to achieve highest heat transfer coefficient diamond nanoparticles should be used. Higher heat transfer coefficients were reported in the case of SiO_2 nanoparticles compared to Cu particles.

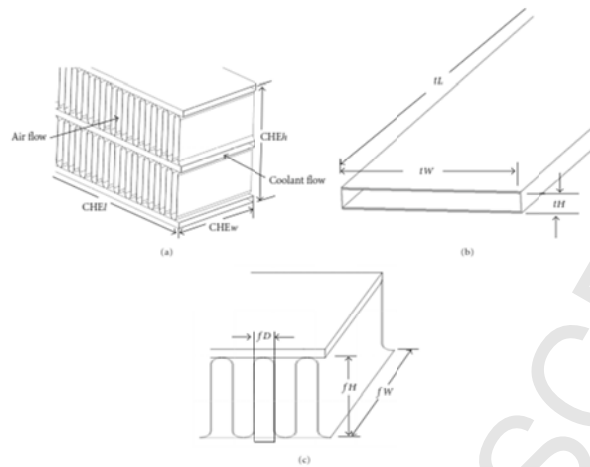


Figure 25. Schematic of the problem considered by Gunnasegaran *et al.* [128] (figure is reprinted with permission from publisher).

Seyf and Feizbakhshi [129] simulated nanofluid flow in micro-pin-fin heat sinks using FVM and single-phase homogenous approach. They used two types of nanoparticles including CuO (in diameters of 28.6 and 29 nm) and Al_2O_3 (in diameters of 38.4 and 47nm) and water as the base fluid. The results were presented for volume fractions of 1 and 4%, and Reynolds numbers between 25 and 75. The authors reported that with increasing the volume fraction of nanoparticles and Re, the heat transfer enhances for both types of nanoparticles and the main reasons may be the Brownian motion of nanoparticles that enhances the micro-convection in the system. It was also concluded that pressure drop increase induced by particle loading is not much significant. The article also investigated the effect of nanoparticle size. It was concluded that with increasing the particle size, the heat transfer enhanced in the case of CuO/water nanofluids while for CuO/water nanofluid an opposite trend was obtained. The reason for this was attributed to thermal conductivity changes that for CuO/water nanofluid with increasing the particle size the thermal conductivity increases, but for $\text{Al}_2\text{O}_3/\text{water}$ the trend was reversed.

Dugaria *et al.* [130] investigated turbulent forced convection of single wall carbon nanohorns (SWCNHs)/water nanofluids in a direct absorption solar receiver. Single-phase homogenous approach was employed in the modeling. Because of low concentrations of nanoparticles, thermophysical properties of nanofluids were assumed to be equal to water and just a function of temperature. Instead, optical properties of nanofluids were involved in the governing equations. It was found that

among nanoparticle concentrations of 0.006, 0.01, 0.02, and 0.05 gr/lit the optimal concentration is 0.02gr/lit in which the receiver thermal efficiency is maximized.

Tiwari et al. [131] experimentally studied turbulent flow of Al_2O_3 /water (with a concentration of 1%) and CeO_2 /water (with a concentration of 0.75%) nanofluids in a plate heat exchanger. Next, CFD simulation was done using single-phase homogenous approach with temperature dependent properties and ANSYS/Fluent software. It was found that CeO_2 /water as the working fluid illustrates better performance.

Mukherjee et al. [132] investigated alumina/water nanofluid turbulent flow with Reynolds numbers between 6,000 and 20,000 in a duct with trapezoidal ribs. Nusselt number enhances by 42.5 and 40.1% for volume fractions of 5 and 3%, respectively. Entropy generation was increased by adding nanoparticles to base fluid.

Hussein et al.[133] simulated forced turbulent convection of TiO_2 /water nanofluid (volume fractions up to 4%) in a duct with application in car radiator using single phase homogenous approach (temperature-independent properties). The results were presented for Reynolds numbers between 10000 and 100000 and inlet temperatures of 60-90 °C. It was found that by using nanofluid with a concentration of 4%, the heat transfer rate can be augmented by 18% although friction factor rises by 12%. As expected Nu increases with increases in inlet temperature and Reynolds number.

Sheikholeslami et al. [134] employed the LBM to study the forced convection of alumina/water nanofluid(volume fractions up to 4%) in a cavity where the bottom wall moves with a constant velocity. The authors investigated the effect of Lorentz force intensity (due to magnetic field), wall velocity, and volume fraction of nanoparticles on heat transfer and fluid flow in the cavity. They indicated that with increasing the Lorentz force the velocity and temperature gradients reduce. It was also found that Nusselt number increased with increases in nanoparticle volume fraction, and velocity of wall but it decreased with raising the Lorentz force. The authors used the KKL model for properties.

Hawwash et al. [135] studied the performance of a flat plate solar collector using alumina/water nanofluids (volume fraction is 0.1-3%) both experimentally and numerically. Single-phase homogenous approach with temperature dependent properties and ANSYS/FLUENT was used for simulation. It was found that the thermal efficiency of solar collector was optimized at a concentration of 0.5%. The

authors stated that adding nanoparticles with volume fractions higher than 0.5% cannot enhance the solar collector's efficiency because of agglomeration of nanoparticles.

Seyf and Nikaaein [136] employed a single-phase homogenous approach (temperature dependent properties) to model the forced convection of nanofluid flow in microchannel heat sinks. Three different types of nanoparticles including CuO, ZnO, and Al₂O₃ suspended in a mixture of water and EG (40:60 by mass) and two sizes of 29 and 77 nm were examined. The authors indicated that CuO nanoparticles provide the highest thermal performance followed by ZnO and Al₂O₃ nanoparticles. It was found that heat transfer ameliorates by increasing the volume fraction of nanoparticles and decreasing the size of nanoparticles. Brownian motion and micro-convection were introduced as the main phenomena for heat transfer enhancement.

Najafabadi and Moraveji [137] solved forced laminar convection flow and entropy generation of Al₂O₃/water nanofluid in a microchannel with converging channels under imposed pressure difference and considering partial slip boundary condition. The problem was solved with OpenFOAM and a single-phase homogenous approach with temperature dependent properties. It was found that if a partial slip boundary condition is considered for the problem, even single-phase approach can predict well the experimental data. It was also noted that with increasing the nanoparticles to base fluid, heat transfer enhances and entropy generation diminishes.

Chen and Cheng [138] studied numerically the potential of TiO₂/water nanofluids (volume fractions up to 3%) for cooling of stove of blast furnace. Single-phase homogenous model was used to predict the turbulent flow (56,800 to 75,700) of nanofluids in the heat exchanger by considering aggregation of nanoparticles. It was found that heat transfer coefficient increases with an increase in the volume fraction of nanoparticles. Also, heat transfer coefficient increases linearly with Re. It was elucidated that with aggregation of nanoparticles heat transfer coefficient reduces while pressure drop increases.

Hybrid nanofluids are a kind of nanofluids which are composed of two different types of nanoparticles. In hybrid nanofluids, usually a kind of nanoparticles with high thermal conductivity (such as CNTs) is added to another kind of nanoparticles with lower thermal conductivity for enhancing the effective thermal conductivity of mixture. Nanoparticles such as CNTs with high thermal conductivity have high price so this mixing them with cheaper nanoparticles is reasonable.

Huminić and Huminić [139] analyzed heat transfer and entropy generation of two different hybrid nanofluids including multi-walled carbon nanotubes + Fe₃O₄/water (with volume fractions of 0.1 and 0.3%) and nanodiamond + Fe₃O₄/water (with volume fractions of 0.1 and 0.2%) in a flattened

tube with application in car radiator. For properties, experimental data available in the literature were used for calculating thermal conductivity and viscosity which were as a function of nanoparticle concentration and temperature. Single-phase homogenous model and ANSYS/CFX software were applied to solve the laminar forced convection problem. It was found that using hybrid nanofluids enhances the heat transfer rate in the system. Moreover, the results revealed that to minimize the entropy generation the use of multi-walled carbon nanotubes + Fe₃O₄/water is more advantageous than nanodiamond + Fe₃O₄/water.

Recently, Alfaryjat et al. [140] studied numerically the effects of different types of nanoparticles including Al₂O₃, CuO, SiO₂ and ZnO in four sizes i.e. 25, 40, 55 and 70 nm suspended in water on flow and heat characteristics of nanofluid forced convection in a hexagonal microchannel heat sink. It was found that heat transfer coefficient was maximized by Al₂O₃ nanoparticles and it was followed by CuO, ZnO, and SiO₂ nanoparticles. Indeed, using nanoparticles with a higher thermal conductivity guaranteed a higher heat transfer coefficient. The heat transfer coefficient augments with increasing the volume fraction of nanoparticles and decreasing the size of nanoparticles. On the other hand, pressure drop increases with reducing the nanoparticle size since the effective viscosity of nanofluids having smaller nanoparticles is higher. For solving the Navier-Stokes equations, ANSYS/FLEUNT was used by assuming a single-phase homogenous model for nanofluids.

Ghasemi and Ranjbar [141] modeled turbulent forced convection in a receiver tube of a parabolic trough collector using CuO/water and Al₂O₃/water nanofluids with volume fractions up to 3% with very high Reynolds numbers i.e. about 754000. To solve the equations, single-phase homogenous model along with temperature-dependent properties were used. For modeling the turbulent flow k-ε model was utilized. The results indicated that using Al₂O₃/water and CuO/water nanofluids at a volume fraction of 3% can enhance the heat transfer coefficient by 28% and 35%, respectively. Since this work dealt with modeling of a parabolic trough collector; it is useful that in future works some results on outlet temperature and efficiency of solar collector are presented.

Karimi and Afrand [142] investigated the influences of hybrid nanofluid, tube arrangement, and tube cross section (circular and elliptical) on the thermal efficiency of an air-cooled heat exchanger. They concluded that the heat exchanger with circular tubes has about 25% smaller pressure drop, while elliptical pipes have 10% larger heat transfer rate. Moreover, they concluded that the Nusselt number and pressure drop increase by nanoparticle loading.

Hosseinzadeh et al. [143] simulated numerically a nanofluid based photovoltaic thermal device. They reported that the thermal energy performance of the ZnO/water nanofluid with a solid volume fraction of 0.03 is about 12.78% higher compared to water.

Toghraie et al. [144] investigated the combined impacts of nanofluid and porous ribs on the efficiency of microchannels. It was concluded that by using the porous ribs, the nanofluid has not a significant effect on heat transfer augmentation.

3.2.2. Studies using the two-phase approach

Khoshvaght-Aliabadi et al. [145] simulated laminar flow of Al_2O_3 /water nanofluid in twisted minichannels having different shapes of cross section including elliptic, half circular, square, rectangular, and triangular. The results were presented for volume concentrations up to 4%, Reynolds numbers less than 1500, and three different ratios of twist pitch to channel length i.e. 0.25, 0.5, and 1. The modeling was done based on two-phase mixture model and FVM. They concluded that by using a nanofluid with a volume fraction of 1% the performance index is maximized.

Zhu et al. [146] numerically studied laminar flow of Al_2O_3 nanoparticles suspended in a binary mixture of ethylene glycol and water inside two microchannel heat sinks with different arrangements of channels. The first microchannel heat sink had straight channels, and the second one had cosine shaped channels. The study was done for volume concentrations up to 4% and Reynolds numbers up to 1,200. It was found that using nanofluid with volume concentrations higher than 1% is not beneficial based on performance index criterion. Although with increasing the nanoparticle concentration, the heat transfer enhances but on the other hand, pressure drop rises, therefore, volume fraction of 4% is not the optimal concentration. A two-phase mixture model was used in the numerical study.

Bizhaem and Abbasi [147] investigated forced convection and entropy generation of alumina/water nanofluid (volume fractions up to 3%) in a helical tube using two-phase mixture model for Reynolds numbers between 200 and 2000. It was found that nanofluid effect on heat transfer enhancement is not visible at entrance region but in fully developed region the heat transfer enhancement is significant. Their results indicated that at $\phi = 3\%$ for $Re = 200$ the heat transfer enhancement (ratio of heat transfer coefficient of nanofluid to water) reaches about 22% while for $Re=2000$ the heat transfer enhancement approaches 15.7%. By adding nanoparticles, heat conduction in the system improves therefore

temperature gradients diminish and consequently entropy generation reduces especially at low Reynolds numbers. As expected, using helical tubes ameliorates heat transfer rate since centrifugal force applies to working fluid.

Delavari and Hashemabadi [148] simulated the cooling of a car radiator using alumina nanoparticles where water and ethylene glycol were considered as base fluids. The numerical study presents a comparison between single-phase and two-phase approaches for both laminar and turbulent regimes. Nusselt number obtained by two-phase model was greater than that of obtained by single-phase approach (between 10-45%). Experimental data confirms that two-phase model results are more reliable and closer to tests. Figure 26 compares experimental data with results of single-phase and two-phase approaches.

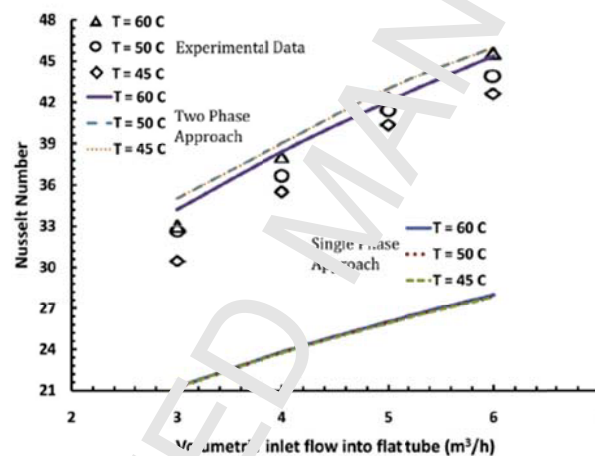


Figure 26. Comparison between experimental data [149] with CFD results obtained by

Delavari and Hashemabadi [148] for 0.7% Al_2O_3 /ethylene glycol (figure is reprinted with permission from publisher).

Ahmed et al. [150] investigated turbulent forced convection of Al_2O_3 and CuO nanoparticles suspended in water flowing in a triangular duct with vortex generators. Two-phase mixture model was considered to solve the nanofluid flow by using ANSYS/FLUENT software. The results were presented for volume fractions up to 3% and Reynolds numbers between 4000 and 24000. It was found that adding nanoparticles even at a low concentration of 0.5% is more effective than equipping the plain duct to vortex generator. The authors showed that alumina/water nanofluid with a volume

fraction of 3% provides the maximum heat transfer rate even more than CuO/water nanofluid with the same concentration. This was attributed to higher Prandtl number of alumina based nanofluids.

Ghale et al. [151] studied laminar forced convection flow of alumina/water nanofluid (with concentrations up to 2%) in a ribbed microchannel heat sink using both single phase (homogenous independent-temperature) and two-phase (mixture model) approaches. It was found that single-phase approach underestimates the heat transfer rate with a high error so that the deviation from experimental data becomes 32.6% and 37.4% for 1% and 2% nanofluid, respectively. However, the two-phase mixture model gives very good prediction of heat transfer rate especially for a volume concentration of 2%. For the case of the mixture model, the deviation from experimental data reduces to 11.39% and 2% for 1% and 2% nanofluids, respectively. It was also indicated that using ribs increases both heat transfer and friction factor. The simulation was done by ANSYS/FLUENT for Reynolds numbers less than 900.

Kumar and Sarkar [152] employed single-phase and two-phase mixture models to study the laminar forced convection flow of Al_2O_3 /water and Al_2O_3 +MWCNTs/water nanofluids in a minichannel heat sink. The numerical results for heat transfer coefficient were compared to experimental results for the case of Al_2O_3 +MWCNTs(9:1)/water nanofluid at Reynolds numbers of 307, 368, and 430 as shown in Fig.27. As seen, in the case of nanofluids, two-phase mixture model overestimates the experimental data while single-phase underestimates the experimental data. Based on the figure, it cannot be stated that two-phase mixture model gives certainly better results since as shown at $\text{Re} = 368$ the results of experimental data and single-phase approach are closer to each other compared to experimental data and two-phase mixture model results. In general, it was also found that by using nanofluid instead of water, heat transfer coefficient increases. The maximum enhancement in the heat transfer coefficient in the case of Al_2O_3 /water nanofluid with a concentration of 0.01 vol.% was about 4% while for the same concentration of hybrid nanofluid (Al_2O_3 +MWCNT) it reached 9%, 12% and 15.6% for the nanoparticle volume ratio of (9:1), (8:2), and (7:3) respectively.

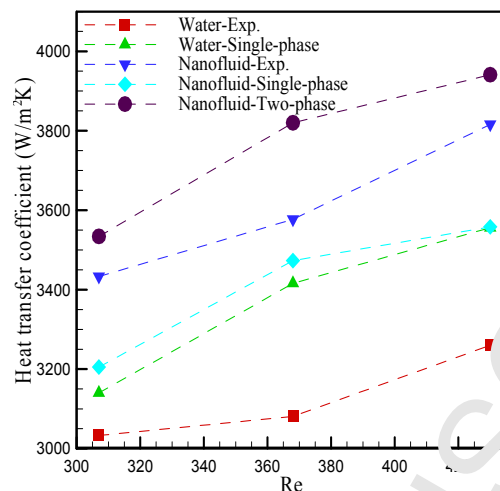


Figure 27. Comparison of single-phase and two-phase approaches with experimental data[152] (figure is reprinted with permission from publisher)..

Yang et al. [153] investigated the turbulent flow of Al_2O_3 /water nanofluids (concentration up to 4 vol.%) in a channel with square cross section where the bottom of channel was equipped to step-shaped turbulators. To solve the governing equations, the turbulence model was implemented while the equations were discretized through finite volume approach. The simulations were conducted for single-phase and mixture models and the results were compared. Both response surface method (RSM) and genetic algorithm (GA) were utilized to optimize the performance factor (the ratio of Nusselt number enhancement to pressure drop increase caused by using nanofluid instead of base liquid) in the channel for Reynolds numbers between 15,000 and 20,000. The results of this study show that single-phase approach estimates the Nusselt number and performance factor lower than that of mixture model (see Fig. 28). The authors stated that since in the mixture model the interactions between nanoparticles as well as the drift velocity between base fluid and solid particles have been considered in the governing equations, so the mixture model provides higher values of Nusselt number. An interesting point that is observed in Fig. 27 is that the same trend of graphs was obtained from both the single-phase and the mixture approaches.

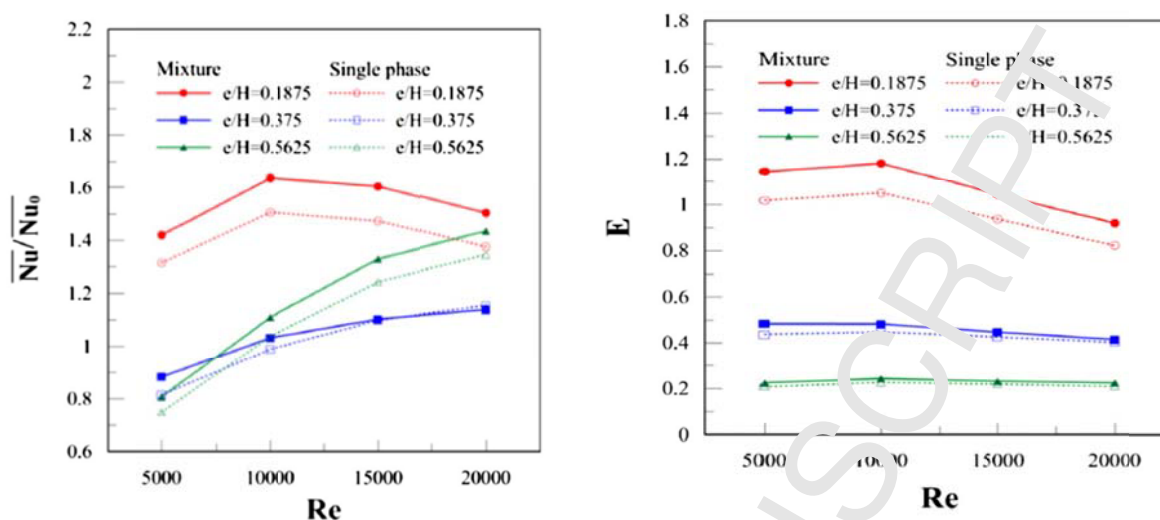


Figure 28. Comparison between single-phase and mixture models (a) Nusselt number enhancement (b) Performance factor (for volume fraction of 4%) [153] (figure is reprinted by permission from publisher).

Later, in a similar work to [153], Yang et al. [154] solved the nanofluid flow in channels with wavy walls where the flow was turbulent and both single and mixture models were applied to obtain the velocity and temperature fields. In this work, they considered different values of amplitude and wavelength for the walls. Similarly, they found mixture model gives higher values for heat transfer.

Bahiraee et al. [155] modeled alumina/water nanofluid flow in a shell and tube heat exchanger with helical baffles applied using both CFD (two phase mixture approach) and a neural network. The authors concluded that although particle loading leads to increases in pressure drop, but heat transfer enhancement is high enough to neglect the pressure drop penalty. For reaching high heat transfer and low pressure drop simultaneously, the helix angle selected should be small.

Ghasemi et al. [156] investigated laminar forced convection of CuO/water nanofluid (for volume fractions up to 0.75%) in heat sinks having channels with two different cross sections i.e. circular and square. Eulerian-Eulerian two phase approach and single-phase homogenous model with temperature-independent thermophysical relations were used for the simulation. In the Eulerian-Eulerian two-phase approach lift force was neglected and just drag force was involved in the equations. They compared the results obtained from single-phase and two-phase approaches by experimental data available in the literature for $400 < Re < 1000$. The comparison showed both approaches overestimated Nusselt number. The maximum deviation between single-phase approach and experimental data was 8% while it reached 6% by using two-phase approach. The results revealed that adding nanoparticles and

increasing Re enhance the Nu number, on the other hand, friction factor increases with increasing nanoparticle volume fraction and reduction of Re . It should be noted that the value of Nu was higher for heat sink with rectangular channels. The authors concluded that using minichannel heat sink with rectangular channels is more advantageous to employ in cooling of electronic devices since the base temperature is lower.

Mashayekhi et al. [157] studied the laminar forced convection flow of $Ag/water$ nanofluids through tubes with staggered and non-staggered inserts as shown in Fig. 29. They solved the governing equation by ANSYS FLUENT and two-phase approach (Eulerian-Lagrangian method) which considers drag, Saffman lift, thermophoretic, and Brownian forces. The results were presented for Reynolds number between 100 and 1500, nanoparticle volume fractions up to 5%, and three types of tube i.e. plain tube (without insert), tube with staggered inserts, and tube with non-staggered inserts. The results indicated that adding nanoparticles enhances both heat transfer rate and friction factor. On the other hand, it was revealed that although non-staggered structure illustrates higher heat transfer rates but staggered structure provides higher Figure of Merit (FoM) because of lower friction factor. With increasing Re the parameter of FoM increases too.

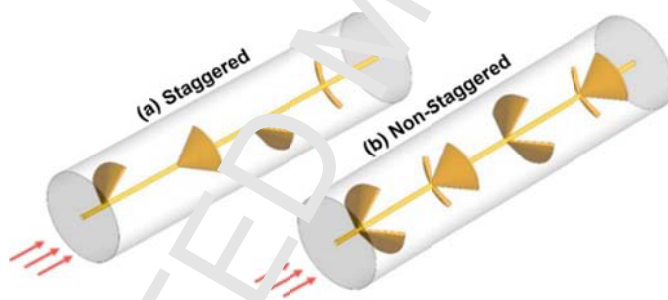


Figure 29. A mixing insert studied by Mashayekhi et al. [157]: (a) staggered alignment and (b) non-staggered alignment (figure is reprinted by permission from publisher).

Naphon and Nakharintra [158] experimentally studied the forced convection flow of $TiO_2/water$ nanofluid with a volume concentration of 0.4% through a minichannel heat sink. Next, they made a 3D model of the system, with both single-phase (homogenous) and two-phase (mixture and VOF) approaches. They found that two-phase approaches are more suitable for modeling the experimental system. The authors used $k-\epsilon$ turbulent model to capture the Brownian motion of nanoparticles while the investigated Reynolds numbers were less than 200.

Rostami and Abbasian [159] simulated laminar forced convection in a wavy microchannel using alumina/water nanofluid (with volume concentrations up to 2%) by both single-phase homogenous and

two-phase Eulerian–Lagrangian approaches. The results were compared with experimental data available in the literature. It was found that the results of two-phase Eulerian–Lagrangian approach is close to experimental data while single-phase approach underestimates the heat transfer rate. Adding 2% nanoparticles to the base fluid led to about 12% heat transfer enhancement and 2.4% increment in pressure drop. It was also concluded that it is more effective to use nanofluids in wavy microchannels instead of microchannels with straight tubes.

Shi et al. [160] modelled laminar forced convection flow in a microchannel using Eulerian-Lagrangian approach and by considering the effects of drag force, the gravity, the Brownian motion, the thermophoretic force, Saffman lift force, and pressure gradient force. It was found that by using nanofluids with a concentration of 2%, the heat transfer coefficient enhances about 8.5% whereas this enhancement for volume fraction of 1% is about 5.9%.

Bhattad et al. [161] experimentally investigated turbulent forced convection flow in a plate heat exchanger using Al_2O_3 /water and Al_2O_3 +MWCNTs/water nanofluids. Next, the experimental data were compared to simulation results of single-phase and two-phase approaches. The main results of this study can be summarized as:

- The comparison between numerical models and experimental data in the case of Al_2O_3 /water nanofluid showed that both single-phase and two-phase approaches underestimate the cold outlet temperature from the heat exchanger, and single phase results are closer to experimental data compared to two-phase approach.
- The comparison between numerical models and experimental data in the case of hybrid nanofluid i.e. Al_2O_3 +MWCNTs/water showed that two-phase approach provides better predictions of cold outlet temperature compared to single-phase approach.
- Hybrid nanofluid showed better performance compared to Al_2O_3 /water nanofluid since adding MWCNTs with high thermal conductivity improves the effective thermal conductivity of mixture.
- By using hybrid nanofluid instead of water, the heat transfer coefficient increases up to 39% while pumping power increases just about 1.2%.

Sheikhalipour and Abbassi [162] used the Buongiorno's two-component non-homogeneous and Eulerian-Lagrangian two-phase models to simulate a nanofluid flowing in a trapezoidal microchannel. Their findings indicated that the Eulerian-Lagrangian approach predicts the heat

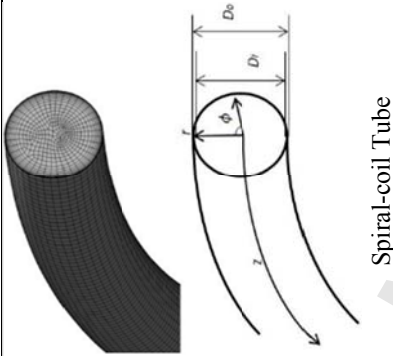
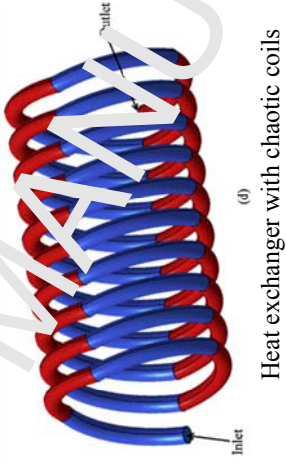
transfer coefficient of microchannel with higher precision. It was found that when the size of nanoparticle reaches 100 nm, the magnitude of thermophoretic force is greater than that of Brownian force at the entrance region of the tube.

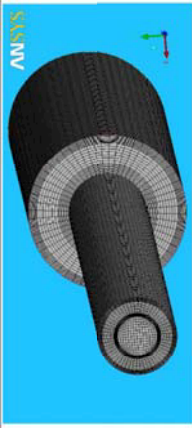
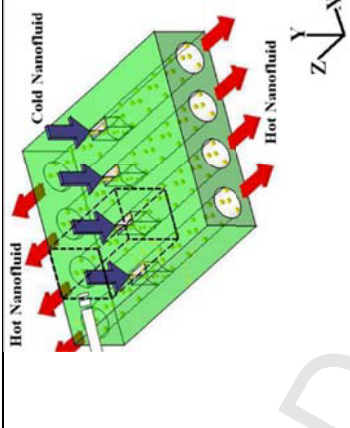
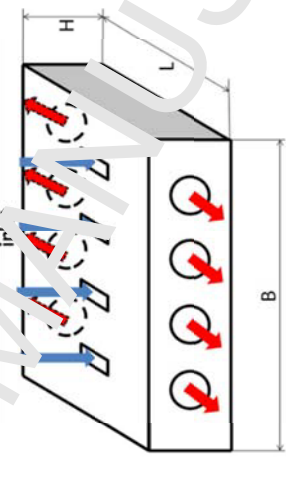
Moraveji et al. [163] employed different methods including single phase, mixture, VOF, and Eulerian methods to simulate dilute heat-dissipating nanofluid flow in the microchannel. They found that there is an acceptable agreement between all methods with the experimental data.

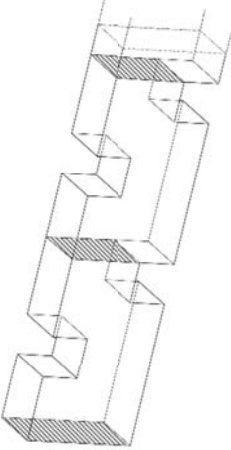
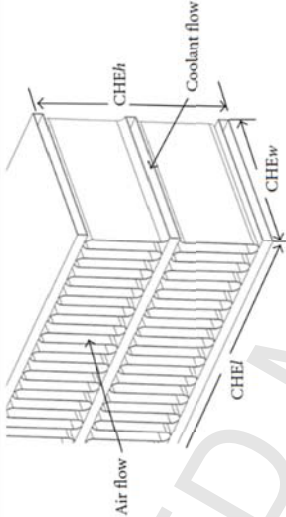
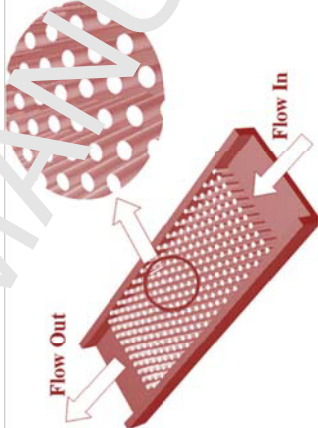
Moreover, they reported that using single-phase method to model the nanofluid flow through the microchannel is more accurate than two-phase methods.

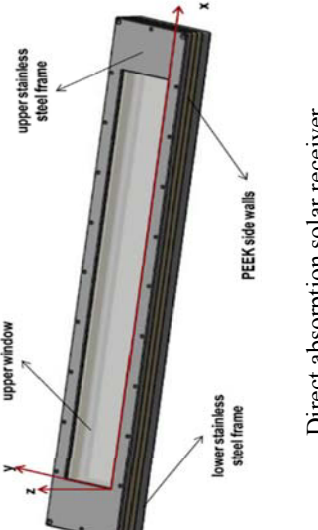
Table 3 provides a summary of studies on 3D simulation of forced convection of nanofluids.

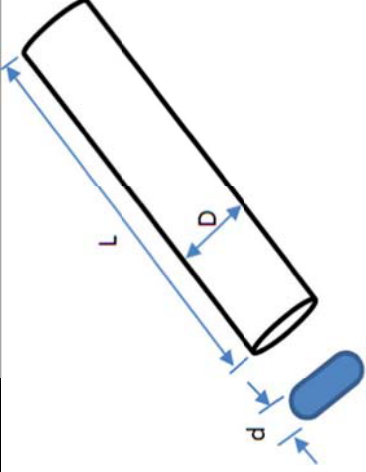
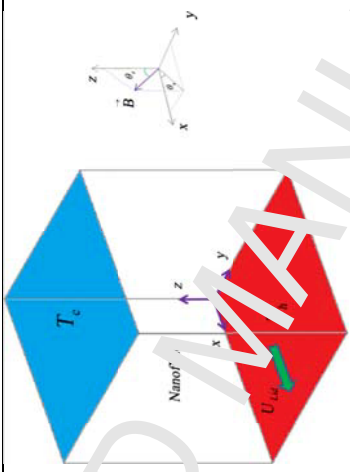
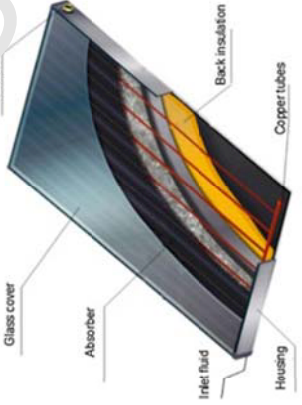
Table 3. Summary of studies on 3D simulation of forced convection of nanofluids (figures are reprinted by permission from publishers).

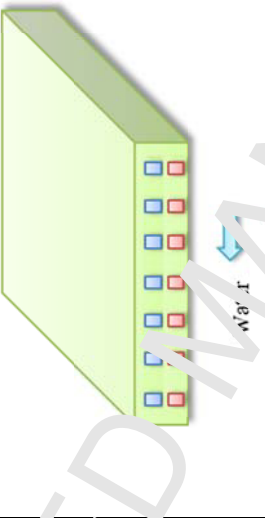
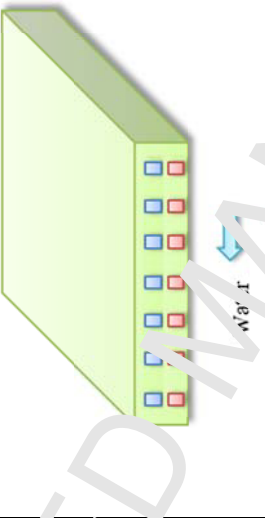
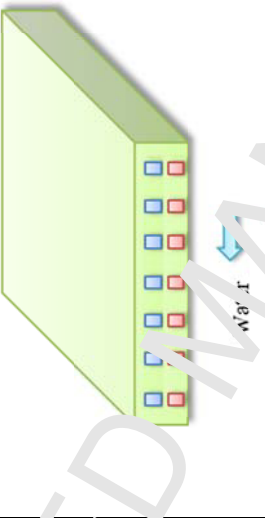
Authors	CFD technique(s) /Software	Physical Model	Nanofluid type	Geometry	Flow regime	Factors of heat transfer enhancement	Concluding remarks
Doshmanzari et al. [122]	FVM	Single phase, homogenous model, Temperature dependent properties	Al ₂ O ₃ /water	 <p>Spiral-coil Tube</p>	Forced convection n-Turbulent	Higher thermal conductivity	-Using nanofluid with a volume fraction of 1.5% instead of water can enhance the heat transfer rate up to 47% for Reynolds numbers between 9000 and 21200.
Tohidi et al. [123]	FEM/ ANSYS FLUENT	Single phase, homogenous model, Temperature dependent properties	CuO/water and Al ₂ O ₃ /water	 <p>Heat exchanger with chaotic coils</p>	Forced convection n-Laminar	Brownian motion and higher thermal conductivity	-CuO nanoparticles gives higher heat transfer enhancement than that of Al ₂ O ₃ . - A water-based heat exchanger with chaotic configuration provides higher heat transfer rates than a nanofluid-based heat exchanger having simple coils

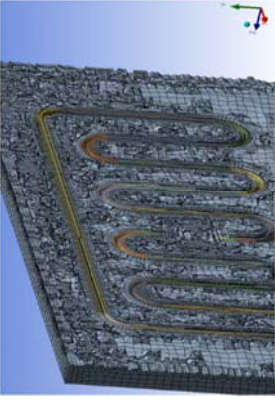
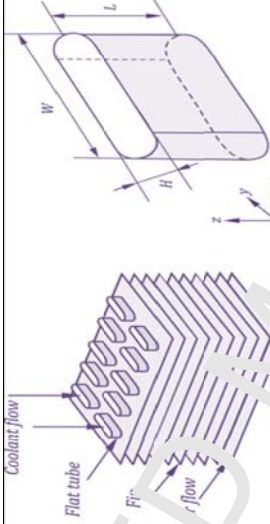
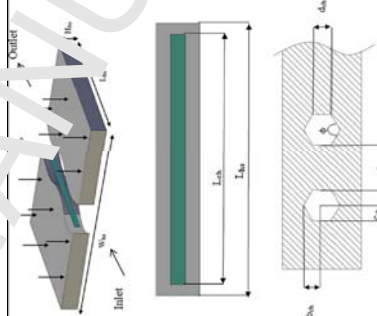
Togun et al. [124]	FVM / ANSYS FLUENT	Single phase, homogenous model, Temperature independent properties	Al_2O_3 /water	 <p>Concentric annular pipe</p>	Forced convection n- Turbulent	Higher thermal conductivity	With increasing the nanoparticle volume fraction along with a larger expansion ratio the surface heat transfer coefficient can be enhanced
Shalchi-Tabrizi and Seyf [125]	FVM	Single phase, homogenous model, Temperature dependent properties	Al_2O_3 /water	 <p>Tangential micro heat sink</p>	Forced convection n- Laminar	Higher thermal conductivity due to Brownian motion	Adding nanoparticles with a smaller size enhances heat transfer rate and decreases the entropy generation rate.
Seyf et al. [126]	FVM	Single phase, homogenous model, Temperature dependent properties	Octadecane / polyalphaolefi (Octadecane used as nanocapsulate d phase change material)	 <p>Microchannel heat sinks</p>	Forced convection n- Laminar	Higher thermal conduction of nanofluids	With increasing the mass concentration of nanoparticles the heat transfer enhances and entropy generation reduces.

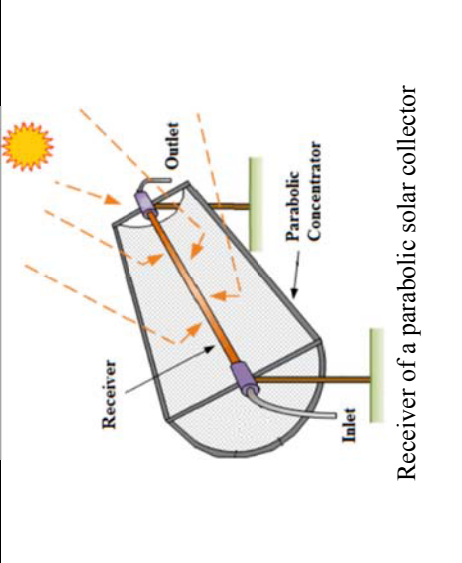
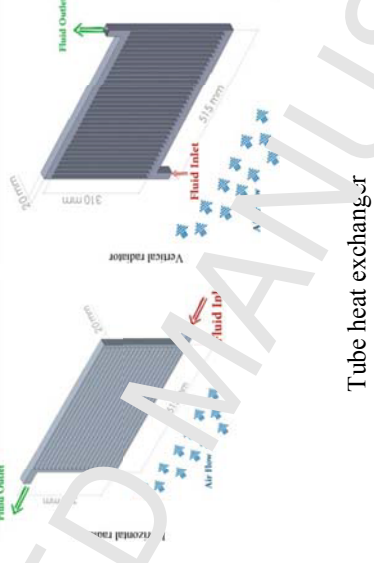
Yavuz and Tokgöz [127]	FVM / ANSYS FLUENT	Single phase, homogenous model, Temperature independent properties	Al ₂ O ₃ /water (with concentration of 8%), argon, and water	 <p>Channel with ribs</p>	Forced convection - Turbulent	-	Argon is the best option.
Gunnasegaran et al. [128]	FVM	Single phase, homogenous model, Temperature independent properties	Cu, diamond, and SiO ₂ /EG	 <p>Compact heat Exchanger</p>	Forced convection - Turbulent	Higher thermal conductivity	-Diamond gives higher heat transfer enhancement - SiO ₂ provides higher heat transfer rate than that of Cu
Seyf and Feizbakhshi [129]	FVM	Single phase, homogenous model, Temperature dependent properties	CuO/water (nano particles with two different sizes i.e. 28.6 and 29 nm) and Al ₂ O ₃ /water (nanoparticles with two different sizes i.e. 38.4 and 47 nm)	 <p>Micro-pin-fin heat sinks</p>	Forced convection - Laminar	Higher thermal conductivity due to Brownian motion	-Al ₂ O ₃ gives higher heat transfer enhancement. - Increasing the size of Al ₂ O ₃ nanoparticles leads to heat transfer enhancement of system but opposite trend was reported for Cu nanoparticles.

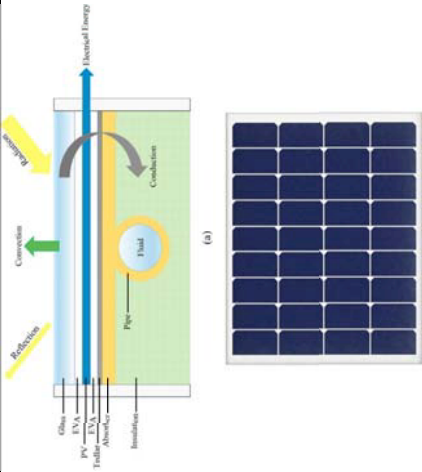
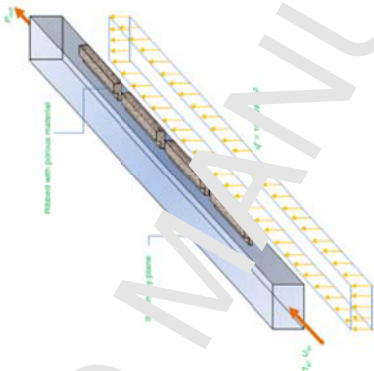
<p>Dugaria et al. [130]</p>	<p>FVM/ ANSYS FLUENT</p>	<p>-Single phase, homogenous model(due to low concentration of nanofluids, properties of nanofluid were assumed the same as water)</p>	<p>SWCNHs/Water</p>	 <p>Direct absorption solar receiver</p>	<p>Forced convection n-Turbulent</p>	<p>Better optical properties</p>	<p>Using nanofluids enhance the thermal efficiency of solar receiver.</p>
<p>Tiwari et al. [131]</p>	<p>FVM/ ANSYS FLUENT</p>	<p>-Single phase, homogenous model, Temperature dependent properties,</p>	<p>CeO₂/water and Al₂O₃/water</p>	 <p>Plate heat exchanger</p>	<p>Forced convection n-Turbulent</p>	<p>-Higher thermal conductivity</p>	<p>-CeO₂/water provides better performance. - The difference between experimental data and simulation does not exceed 4%.</p>
<p>Mukherjee et al. [132]</p>	<p>FVM/ ANSYS FLUENT</p>	<p>-Single phase, homogenous model, Temperature independent properties,</p>	<p>Al₂O₃/water</p>	 <p>Rectangular duct with protrusions</p>	<p>Forced convection n-Turbulent</p>	<p>-Higher thermal conductivity</p>	<p>-Nu increases 42.5% for volume fraction of 5%. - Entropy generation increases with increasing volume fraction.</p>

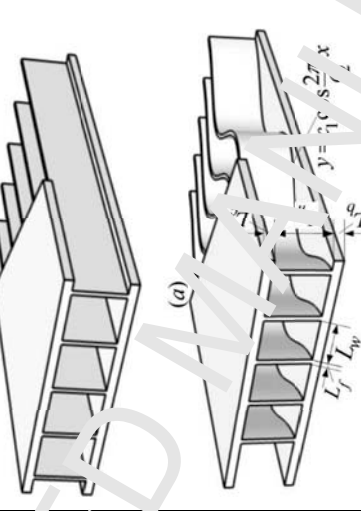
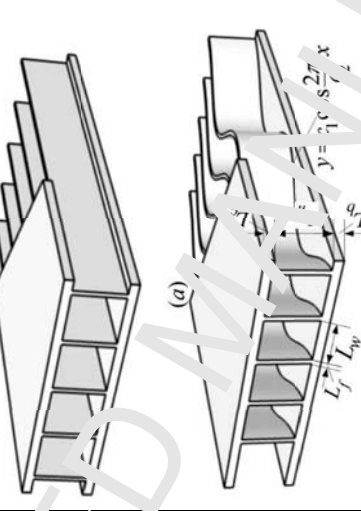
Hussein et al. [133]	FVM/ ANSYS FLUENT	-Single phase, homogenous model, Temperature independent properties	TiO ₂ / water	 <p>Automotive cooling system</p>	Forced convection- Turbulent	Higher thermal conductivity	Heat transfer rate can be ameliorated by 18% although friction factor rises by 12%
Sheikholeslami et al. [134]	LBM	-Single phase, homogenous model, Temperature dependent properties	Al ₂ O ₃ /water	 <p>Cubic cavity</p>	Forced convection- Laminar	Higher thermal conductivity, Brownian motion, Moving the bottom wall	Nusselt number increased with increases in nanoparticle volume fraction, and velocity of wall but it decreased with raising the Lorentz force.
Hawwash et al. [135]	FEM/ ANSYS FLUENT	-Single phase, homogenous model, Temperature dependent properties,	Al ₂ O ₃ /water	 <p>Flat Plate Solar Collector</p>	Forced convection- Laminar	-Higher thermal conductivity may enhance the thermal efficiency but agglomeration of nanoparticles reduces the efficiency.	For nanoparticle volume fractions in the range of 0.1 and 3%, the optimum volume fraction was obtained less than 0.5% to reach maximum thermal efficiency.

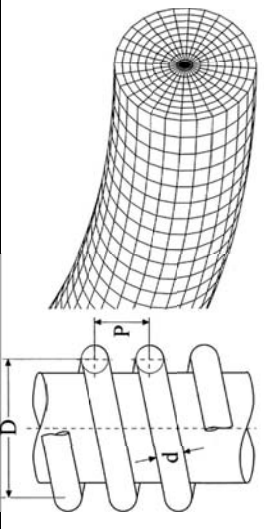
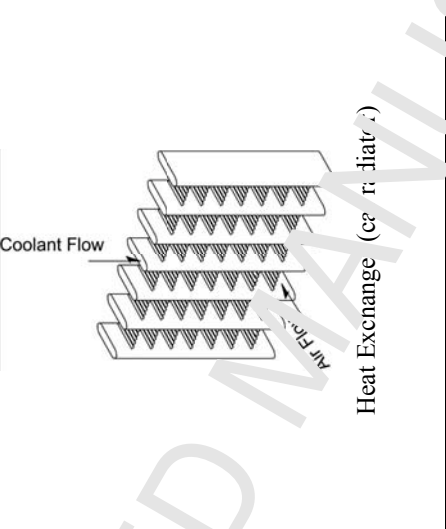
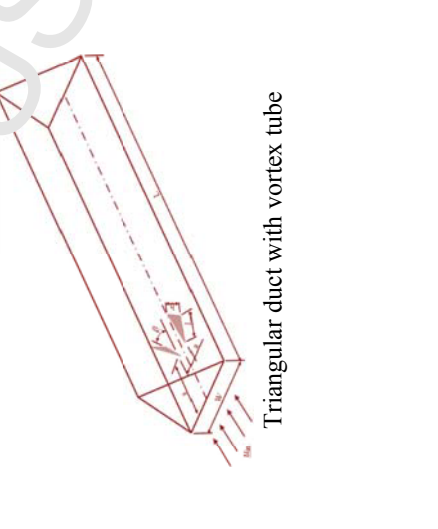
Seyf and Nikaein [136]	FVM	-Single phase, homogenous model, Temperature dependent properties	ZnO, CuO and Al ₂ O ₃ nanoparticles suspended in EG-water(60:40 by mass) and two sizes of 29 and 77 nm.	 <p>Microchannel heat sink</p>	Forced convection n-Laminar	-Higher thermal conductivity -Brownian motion and micro-convection	-CuO and Al ₂ O ₃ nanoparticles provide highest and lowest thermal performance, respectively. - With increasing size the thermal performance decreases.
Mohammadian et al. [13]	FVM	-Single phase, homogenous model, Temperature dependent properties	Al ₂ O ₃ /water (three different sizes including 29, 36.4, and 47 nm)	 <p>Nanofluid Exchanger</p>	Forced convection n-Laminar	Higher thermal conductivity Brownian motion	- With decreasing size of nanoparticles the thermal performance enhances. - Entropy generation reduces with increasing volume fraction and decreasing nanoparticle size.
Najafabadi and Moraveji [137]	FVM/ OpenFOAM	-Single phase, homogenous model, Temperature dependent properties,	Al ₂ O ₃ /water	 <p>Converging microchannel</p>	Forced convection n-Laminar	-	By considering partial slip boundary condition, even single-phase homogenous approach can predict well experimental data

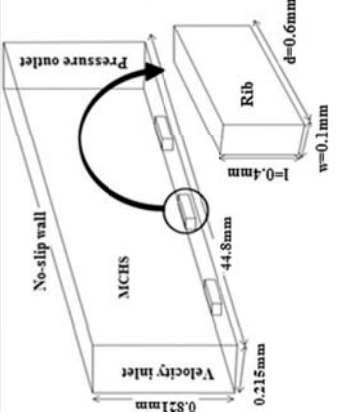
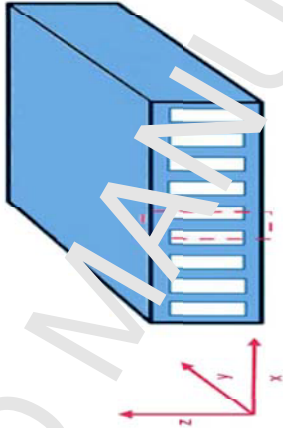
Chen and Cheng [138]	FVM/ ANSYS FLUENT	-Single phase, homogenous model, Temperature independent properties,	TiO ₂ /water	 Stave of blast furnace	Forced convection- Turbulent	- Aggregation of nanoparticles reduces heat transfer rate.	Heat transfer coefficient increases linearly with increasing Re.
Huminić and Huminić [139]	CVFEM/ ANSYS CFX	-Single phase, homogenous model, Temperature dependent properties,	MWCNT + Fe ₃ O ₄ /water + nanodiamond + Fe ₃ O ₄ /water	 Flat tube	Forced convection- Laminar	-Higher thermal conductivity	MWCNT + Fe ₃ O ₄ /water hybrid nanofluids exhibited higher enhancement and lower entropy generation.
Alfaryjat et al. [140]	FVM/ ANSYS FLUENT	-Single phase, homogenous model, Temperature dependent properties	Al ₂ O ₃ , CuO, SiO ₂ and ZnO nanoparticles suspended in water (nanoparticles with four different sizes i.e. 25, 40, 55 and 70 nm	 Hexagonal microchannel heat sink	Forced convection- laminar	Higher thermal conductivity Brownian motion	-Al ₂ O ₃ gives higher heat transfer enhancement. -With decreasing the size of nanoparticles the heat transfer rate enhances but pressure drop increases.

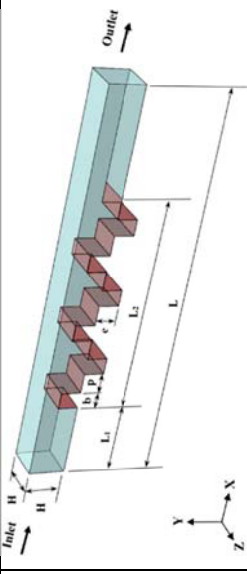
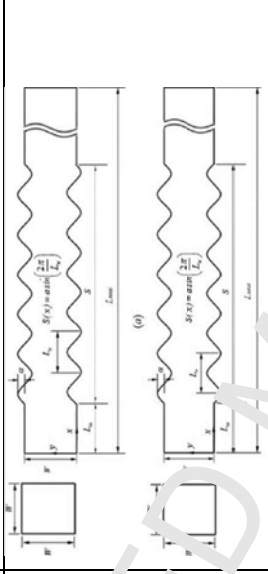
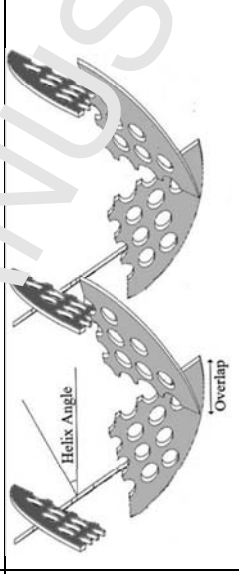
Ghasemi and Ranjbar [141]	FVM/ ANSYS FLUENT	-Single phase, homogenous model, Temperature dependent properties,	CuO/water and Al ₂ O ₃ /water	 <p>Receiver of a parabolic solar collector</p>	Forced convection n-Turbulent	-Higher thermal conductivity -Brownian force	The increase of thermal performance of solar parabolic trough collector was higher for the CuO-water nanofluid
Karimi and Afrand [142]	FVM	-Single phase, homogenous model, Temperature dependent properties	MgO-MWCNTs/E Water	 <p>Tube heat exchanger</p>	Forced convection n-Laminar	-Higher thermal conductivity	Solid volume fraction of nanoparticle is the dominant factor for increasing both temperature elevation and friction.

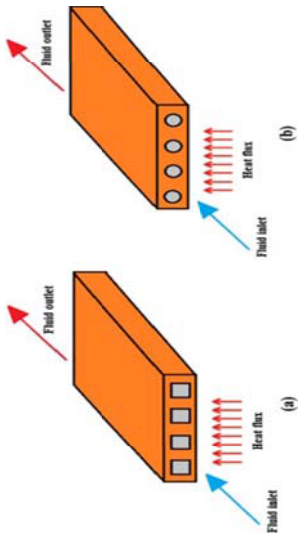
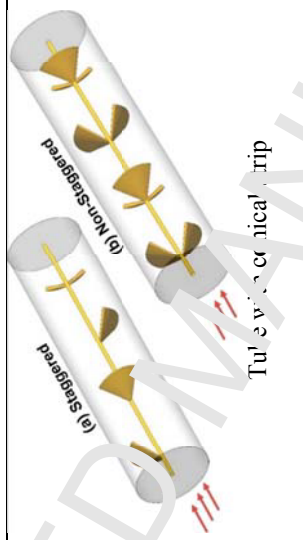
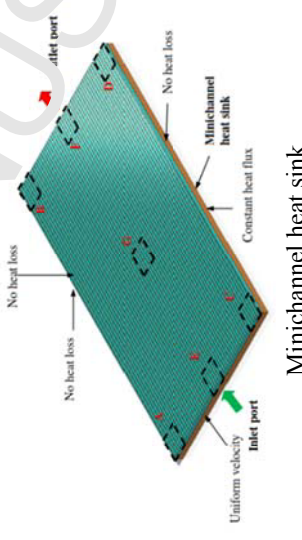
Hosseinzadeh et al. [143]	FVM, ANSYS Fluent software	-Single phase, homogenous model, Temperature dependent properties	ZnO/water	 <p>Photovoltaic system</p>	Forced convection n-Laminar	Higher thermal conductivity	The thermal energy performance of the ZnO/water nanofluid with a solid volume fraction of 0.03 is 12.78% higher than water.
Toghraie et al. [144]	FVM	-Single phase, homogenous model, Temperature independent properties	CuC Water	 <p>Microchannel</p>	Forced convection n-Laminar	Higher thermal conductivity	In the presence of porous ribs, the nanofluid has not a considerable influence on heat transfer enhancement.

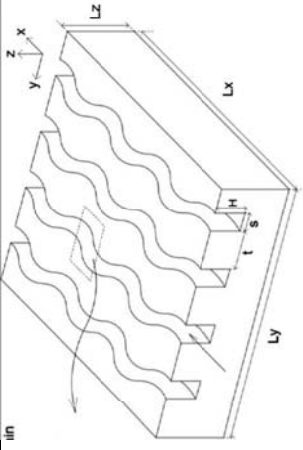
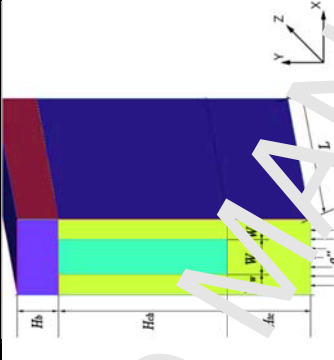
<p>Khoshvaght-Aliabadi [145]</p>	<p>FVM /ANSYS FLUENT</p>	<p>Two-phase, mixture model</p>	<p>Al_2O_3/water</p>	 <p>A part of TMC with elliptic cross section 360° twist Circular Elliptic Half-circular Square Rectangular Triangular</p> <p>Twisted minichannel</p>	<p>Forced convective n-Laminar</p> <p>Higher thermal conductivity</p> <p>Nanofluids with concentrations up to 4% investigated but it was found the performance index is maximized at a volume fraction of 1%.</p>
<p>Zhu et al. [146]</p>	<p>FVM</p>	<p>Two-phase mixture model</p>	<p>Al_2O_3/EC/water</p>	 <p>(a) (b)</p> <p>Wavy-finned heat sink</p> <p>$y = 1 + c \sin \frac{2\pi}{L_f} x$</p>	<p>Forced convective n-Laminar</p> <p>Higher thermal conductivity and Brownian motion</p> <p>Nanofluids with concentrations up to 4% investigated but it was found the performance index is maximized at a volume fraction of 1%.</p>

<p>Bizhaem and Abbassi [147]</p>	<p>FVM/ ANSYS FLUENT</p>	<p>Two-phase mixture model</p>	<p>Al₂O₃/water</p>	 <p>Helical tube</p>	<p>Forced convection n- Laminar</p>	<p>Brownian of nanoparticles and centrifugal force due to curvature of pipe.</p>	<p>- Heat transfer enhances by adding nanoparticles especially at low Reynolds numbers. - Entropy generation reduces with particle loading due to the increase of heat conduction in the system.</p>
<p>Delavari and Hashemabadi [148]</p>	<p>FVM</p>	<p>-Single phase, homogenous model, Temperature dependent properties and -Two-phase, mixture model</p>	<p>Al₂O₃/water and Al₂O₃/EG</p>	 <p>Heat Exchange (coefficient)</p> <p>Triangular duct with vortex tube</p>	<p>Forced convection n- Both Turbulent and Laminar</p>	<p>Higher thermal conductivity Brownian motion</p>	<p>-Nusselt number obtained by two-phase model was greater than that of obtained by single-phase approach (between 10-45%). - Experimental data confirms that two-phase model results are more reliable and closer to tests</p>
<p>Ahmed et al. [150]</p>	<p>FVM/ ANSYS FLUENT</p>	<p>Two-phase mixture model</p>	<p>CuO/water and Al₂O₃/water</p>	 <p>Triangular duct with vortex tube</p>	<p>Forced convection n- Turbulent</p>	<p>Higher Prandtl number</p>	<p>-Al₂O₃ gives higher heat transfer enhancement than that of CuO because of higher Pr number. Using nanofluids even a low concentration of 0.1% is more effective than equipping the tube with vortex generator.</p>

<p>Ghale et al. [151]</p>	<p>FVM/ ANSYS FLUENT</p>	<p>-Single phase, homogenous model, Temperature independent properties -Two-phase, mixture</p>	<p>Al₂O₃/water</p>	 <p>Ribbed microchannel heat sink</p>	<p>Forced convection- Laminar</p>	<p>Brownian motion of nanoparticles</p>	<p>Average relative error between experimental data and single-phase model is 32.6% and 37.4% for 1% and 2% nanofluid, while based on two-phase mixture model the deviation reduces to 11.39% and 2% for 1% and 2% nanofluids, respectively.</p>
<p>Kumar and Sarkar [152]</p>	<p>FVM/ ANSYS FLUENT</p>	<p>-Single phase, homogenous model, Temperature independent properties, - Two-phase mixture model</p>	<p>-Al₂O₃/water, Al₂O₃+MWCNT /water (hybrid nano fluid) -For hybrid nanofluids, three volume ratio of nanoparticles were investigated including (9:1), (8:2), and (7:3).</p>	 <p>Minichannel heat sink</p>	<p>Forced convection- Laminar</p>	<p>Higher thermal conductivity</p>	<p>- By using Al₂O₃/water at = 0.01%, the heat transfer coefficient enhances 4% while the enhancement reached 15.6% for hybrid nanofluid (with volume ratio of 7:3). -Two-phase mixture model overestimates experimental data while single-phase underestimates the data.</p>

Yang et al. [153]	FDM	-Single phase, homogenous model, Temperature independent properties and two-phase, mixture model	Al_2O_3 /water	 <p>Rib-grooved channel</p>	Forced convective n-Turbulent	Higher thermal conductivity and Brownian motion of particles	Single-phase model gives lower estimations of the Nusselt number compared to mixture model
Yang et al. [154]	FDM	-Single phase, homogenous model, Temperature independent properties and -Two-phase, mixture model	Al_2O_3 /water	 <p>Wavy channel</p>	Forced convective n-Turbulent	Higher thermal conductivity and Brownian motion of particles	Single-phase model gives lower estimations of the Nusselt number compared to mixture model
Bahiraee et al. [155]	FVM	Two-phase, mixture model	Al_2O_3 /water	 <p>Heat Exchanger with baffles</p>	Forced convective n-Turbulent	Brownian motion of nanoparticles	Heat transfer enhancement is high enough to neglect the pressure drop penalty.

Ghasemi et al. [156]	FVM/ ANSYS FLUENT	-Single phase, homogenous model, Temperature independent properties, Two-phase model, Eulerian-Eulerian	CuO/water	 <p>Minichannel heat sinks</p>	Forced convection n-Laminar	-Higher thermal conductivity, -drag force -Brownian motion	-Single-phase and two-phase approaches overestimated the experimental Nu number. -Two-phase model is more accurate. -Heat sink with rectangular channel is better.
Mashayekhi et al. [157]	FVM/ ANSYS FLUENT	Two-phase, Eulerian-Lagrangian	Ag/water	 <p>Tube with mechanical stirrer</p>	Forced convection n-Laminar	Higher thermal conductivity, drag force, thermophoretic force, Brownian force, lift force	-Staggered structure provides higher FoM. -Adding nanoparticles enhances the heat transfer enhancement.
Naphon and Nakharinr [158]	FVM/ ANSYS FLUENT	-Single phase, homogenous model, Temperature independent properties -Two-phase, mixture -Two-phase, (VOF)	TiO ₂ / water	 <p>Minichannel heat sink</p>	Forced convection n-Turbulent (although the average of Re is associate with laminar flow)	Higher thermal conductivity	Two phase models (mixture two phase and VOF) are more appropriate than homogeneous model (single phase)

Rostami and Abbassi [159]	FVM	Single-phase homogenous temperature dependent and Two-phase Eulerian-Lagrangian	Al_2O_3 /water	 <p>Wavy microchannel</p>	Forced convection n-Laminar	Higher thermal conductivity Brownian force drag force	-Two-phase Eulerian-Lagrangian approach is closer to experimental data -Adding 2% nanoparticles to the base fluid led to about 12% heat transfer enhancement and 2.4% increment in pressure drop
Shi et al. [160]	FVM/ ANSYS FLUENT	Two-phase, Eulerian-Lagrangian	Al_2O_3 /water	 <p>Microchannel</p>	Forced convection n-Laminar	The following forces were included : drag, the gravity, the Brownian motion, the thermophoretic, Saffman lift, pressure gradient	By using nanofluids with a concentration of 2%, the heat transfer coefficient enhances about 8.5% whereas this enhancement for volume fraction of 1% is about 5.9%.

3.3. Mixed convection of nanofluid flow

Although they are not as common, some studies have been done to simulate nanofluids in the mixed convection mode. As before, these studies can be roughly classified into two main groups – (i) single-phase and (ii) two-phase approaches.

3.3.1. Studies using the single-phase approach

One major applications of nanofluids is in nuclear reactor. Nanofluids with higher thermal conductivity compared to base fluids can be good coolants for nuclear reactors. In this regard, Mousavizadeh et al. [164] investigated the effect of using TiO_2 /water nanofluid on cooling rate of VVER-1000 nuclear reactor with a height about 3.8 m by modeling the mixed turbulent nanofluid flow as a single-phase and homogenous working fluid. It was found that adding nanoparticles with a volume concentration of 1.5% can reduce the clad temperature about 1 °C. Maybe testing other working fluids such as CNTs based nanofluids could show better cooling performance since the effective thermal conductivity of TiO_2 /water is not much high. It was also found that with decreasing the size of nanoparticles and the increase of nanoparticle concentration, the heat transfer coefficient increases.

Dawood et al. [165] studied laminar mixed convection of different nanofluids containing Al_2O_3 , CuO , SiO_2 and ZnO nanoparticles in an elliptic annulus under constant heat flux (5000W/m^2) using finite volume method. The effects of nanoparticle type, nanoparticle size (from 20 to 80 nm), volume concentration (up to 4%), Reynolds number (between 200 and 1000), and different base fluids including water, glycerin, engine oil, and ethylene glycol have been investigated in the study. They concluded that although SiO_2 nanoparticles have the lowest thermal conductivity among the above mentioned particles but using SiO_2 based nanofluids provides the highest Nusselt number (followed by Al_2O_3 , ZnO , and CuO nanoparticles). It was found glycerin was the best base fluid for heat transfer enhancement. In addition, heat transfer rate was maximized at Reynolds number of 1000, nanoparticle size of 20 nm and volume fraction of 4%.

Kherbeet et al. [166] solved laminar mixed convection flow (at $\text{Re} = 35$) of four different ethylene glycol based nanofluids containing SiO_2 , Al_2O_3 , CuO , and ZnO nanoparticles over a micro forward-facing step through single-phase approach. The results were presented for nanoparticle sizes ranging

from 25 to 70 nm and volume fraction up to 4%. They found that using SiO₂ nanoparticles provides highest Nusselt number. Moreover, an increase in volume fraction ameliorates the Nusselt number while using nanoparticles with a smaller size is more advantageous from the heat transfer enhancement viewpoint.

Al-Rashed et al. [167] simulated mixed convection of CNTs/water nanofluids in a cavity equipped with adiabatic-driven baffle for Rayleigh numbers between 10³ and 10⁵ using FVM and single phase approach. They investigated the effect of baffle motion in different directions on heat transfer and entropy generation inside the partially heated cavity.

Al-Rashed et al. [168] later applied the FVM to model the single phase mixed convection flow and entropy generation of Al₂O₃/water nanofluid inside a cavity where a hot block was installed in its center. Maxwell and Brinkman models were used for thermal conductivity and viscosity estimation. It was found that Nusselt number and entropy generation increase with increasing the nanoparticle volume fraction. The results also revealed that with increasing the block size, Nu number and entropy generation increase.

Zhou et al. [60] analyzed mixed convection in a cavity filled with alumina/water nanofluid using LBM (D3Q19 model) and employing Maxwell and Brinkman models for thermal conductivity and viscosity. The cavity was under an external magnetic field and the cold side wall moved with a constant velocity towards up. The effects of nanoparticle volume fractions up to 6%, orientation of magnetic field (angles between 0 to 180°), Richardson numbers between 0.1 and 10, and Rayleigh numbers of 10⁴-10⁶ on flow and heat characteristics inside the cavity. The results of this study show that the average Nusselt number decreases with increasing the Ri number, and the average Nusselt number increases with an increase in volume fraction of nanoparticles at Ra numbers of 10⁴ and 10⁵, however, for Ra = 10⁶ in which the convection mode is dominant an optimal value for volume fraction can be found in which Nusselt number is maximized. It was also found that with increasing the magnetic field intensity the average Nu decreases, on the other hand Nu is minimized when the magnetic field is applied to the cavity perpendicularly. This study suggests that for increasing the average Nusselt number, nanoparticles can be added to the base fluid, especially at low Ra numbers, also the direction that magnetic force applies to the cavity should be parallel to the cavity as much as possible.

Karbasifar et al. [169] studied the effect of adding alumina/water nanoparticles to water on flow and heat transfer patterns in an inclined cavity where a hot cylinder was installed in its center and the top wall could move with a constant velocity. The results were presented for Richardson numbers of 0.1, 1, 10, 100, nanoparticle volume fractions of up to 0.2%, inclination angles of 0°, 15°, and 45°, and a temperature difference of 50°C between hot cylinder and cold side walls. Their results show that at a given Ri number, the Nu number increases with nanoparticle volume fraction linearly, the authors attributed this increase to Brownian motion and clustering phenomenon. However, the authors used the Maxwell and Brinkman models to calculate the thermal conductivity and viscosity, and on the other hand, single phase approach was used to solve the problem, therefore the heat transfer enhancement due to particle adding is not because of Brownian motion and clustering phenomenon since they were not considered in governing equations and thermo-physical models. The only factor that can be considered in this problem as responsible factor for heat transfer enhancement is enhancement in thermal conductivity.

Mohammed et al. [170] simulated the laminar mixed convection of Ag, Au, CuO, diamond, and SiO₂ nanoparticles suspended in water in a vertical channel with square cross section by considering radiation effect. The simulation was done for volume concentrations up to 3%, Reynolds numbers between 200 and 1000, and Rayleigh numbers between 2×10^6 and 2×10^7 . It was concluded that despite the lowest effective thermal conductivity of SiO₂/water nanofluids, they give highest values of Nusselt number. On the other hand, Au nanoparticles provide lowest Nusselt numbers.

Selimefendigil and Oztop [171] considered mixed convection in a cavity where two cylinders rotate with a constant angular velocity inside the cavity. Three different water based nanofluids containing Al₂O₃, TiO₂, and Cu nanoparticles have been selected as the working fluid. The results were provided for Ra numbers between 10^4 and 10^6 and volume fractions up to 4%. Single phase approach and FEM were used to solve the problem while Maxwell and Brinkman models were selected to estimate the thermal conductivity and viscosity. Figure 30 shows the variation of Nu number versus different values of dimensionless angular velocity of cylinders for volume fraction of 2% and Rayleigh number of 10^5 . As seen, adding Cu nanoparticles with highest thermal conductivity to the base fluid leads to highest heat transfer enhancement. Indeed, the heat transfer enhancement has direct relationship with thermal conductivity of nanoparticles. Also, the maximum heat transfer happens when the cylinders rotate with maximum speed and in counter-clockwise direction.

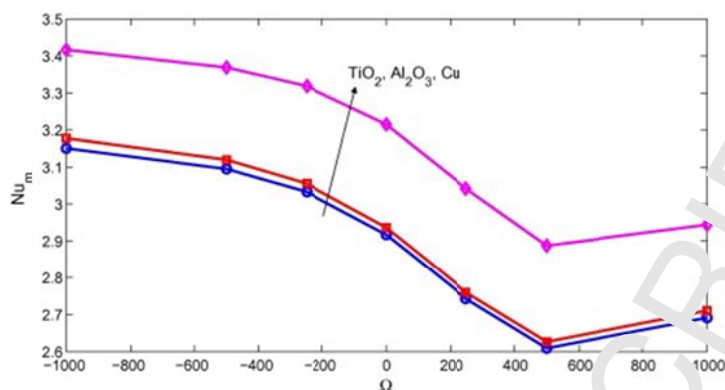


Figure 30. Variations of Nusselt number versus relative angular velocity of cylinders [171] (figure is reprinted by permission from publisher)

Akbari and Behzadmehr [172] employed finite volume approach to solve the laminar mixed convection developing flow of Al₂O₃/water nanofluid (with volume fractions up to 4%) in a horizontal tube under constant heat flux. It was concluded that adding nanoparticles has no a significant effect on the velocity field. However, the temperature field is affected clearly by particle loading because of heat diffusion induced by solid nanoparticles. It was reported that local heat transfer coefficient ameliorates by adding nanoparticles.

3.3.2. Studies using the two-phase approach

Safikhani et al. [173] first simulated laminar mixed convection flow of Al₂O₃/water nanofluid in ducts with circular and flatted cross sections using two-phase mixture model, next using CFD results, optimization was done to reach the maximum heat transfer coefficient and the least pressure drop. Multi-objective optimization was carried out by employing artificial neural networks (ANN) and genetic algorithms (GA) by involving different parameters including heat flux, size of nanoparticles, volume flow rate, nanoparticle concentration and geometry specifications.

Mohammed et al. [174] investigated laminar and turbulent mixed convection of ZnBr₂/acetone nanofluid by employing two-phase mixture model. It was found that adding 1.5% nanoparticles enhanced heat transfer coefficient up to 180% mainly due to Brownian motion of nanoparticles.

Shariat et al. [175] studied laminar mixed convection flow of alumina/water nanofluids in ducts with elliptical cross section under constant heat flux using two-phase mixture model. They used

temperature-dependent models to estimate the thermal conductivity and viscosity. The authors indicated that adding nanoparticles boosts the secondary flows, and, consequently, the heat transfer rate enhances. In addition, the authors showed that a duct with elliptical cross section can be more advantageous than a circular tube by considering both maximum heat transfer rate and minimum friction factor.

Zonouzi et al.[176] simulated 3D dimensional mixed convection flow of $\text{Al}_2\text{O}_3/\text{water}$ nanofluid (laminar and developing flow) in helical tubes using mixture model and FVM. Temperature dependent models for properties were employed in the simulation where the maximum concentration of nanofluids was 3%. Effects of different forces including centrifugal and gravitational as well as nanofluid concentration and structure of tubes on heat transfer rate of the system were investigated. The authors concluded that although adding nanoparticles continuously increases the heat transfer rate but for concentrations higher than 2% the heat transfer enhancement is not significant and is not economic.

Sivasankaran and Narrein [177] numerically investigated laminar flow of alumina/water nanofluid (with volume fraction of 1%) in a helical and porous microchannel using two-phase mixture model. Modified models for thermal conductivity and viscosity were used. In the study, there is no discussion on the effect of working fluid type and concentration; instead the focus was on the inlet velocity profile to microchannel.

Kareem and Gao [178] studied turbulent mixed convection in a double lid-driven cavity using ANSYS/FLUENT (based on FVM) and two-phase mixture model. The results were presented for three different nanoparticles including SiO_2 , ZnO and CuO suspended in water as well as Reynolds numbers between 5000 and 30000, nanoparticle diameters between 25 and 85 nm, and nanoparticle volume fractions up to 8%. For modeling the turbulence flow, Unsteady Reynolds-Averaged Navier-Stokes (URANS) and Large Eddy Simulation (LES) techniques were employed. The results of this study can be summarized as follows.

- The maximum heat transfer enhancement was obtained by using SiO_2 nanoparticles because of their lowest density and hence the maximum velocity. Following SiO_2 , ZnO and CuO are in the next rank.
- Large Eddy Simulation method gives more details of velocity and temperature fields compared to URANS. Although their prediction of Nusselt number is not much different.

- With increasing volume fraction of nanoparticles and Reynolds number and on the other hand decreasing the size of nanoparticles, Nusselt number enhances.
- It can be observed from Fig. 31 that the effect of adding nanoparticle to base fluid on Nu is not much visible at low volume fractions (i.e. 2%), however, with increasing the nanoparticle loading the heat transfer enhancement highlighted (for volume fraction from 2 to 8%).

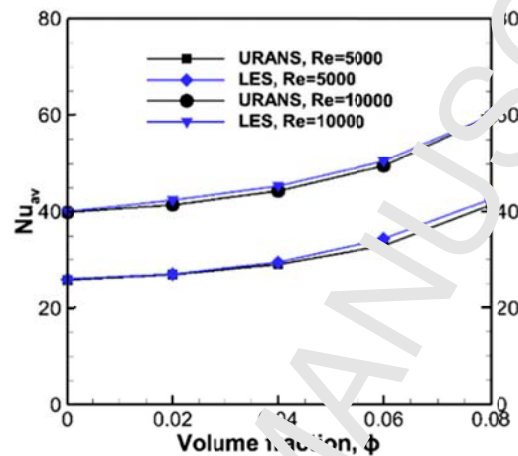
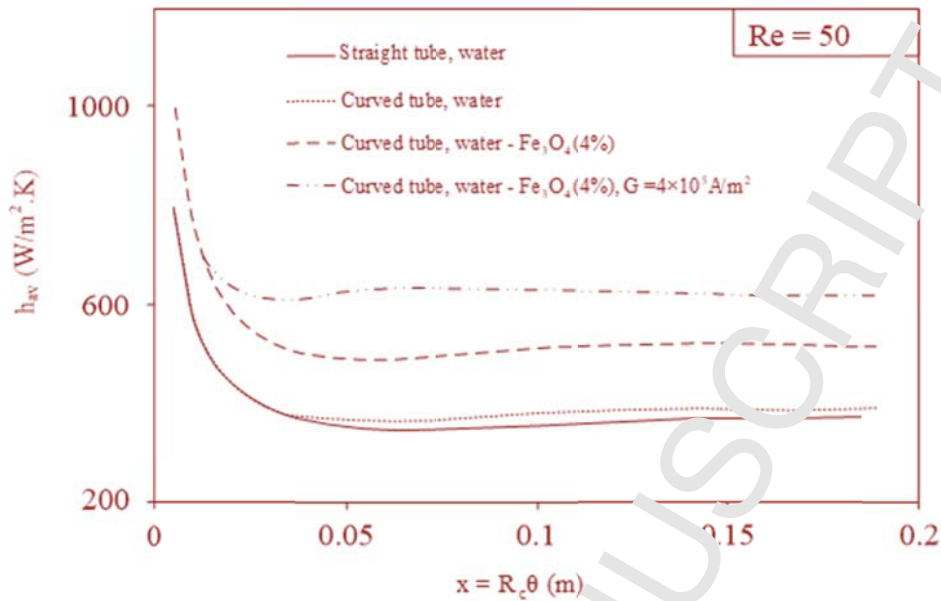
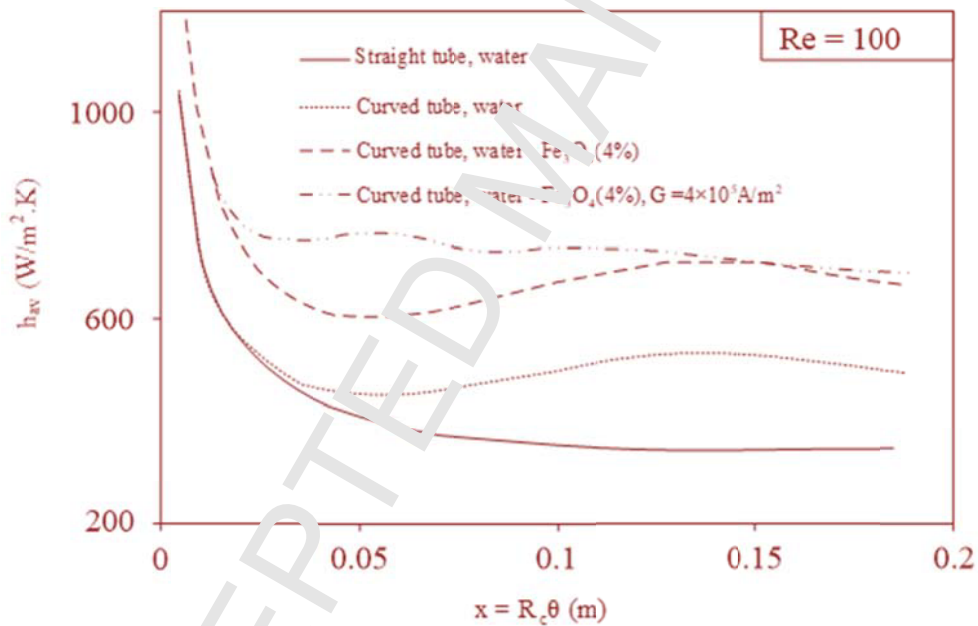


Figure 31. Comparison of average Nusselt numbers for different Reynolds numbers and volume fractions [178] (figure is reprinted by permission from publisher).

Aminfar et al. [179] investigated the potential of passive and active techniques for heat transfer enhancement in a straight tube where water flows inside it with a laminar regime and mixed convection mode. Using a curved tube instead of straight tube (passive), loading Fe_3O_4 nanoparticles to water with a volume fraction of 4% (passive), and applying a non-uniform magnetic field (active) were the three used techniques. Figure 32 shows the effect of applying different techniques on the value of local heat transfer coefficient for Reynolds numbers of 50 and 100. As seen, using the three techniques can enhance the heat transfer significantly (especially nanoparticles loading and applying magnetic field). The authors used two-phase mixture approach to model the problem.



(a)



(b)

Figure 32. Effects of the geometric curvature, magnetic nanoparticles and magnetic field on the heat transfer coefficient behavior along the tube (a) $Re = 50$ (b) $Re = 100$ [179] (figure is reprinted and modified by permission from publisher).

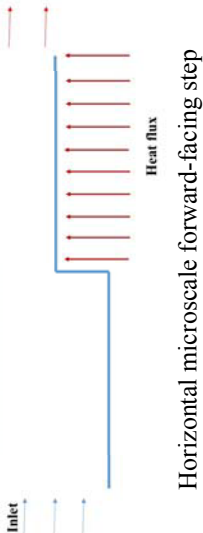
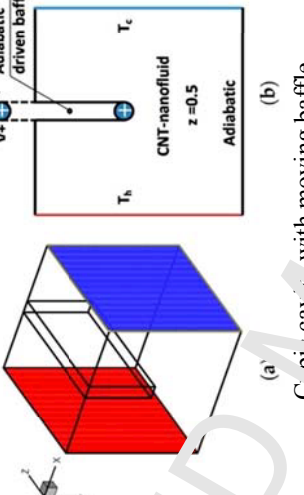
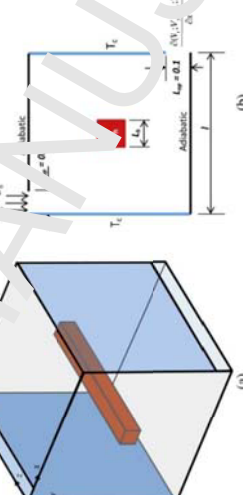
Shahmohammadi and Jafari [180] studied mixed convection of alumina/water nanofluids in a horizontal tube under constant heat flux and equipped to baffles. The simulations were carried for both laminar and turbulent flow regimes (Reynolds numbers up to 3000) and two volume concentrations of

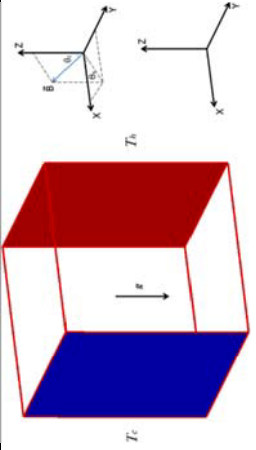
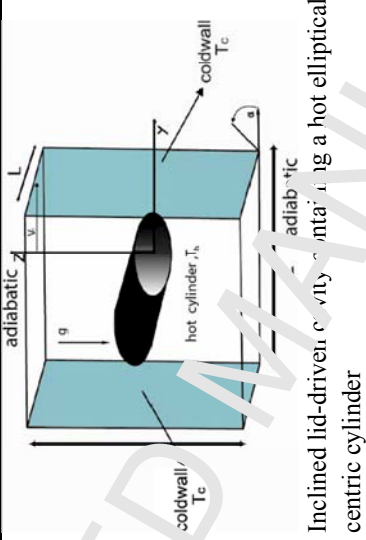

1 and 1.6%. The authors employed homogenous single-phase approach (temperature-independent properties) and three two-phase approaches including Eulerian, mixture, and VOF. The results were compared with experimental data and it was found that single-phase approach results are relatively far from experimental data, so a two-phase approach should be used to capture the experimental data. Among the three two-phase approaches, mixture model was proposed as the most suitable because of easiness of application. It was also found that effects of adding nanoparticles to base fluid is more visible at $Re < 1000$, while equipping the tube to baffle is more advantageous at $Re > 2000$. Yousofvand et al. [181] studied the first and second laws of thermodynamics for an electromagnetic pump using nanofluids by employing LBM.

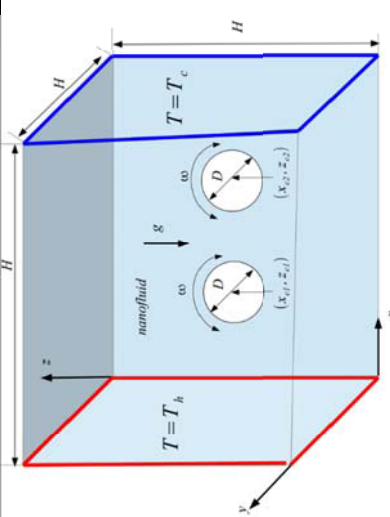
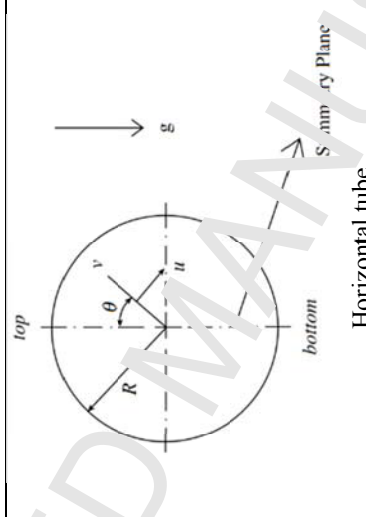
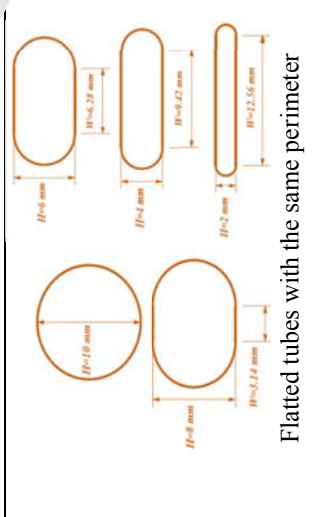
Table 4 gives a summary of studies on 3D simulation of mixed convection of nanofluids.

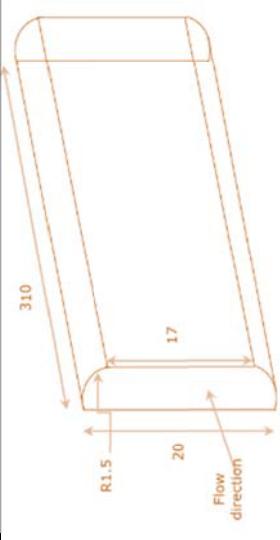
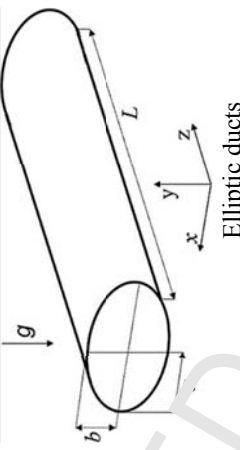
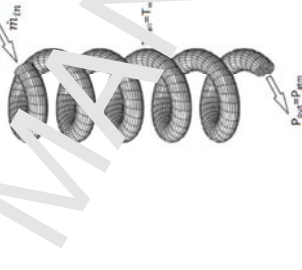
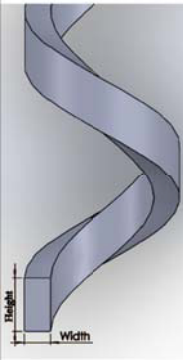
Table 4. Summary of studies on 3D simulation of mixed convection of nanofluids (figures are reprinted by permission from publishers).

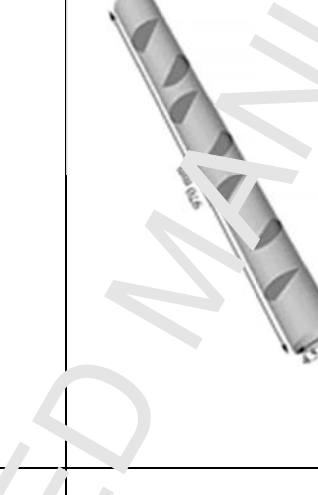

Mousavizadeh et al. [164]	FVM/ ANSYS FLUENT	-Single phase, homogenous model, Temperature dependent properties,	TiO ₂ /water (nanoparticles in two different sizes of 10 and 20nm were tested)	<p style="text-align: center;">Nuclear reactor</p>	Mixed convection- Turbulent	-Higher thermal conductivity -Brownian Motion	Adding nanoparticles has considerable effect on heat transfer coefficient while there is no significant effect on temperatures like clad temperature.
Dawood et al. [165]	FVM / ANSYS FLUENT	Single phase, homogenous model, Temperature dependent properties	SiO ₂ , Al ₂ O ₃ , ZnO, and CuO nanoparticles, and four base fluids i.e. water, glycerin, engine oil, and EG.	<p style="text-align: center;">Elliptic annulus</p>	Mixed convection- Laminar	Higher velocity of nanofluid	<ul style="list-style-type: none"> - SiO₂ provides highest heat transfer enhancement because of its lowest density and consequently highest velocity. - Nanoparticles with minimum size and maximum volume fraction give maximum heat transfer rate. -Glycerin is the best performing fluid.

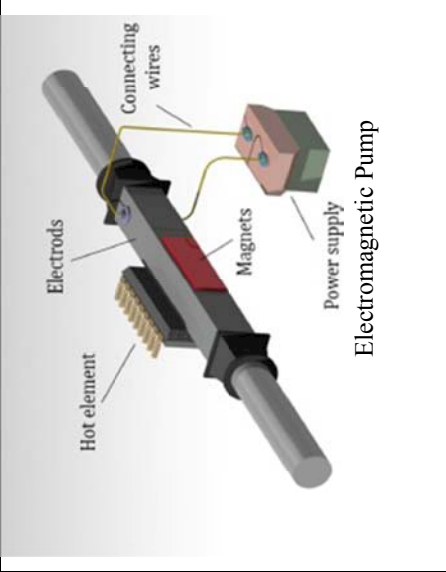
Kherbeet et al. [166]	FVM	Single phase, homogenous model, Temperature dependent properties	SiO ₂ , Al ₂ O ₃ , CuO, and ZnO nanoparticles suspended in EG.	 <p>Horizontal microscale forward-facing step</p>	Mixed convective n-Laminar	Lower density and higher velocity	-SiO ₂ gives higher heat transfer enhancement - Smallest size and highest volume fraction of nanoparticles provide highest heat transfer rate.
Al-Rashed et al [167]	FVM	-Single phase, homogenous model, Temperature independent properties	CNT/water	 <p>(a) Cubic cavity with moving baffle</p> <p>(b)</p>	Mixed convective n-Laminar	Higher thermal conductivity and using the baffle at high Ra numbers.	Effect of baffle motion in different directions on heat transfer and entropy generation inside the cavity was investigated.
Al-Rashed et al [168]	FVM	-Single phase, homogenous model, Temperature independent properties	CNT/water	 <p>(a) Cubic Open Cavity with a Central Isothermal Block</p> <p>(b)</p>	Mixed convective n-Laminar	Higher thermal conductivity	With increasing the block size and concentration of nanofluids both Nu number and entropy generation increase.

Zhou et al. [60]	LBM	-Single phase, homogenous model, Temperature independent properties	Al_2O_3 /water	 <p>Cubic cavity under magnetic field</p>	Mixed convection n-Laminar	Higher thermal conductivity, decreasing the intensity of magnetic field increases the Nu number	<ul style="list-style-type: none"> Nu increases with an increase in volume fraction of nanoparticles at Ra numbers of 10^4 and 10^5, however, at Ra = 10^6 an optimal value for volume fraction can be found in which Nusselt number is maximized.
Karbasifard et al. [169]	FVM	-Single phase, homogenous model, Temperature independent properties,	Al_2O_3 /water	 <p>Inclined lid-driven cavity containing a hot elliptical centric cylinder</p>	Mixed convection n-Laminar	-Higher thermal conductivity	Nu number increases with nanoparticle volume fraction linearly
Mohammed et al. [170]	FVM	Single phase, homogenous model, Temperature dependent properties	Ag, Au, CuO, diamond, and SiO_2 /water	 <p>Vertical Rectangular Duct</p>	Mixed convection n-Laminar	Lower density and higher velocity	SiO_2 gives higher heat transfer enhancement

Selimefendigil and Öztop [171]	FEM	-Single phase, homogenous model, Temperature independent properties	Cu, Al ₂ O ₃ and TiO ₂ /water	 <p>Cavity with two adiabatic inner rotating cylinders</p>	Mixed convective n-Laminar	-Higher thermal conductivity	-When the cylinders rotate counter-clockwise the heat transfer rate is maximized. - Using nanoparticles with highest thermal conductivity provides the highest heat transfer enhancement.
Akbari and Behzadmehr [172]	FVM	Single-phase, homogenous model, Temperature independent properties	Al ₂ O ₃ /water	 <p>Horizontal tube</p>	Mixed convective n-Laminar	Heat diffusion induced by nanoparticle s.	Adding particles has no significant effect on velocity field but considerably affect the temperature field.
Safikhani et al. [173]	FVM	Two-phase mixture model	Al ₂ O ₃ /water	 <p>Flattened tubes with the same perimeter</p>	fixed convective n-Laminar	-	CFD results were used to carry out multi-objective optimization by employing ANN and GA.

Mohammed et al. [174]	FVM/ ANSYS FLUENT	Two-phase, mixture model	ZnBr/acetone	 <p>Flat tube</p>	Mixed convective n-Laminar and turbulent	-Higher thermal conductivity, Brownian Motion	Adding 1.5% nanoparticles enhanced heat transfer coefficient up to 180%.
Shariat et al. [175]	FVM	Two-phase, mixture model	Al_2O_3 /water	 <p>Elliptic ducts</p>	Mixed convective n-Laminar	Higher thermal conductivity and Brownian motion	Adding nanoparticles boosts the secondary flows, and, consequently, the heat transfer rate enhances
Zonouzi et al. [176]	FVM	Two-phase, mixture model	Al_2O_3 /water	 <p>Helical Tube</p>	Mixed convective n-Laminar	Brownian motion of nanoparticles and centrifugal force due to tube structure.	Heat transfer rate decreases with increasing nanoparticle volume fraction.
Sivasankaran and Narrein [177]	FVM/ ANSYS FLUENT	Two-phase, mixture model	Al_2O_3 /water	 <p>Helical microchannel</p>	Mixed convective n-Laminar	-	Effect of inlet velocity profile on heat transfer rate was investigated.

<p>Kareem and Gao [178]</p>	<p>FVM/ ANSYS FLUENT</p>	<p>Two-phase, mixture model</p>	<p>CuO, ZnO, SiO₂ nanoparticles suspended in water</p>	 <p>Closed lid-driven cavity</p>	<p>Mixed convection- Turbulent</p>	<p>Higher velocity (lower density) and thermal conductivity</p>	<p>-SiO₂ gives higher heat transfer enhancement. - With increasing concentration and Re, and decreasing size of nanoparticles the heat transfer enhances. - Large eddy simulation approach is suggested to use instead of Unsteady Reynolds-Averaged Navier-Stokes approach.</p>
<p>Shahmohammadi and Jafari [180]</p>	<p>FVM/ ANSYS FLUENT</p>	<p>-Single phase, homogenous model, Temperature independent properties -Two-phase Eulerian-Eulerian, Mixture, VOF</p>	<p>Al₂O₃/water</p>	 <p>Tube with baffles</p>	<p>Mixed convection- Laminar and Turbulent</p>	<p>Brownian motion of nanoparticles and heat diffusion of nanoparticles. Using baffle to increase turbulence</p>	<p>Two-phase models are more precise than single-phase model. Among the three two-phase models, the mixture model is suggested to use because it is simplest to apply.</p>

Yousofvand et al. [181]	LBM	Single-phase, homogenous model, Temperature dependent properties	Cu/water		Mixed convection n-Laminar	-	Optimal value for volume fraction of nanoparticles can be found at high Rayleigh numbers so that the cooling rate is highest.
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4. Conclusions

Nanofluids are attracting increasing attention due both to their highly attractive thermal properties and to the fact that the nanofluid properties can, to some extent, be tailored to specific applications. Nanofluids are used in wide-ranging applications, from automobiles to solar thermal collectors—any applications which require efficient heat and mass transfer. This comprehensive review built upon Part I [1], which dealt with fundamental and theoretical physics of nanofluids, including various models presented to describe nanofluid flow on different scales. As such, the present review, Part II, discussed the primary CFD approaches for solving the transport equations given in part I [1]. These approaches were categorized into three general groups i.e. macroscale-based, mesoscale-based, and microscale-based techniques. To examine the state-of-the-art in this field, recent three-dimensional studies on modeling nanofluid flow were reviewed, indicating the range of flow regimes, geometries, the nanofluid types, solution methods, and the physical phenomena incorporated in each model. Although underlying phenomena can be applied to many applications, it was found that these three-dimensional studies have most frequently been used to analyze thermal systems such as solar collectors and microchannels. The main points that have been revealed in Part II of this review can be summarized as:

- FVM and ANSYS/FLUENT are the most used numerical method and software package, respectively, for solving problems in nanofluid flow. However, as with all fluid mechanics problems, the choice of numerical method depends critically on the specifics of the problem.
- Several studies look at the natural convection of nanofluids in cavities, and the difference between 2D and 3D models diverge with increasing Rayleigh number and nanoparticle volume fraction (see, for example, Ref. [87])
- The aspect ratio of a cavity may cause differences between predictions of 2D and 3D models.
- For a specific problem, the single-phase approach may results which match experimental better data compared to a two-phase approach, but by changing the nanofluid this trend may reverse (as mentioned in Ref. [161]).

- Brownian motion of nanoparticles has often been cited as the most important phenomenon responsible for heat transfer enhancement of nanofluids, especially in studies that employed the single-phase homogenous approach with temperature-dependent properties.
- Using nanoparticles with a higher thermal conductivity (for example Al_2O_3 with a thermal conductivity of about 40) does not always provide a higher heat transfer rate. Sometimes using nanoparticles with a lower density (such as SiO_2 with a thermal conductivity of 1.4 W/mK) yields a higher heat transfer rates because a lower density nanofluid can have a higher velocity. In other words, the velocity of nanofluid is also an important—but often overlooked—parameter in the determination of the heat transfer rate.
- Entropy generation in thermal systems where irreversibility due to heat transfer is dominant usually decreases by adding nanoparticles, since nanoparticles enhance heat conduction in the system and consequently reduce temperature gradients.
- Considering a partial slip boundary condition in microchannels increases the accuracy of the single-phase homogenous approach in comparison with experimental data (see for example [137]).
- The effects of adding nanoparticles on heat transfer coefficient is not high at the entrance region of tube in cases of forced and mixed convection, but when the flow becomes fully developed the effect of nanoparticle loading on heat transfer coefficient is more pronounced.

5- Gaps and future work

The present review revealed that there are still many unsolved questions in the field of nanofluid flow simulation, particularly when three-dimensions must be considered. The literature still needs comprehensive studies that consider all involved physical phenomena in nanofluid flow and heat transfer problems without excessive simplifications. Figure 33 shows the most common keywords in studies on three-dimensional modeling of nanofluids, and may help to better identify the gaps in this area. The following points can be found from the figure:

- As seen, among nanoparticles, Al_2O_3 nanoparticles are the most used. Other nanoparticles such as CuO , TiO_2 , SiO_2 and CNTs are other materials that used frequently. Moreover, almost all nanoparticles used in the studies are spherical and cylindrical in shape. Therefore, one

suggestion may be developing experimental and theoretical models for prediction the properties of nanofluids having novel nanoparticles and shapes, and their evaluation of their potential for heat transfer enhancement in different systems.

- Most studies use homogenous models (either temperature-dependent or temperature-independent) or the mixture model. More studies, both numerical and experimental, are needed to determine the optimum approach for modeling and the conditions under which the single-phase homogenous approach (the easiest technique for modeling of nanofluid flow) gives acceptable results.
- Besides laminar and turbulent regimes, nanofluids flow in the transition regime is an area that requires more investigation.
- The FVM was the most used numerical approach for simulation of nanofluid flow. Comprehensive studies are still lacking that compare the capabilities of different numerical approaches.



Figure 33. Word cloud of most used terms in three-dimensional modeling of nanofluids

In the following some other suggestions have been given for future work:

- Three dimensional modeling of nanofluids in thermal systems having vibration may be interesting since in practice vibration might keep the particles suspended better and create mixing at the same time.
- More attempts should be done on three dimensional modeling of thermal systems by considering particle sedimentation during the computational time interval.
- Erosion and corrosion due to nanofluid flow may be another topic of interest.

- Using experimental-based correlations or more advanced models for thermophysical properties may reduce the difference between heat transfer rate predicted by the single-phase homogenous approach and experimental data.
- Using two-phase approaches for hybrid nanofluids, where the non-uniformity in shape and size of nanoparticles are included simultaneously, could be a challenge.
- Studies can be done on the determination of the exact contribution of different forces, including common forces such as Brownian and thermophoretic on one hand, and on the other hand interparticle forces such as double layer and van der Waals.
- Solidification and melting phenomena may be considered for 2D modeling using two-phase approaches.
- Three dimensional analysis of solar desalination systems [182] could be of interest due to the present crisis of fresh water in the world.
- Determination of dominant forces in nano-refrigerant flows and modeling of this type of flow in various configurations and regimes could be of interest.
- There are only a few studies on CFD simulation of bubbles dynamics in pool boiling of nanofluids (for example see Ref. [183]) so three dimensional modeling of pool boiling of nanofluids could be of interest going forward.

Acknowledgment

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References

- [1] Omid Mahian; Lioua Kolsi; Patrice Estellé; Goodarz Ahmadi; Clement Heitschreuer; Jeffrey S Marshall; Majid Siavashi; Robert A Taylor; Hamid Niazmand; Somchai Wongwises; Tasawar Hayat; Arun Kolarjyil; Alibakhsh Kasaeian; Ioan Pop, Recent Advances in Modeling and Simulation of Nanofluid Flows-Part I: Fundamental and Theory. *Phys. Rep.* (n.d.).
- [2] A. A. Mohammed, Lattice Boltzmann Method: Fundamentals and Engineering Applications with Computer Codes, 2012. doi:10.2514/1.J051744.
- [3] J.H. Ferziger, M. Peric, Computational methods for fluid dynamics, 2003. doi:10.1016/S0898-1221(03)90046-0.
- [4] D. Kuzmin, A Guide to Numerical Methods for Transport Equations, 2010. doi:10.1007/978-3-642-11640-7_3.
- [5] M.S. Astanina, M.A. Sheremet, H.F. Oztop, N. Abu-Hamdeh, Mixed convection of Al₂O₃-water nanofluid in a lid-driven cavity having two porous layers, *Int. J. Heat Mass Transf.* 118 (2018) 527–537. doi:10.1016/j.ijheatmasstransfer.2017.11.018.
- [6] M.A. Sheremet, I. Pop, O. Mahian, Natural convection in an inclined cavity with time-periodic temperature boundary conditions using nanofluids: Application in solar collectors, *Int. J. Heat Mass Transf.* 116 (2018). doi:10.1016/j.ijheatmasstransfer.2017.09.070.
- [7] M.S. Astanina, M. Kame' Rishi, E. Abu-Nada, M.A. Sheremet, Magneto-hydrodynamic in partially heated square cavity with variable properties: Discrepancy in experimental and theoretical conductivity correlations, *Int. J. Heat Mass Transf.* 116 (2018) 532–548. doi:10.1016/j.ijheatmasstransfer.2017.09.050.
- [8] M.A. Ismael, E. Abu-Nada, A.J. Chamkha, Mixed convection in a square cavity filled with CuO-water nanofluid heated by corner heater, *Int. J. Mech. Sci.* 133 (2017) 42–50. doi:10.1016/j.imecsci.2017.08.029.
- [9] A.I. Alsabery, A.J. Chamkha, H. Saleh, I. Hashim, Heatline visualization of conjugate natural convection in a square cavity filled with nanofluid with sinusoidal temperature variations on

- both horizontal walls, *Int. J. Heat Mass Transf.* 100 (2016) 835–850.
doi:10.1016/j.ijheatmasstransfer.2016.05.031.
- [10] H. Hassan, S. Harmand, Study of the parameters and characteristics of flat heat pipe with nanofluids subjected to periodic heat load on its performance, *Int. J. Therm. Sci.* 97 (2015) 126–142. doi:10.1016/j.ijthermalsci.2015.06.009.
- [11] C.C. Cho, C.L. Chen, C.K. Chen, Natural convection heat transfer performance in complex-wavy-wall enclosed cavity filled with nanofluid, *Int. J. Therm. Sci.* 60 (2012) 255–263. doi:10.1016/j.ijthermalsci.2012.05.001.
- [12] M.A. Ismael, M.A. Mansour, A.J. Chamkha, A.M. Rashad, Mixed convection in a nanofluid filled-cavity with partial slip subjected to constant heat flux and inclined magnetic field, *J. Magn. Magn. Mater.* 416 (2016) 25–36. doi:10.1016/j.jmmm.2016.05.006.
- [13] S.K. Mohammadian, H. Reza Seyf, Y. Zhang, Performance Augmentation and Optimization of Aluminum Oxide-Water Nanofluid Flow in a Two-Fluid Microchannel Heat Exchanger, *J. Heat Transfer.* 136 (2013) 21701. doi:10.1115/1.4023431.
- [14] W. Tang, M. Hatami, J. Zhou, D. Jing, Natural convection heat transfer in a nanofluid-filled cavity with double sinusoidal wavy walls of various phase deviations, *Int. J. Heat Mass Transf.* 115 (2017) 430–440. doi:10.1016/j.ijheatmasstransfer.2017.07.057.
- [15] A.A. Minea, W.M. El-Maghlany, Natural convection heat transfer utilizing ionic nanofluids with temperature-dependent thermo-physical properties, *Chem. Eng. Sci.* 174 (2017) 13–24. doi:10.1016/j.ces.2017.03.023.
- [16] A.A. Minea, W.M. El-Maghlany, Influence of hybrid nanofluids on the performance of parabolic trough collector in solar thermal systems: Recent findings and numerical comparison, *Renew. Energy.* 120 (2018) 350–364. doi:10.1016/j.renene.2017.12.093.
- [17] J.A. Esfahani, M. Arabzadeh, S. Rashidi, M.A. Rosen, R. Ellahi, Influences of wavy wall and nanoparticles on entropy generation over heat exchanger plate, *Int. J. Heat Mass Transf.* 109 (2017) 1162–1171. doi:10.1016/j.ijheatmasstransfer.2017.03.006.
- [18] A.A. Minea, Simulation of Nanofluids Turbulent Forced Convection at High Reynolds Number:

- A Comparison Study of Thermophysical Properties Influence on Heat Transfer Enhancement, *Flow, Turbul. Combust.* 94 (2014) 555–575. doi:10.1007/s10494-014-9590-9.
- [19] A.A. Minea, G. Lorenzini, A numerical study on ZnO based nanofluids behavior on natural convection, *Int. J. Heat Mass Transf.* 114 (2017) 286–296. doi:10.1016/j.ijheatmasstransfer.2017.06.069.
- [20] A.A. Minea, A study on Brinkman number variation on water based nanofluid heat transfer in partially heated tubes, *Mech. Res. Commun.* 73 (2016) 7–11. doi:10.1016/j.mechrescom.2016.01.013.
- [21] E.I. Chereches, K.V. Sharma, A.A. Minea, A numerical approach in describing ionanofluids behavior in laminar and turbulent flow, *Contin. Mech. Thermodyn.* 30 (2018) 657–666. doi:10.1007/s00161-018-0634-x.
- [22] E. Kaloudis, E. Papanicolaou, V. Belessiotis, Numerical simulations of a parabolic trough solar collector with nanofluid using a two-phase model, *Renew. Energy.* 97 (2016) 218–229. doi:10.1016/j.renene.2016.05.046.
- [23] X. Meng, X. Zhang, Q. Li, Numerical investigation of nanofluid natural convection coupling with nanoparticles sedimentation, *Appl. Therm. Eng.* 95 (2016) 411–420. doi:10.1016/j.applthermaleng.2015.10.089.
- [24] R. Nasrin, N.A. Rahim, H. Fayaz, M. Masanuzzaman, Water/MWCNT nanofluid based cooling system of PVT: Experimental and numerical research, *Renew. Energy.* (2018). doi:10.1016/j.renene.2018.01.014.
- [25] V. Bianco, F. Scarpà, L. A. Tagliafico, Numerical analysis of the Al₂O₃-water nanofluid forced laminar convection in an asymmetric heated channel for application in flat plate PV/T collector, *Renew. Energy.* 116 (2018) 9–21. doi:10.1016/j.renene.2017.09.067.
- [26] P.S.R. Dawid Gude, Gunjo, Smruti Ranjan Jena, Pinakeswar Mahanta, Melting enhancement of a latent heat storage with dispersed Cu, CuO and Al₂O₃ nanoparticles for solar thermal application, *Renew. Energy.* (2018). doi:https://doi.org/10.1016/j.renene.2018.01.013.
- [27] M. Hatami, D. Jing, Evaluation of wavy direct absorption solar collector (DASC) performance

- using different nanofluids, *J. Mol. Liq.* 229 (2017) 203–211. doi:10.1016/j.molliq.2016.12.072.
- [28] M. Hatami, D. Jing, Optimization of wavy direct absorber solar collector (WDASC) using Al_2O_3 -water nanofluid and RSM analysis, *Appl. Therm. Eng.* 121 (2017) 1049–1050. doi:10.1016/j.applthermaleng.2017.04.137.
- [29] E. Tombarević, V.R. Voller, I. Vusanović, Detailed CVFEM algorithm for three dimensional advection-diffusion problems, *C. - Comput. Model. Eng. Sci.* 96 (2013) 1–29. doi:10.3970/cmescs.2013.096.001.
- [30] M. Sheikholeslami, T. Hayat, A. Alsaedi, On simulation of nanofluid radiation and natural convection in an enclosure with elliptical cylinders, *Int. J. Heat Mass Transf.* 115 (2017) 981–991. doi:10.1016/j.ijheatmasstransfer.2017.07.119.
- [31] M. Sheikholeslami, T. Hayat, A. Alsaedi, MHD free convection of Al_2O_3 -water nanofluid considering thermal radiation: A numerical study, *Int. J. Heat Mass Transf.* 96 (2016) 513–524. doi:10.1016/j.ijheatmasstransfer.2016.01.051.
- [32] M. Sheikholeslami, M.M. Rashidi, T. Hayat, D.D. Ganji, Free convection of magnetic nanofluid considering MFD viscosity effect, *J. Mol. Liq.* 218 (2016) 393–399. doi:10.1016/j.molliq.2016.02.093.
- [33] M. Sheikholeslami, T. Hayat, A. Alsaedi, S. Abelman, Numerical analysis of EHD nanofluid force convective heat transfer considering electric field dependent viscosity, *Int. J. Heat Mass Transf.* 108 (2017) 2558–2565. doi:10.1016/j.ijheatmasstransfer.2016.10.099.
- [34] M. Sheikholeslami, T. Hayat, A. Alsaedi, Numerical study for external magnetic source influence on water based nanofluid convective heat transfer, *Int. J. Heat Mass Transf.* 106 (2017) 745–755. doi:10.1016/j.ijheatmasstransfer.2016.09.077.
- [35] M. Sheikholeslami, D.D. Ganji, Influence of electric field on Fe_3O_4 - water nanofluid radiative and convective heat transfer in a permeable enclosure, *J. Mol. Liq.* 250 (2018) 404–412. doi:10.1016/j.molliq.2017.12.028.
- [36] M. Sheikholeslami, Numerical simulation of magnetic nanofluid natural convection in porous media, *Phys. Lett. A.* 381 (2017) 494–503. doi:10.1016/j.physleta.2016.11.042.

- [37] M. Sheikholeslami, Numerical simulation of magnetic nanofluid natural convection in porous media, *Phys. Lett. Sect. A Gen. At. Solid State Phys.* 381 (2017) 494–503. doi:10.1016/j.physleta.2016.11.042.
- [38] M. Costabel, Principles of boundary element methods, *Comput. Phys. Reports.* 6 (1987) 243–274. doi:10.1016/0167-7977(87)90014-1.
- [39] F.S. Oğlakkaya, C. Bozkaya, MHD natural convection in a semi-annulus enclosure filled with water-based nanofluid using DRBEM, *Eng. Anal. Bound. Elem.* 71 (2016) 151–163. doi:10.1016/j.enganabound.2016.07.013.
- [40] D. Gottlieb, S. Gottlieb, Spectral methods, *Scholarpedia.* 4 (2009) 7504. doi:10.4249/scholarpedia.7504.
- [41] A. Wakif, Z. Boulahia, R. Sehaqui, Numerical analysis of the onset of longitudinal convective rolls in a porous medium saturated by an electrically conducting nanofluid in the presence of an external magnetic field, *Results Phys.* 7 (2017) 2151–2152. doi:10.1016/j.rinp.2017.06.003.
- [42] G.R. Liu, Y.T. Gu, *An introduction to meshfree methods and their programming*, 2005. doi:10.1007/1-4020-3468-7.
- [43] A.J. Katz, *Meshless Methods for Computational Fluid Dynamics*, Stanford University, 2009.
- [44] P.J. Hoogerbrugge, J.M.V.A. Koelman, Simulating microscopic hydrodynamic phenomena with dissipative particle dynamics, *JPL.* 19 (1992) 155–160. doi:10.1209/0295-5075/19/3/001.
- [45] M.B. Liu, G.R. Liu, L.W. Zhou, J.Z. Chang, Dissipative Particle Dynamics (DPD): An Overview and Recent Developments, *Arch. Comput. Methods Eng.* 22 (2015) 529–556. doi:10.1007/s11831-014-9114-x.
- [46] E. Abu-Nada, Mixed convection simulation using dissipative particle dynamics, *Numer. Heat Transf. Part A Appl.* 67 (2015) 808–825. doi:10.1080/10407782.2014.949178.
- [47] M.B. Liu, G.R. Liu, Smoothed particle hydrodynamics (SPH): An overview and recent developments, *Arch. Comput. Methods Eng.* 17 (2010) 25–76. doi:10.1007/s11831-010-9040-7.
- [48] N. Filipovic, M. Ivanovic, M. Kojic, A comparative numerical study between dissipative

particle dynamics and smoothed particle hydrodynamics when applied to simple unsteady flows in microfluidics, *Microfluid. Nanofluidics*. 7 (2009) 227–235. doi:10.1007/s10404-008-0379-0.

- [49] A.V. Potapov, M.L. Hunt, C.S. Campbell, Liquid-solid flows using smoothed particle hydrodynamics and the discrete element method, *Powder Technol.* 116 (2001). doi:10.1016/S0032-5910(00)00395-8.
- [50] M. Ellero, P. Español, Everything you always wanted to know about SDD* (*but were afraid to ask), *Appl. Math. Mech.* 39 (2018) 103–124. doi:10.1007/s10483-018-2255-6.
- [51] M. Nikfar, M. Mahmoodi, Meshless local Petrov-Galerkin analysis of free convection of nanofluid in a cavity with wavy side walls, *Eng. Anal. Bound. Elem.* 36 (2012) 433–445. doi:10.1016/j.enganabound.2011.09.017.
- [52] E.H. Ooi, V. Popov, Numerical study of influence of nanoparticle shape on the natural convection in Cu-water nanofluid, *Int. J. Therm. Sci.* 65 (2013) 178–188. doi:10.1016/j.ijthermalsci.2012.10.020.
- [53] G.C. Bourantas, E.D. Skouras, V.C. Loukopoulos, V.N. Burganos, Heat transfer and natural convection of nanofluids in porous media, *Eur. J. Mech. B/Fluids*. 43 (2014) 45–56. doi:10.1016/j.euromechflu.2013.06.013.
- [54] Y. Xuan, Z. Yao, Lattice boltzmann model for nanofluids, *Heat Mass Transf. Und Stoffuebertragung*. 41 (2005) 199–207. doi:10.1007/s00231-004-0539-z.
- [55] A. Montessori, P. Prestinazzi, M. La Rocca, S. Succi, Lattice Boltzmann approach for complex nonequilibrium flows, *Phys. Rev. E - Stat. Nonlinear, Soft Matter Phys.* 92 (2015). doi:10.1103/PhysRevE.92.043308.
- [56] N.A. Che Sidik, S. Aisyah Razali, Lattice Boltzmann method for convective heat transfer of nanofluids - A review, *Renew. Sustain. Energy Rev.* 38 (2014) 864–875. doi:10.1016/j.rser.2014.07.001.
- [57] N.A.C. Sidik, R. Mamat, Recent progress on lattice Boltzmann simulation of nanofluids: A review, *Int. Commun. Heat Mass Transf.* 66 (2015) 11–22. doi:10.1016/j.icheatmasstransfer.2015.05.010.

- [58] M. Kalteh, H. Hasani, Lattice Boltzmann simulation of nanofluid free convection heat transfer in an L-shaped enclosure, *Superlattices Microstruct.* 66 (2014) 112–128. doi:10.1016/j.spmi.2013.12.004.
- [59] B. Mliki, M.A. Abbassi, A. Omri, Z. Belkacem, Lattice Boltzmann analysis of MHD natural convection of CuO-water nanofluid in inclined C-shaped enclosure under the effect of nanoparticles Brownian motion, *Powder Technol.* 308 (2017) 70–83. doi:10.1016/j.powtec.2016.11.054.
- [60] W. Zhou, Y. Yan, Y. Xie, B. Liu, Three dimensional lattice Boltzmann simulation for mixed convection of nanofluids in the presence of magnetic field, *Int. Commun. Heat Mass Transf.* 80 (2017) 1–9. doi:10.1016/j.icheatmasstransfer.2016.11.012.
- [61] R. Mohebbi, M.M. Rashidi, M. Izadi, N.A.C. Sidik, H.W. Hsian, Forced convection of nanofluids in an extended surfaces channel using lattice Boltzmann method, *Int. J. Heat Mass Transf.* 117 (2018). doi:10.1016/j.ijheatmasstransfer.2017.10.063.
- [62] K. Ghasemi, M. Siavashi, Lattice Boltzmann numerical simulation and entropy generation analysis of natural convection of nanofluid in a porous cavity with different linear temperature distributions on side walls, *J. Mol. Liq.* 233 (2017) 415–430. doi:10.1016/j.molliq.2017.03.016.
- [63] R.D. Groot, P.B. Warren, Dissipative particle dynamics: Bridging the gap between atomistic and mesoscopic simulation, *J. Chem. Phys.* 107 (1997) 4423–4435.
- [64] E. Abu-Nada, Modeling of various heat transfer problems using dissipative particle dynamics, *Numer. Heat Transf. Part A Appl.* 58 (2010) 660–679. doi:10.1080/10407782.2010.516681.
- [65] E. Abu-Nada, Heat transfer simulation using energy conservative dissipative particle dynamics, *Mol. Simul.* 36 (2010) 382–390. doi:10.1080/08927020903515337.
- [66] E. Abu-Nada, Energy Conservative Dissipative Particle Dynamics Simulation of Natural Convection in Liquids, *J. Heat Transfer.* 133 (2011) 112502. doi:10.1115/1.4004347.
- [67] E. Abu-Nada, Natural convection heat transfer simulation using energy conservative dissipative particle dynamics, *Phys. Rev. E - Stat. Nonlinear, Soft Matter Phys.* 81 (2010). doi:10.1103/PhysRevE.81.056704.

- [68] E. Abu-Nada, Application of dissipative particle dynamics to natural convection in differentially heated enclosures, *Mol. Simul.* 37 (2011) 135–152.
doi:10.1080/08927022.2010.533272.
- [69] E. Abu-Nada, Dissipative particle dynamics simulation of natural convection using variable thermal properties, *Int. Commun. Heat Mass Transf.* 69 (2015) 84–93.
doi:10.1016/j.icheatmasstransfer.2015.10.008.
- [70] E. Abu-Nada, Simulation of heat transfer enhancement in nanofluids using dissipative particle dynamics, *Int. Commun. Heat Mass Transf.* 85 (2017) 1–11.
doi:10.1016/j.icheatmasstransfer.2017.04.008.
- [71] E. Abu-Nada, Dissipative particle dynamics investigation of heat transfer mechanisms in Al₂O₃-water nanofluid, *Int. J. Therm. Sci.* 123 (2018) 59–72.
doi:10.1016/j.ijthermalsci.2017.09.005.
- [72] X. Fan, N. Phan-Thien, S. Chen, X. Wu, T.Y. Ng, Simulating flow of DNA suspension using dissipative particle dynamics, *Phys. Fluids* 18 (2006). doi:10.1063/1.2206595.
- [73] P. Español, Dissipative particle dynamics with energy conservation, *Europhys. Lett.* 40 (1997) 631–636. doi:10.1209/epl/i1997-00015-8.
- [74] Y. Li, J. Xu, D. Li, Molecular dynamics simulation of nanoscale liquid flows, *Microfluid. Nanofluidics*. 9 (2010) 1011–1031. doi:10.1007/s10404-010-0612-5.
- [75] F. Jabbari, A. Rajabpour, G. Gaedodin, Thermal conductivity and viscosity of nanofluids: A review of recent molecular dynamics studies, *Chem. Eng. Sci.* 174 (2017) 67–81.
doi:10.1016/j.ces.2017.05.034.
- [76] S.L. Lee, R. Saidur, M.F.M. Sabri, T.K. Min, Effects of the particle size and temperature on the efficiency of nanofluids using molecular dynamic simulation, *Numer. Heat Transf. Part A Appl.* 69 (2016). doi:10.1080/10407782.2015.1109369.
- [77] M. Wei, C. Song, Y. Wang, Heat transfer of nanofluidics in hydrophilic pores: Insights from molecular dynamics simulations, *Chinese J. Chem. Eng.* 24 (2016) 1117–1121.
doi:10.1016/j.cjche.2016.04.036.

- [78] T. Fu, Y. Mao, Y. Tang, Y. Zhang, W. Yuan, Effect of nanostructure on rapid boiling of water on a hot copper plate: a molecular dynamics study, *Heat Mass Transf. Und Stoffübertragung*. 52 (2016) 1469–1478. doi:10.1007/s00231-015-1668-2.
- [79] S.S. Azimi, M. Kalbasi, A Molecular Dynamics Simulation of Brownian Motion of a Nanoparticle in a Nanofluid, *Nanoscale Microscale Thermophys. Eng.* 21 (2017) 263–277. doi:10.1080/15567265.2017.1286420.
- [80] D. Toghraie, M. Mokhtari, M. Afrand, Molecular dynamic simulation of Copper and Platinum nanoparticles Poiseuille flow in a nanochannels, *Phys. E Low-Dimensional Syst. Nanostructures*. 84 (2016) 152–161. doi:10.1016/j.physe.2016.06.006.
- [81] W. Cui, Z. Shen, J. Yang, S. Wu, Effect of chaotic movement of nanoparticles for nanofluid heat transfer augmentation by molecular dynamics simulation, *Appl. Therm. Eng.* 76 (2015) 261–271. doi:10.1016/j.applthermaleng.2014.11.030.
- [82] Y. G., Evans, J., Blackledge, P., *Numerical Methods for Partial Differential Equations*, Springer-Verlag London, 2000.
- [83] M. Stynes, Finite volume methods for convection-diffusion problems, *J. Comput. Appl. Math.* 63 (1995) 83–90. doi:10.1016/0377-0427(95)00056-9.
- [84] V.M. Puri, R.C. Anantheswarar, The finite-element method in food processing: A review, *J. Food Eng.* 19 (1993) 247–277. doi:10.1016/0260-8774(93)90046-M.
- [85] C.A. Brebbia, *The Boundary Element Method For Engineers*, Pentech Press, 1984. <https://books.google.co.uk/books?id=HuoeAQAAIAAJ>.
- [86] I. Rashidi, O. Mahian, G. Lorenzini, C. Biserni, S. Wongwises, Natural convection of Al_2O_3 /water nanofluid in a square cavity: Effects of heterogeneous heating, *Int. J. Heat Mass Transf.* 74 (2014) 291–302. doi:https://doi.org/10.1016/j.ijheatmasstransfer.2014.03.030.
- [87] A. Boutra, K. Ragu, N. Labsi, R. Bennacer, Y.K. Benkahla, Natural Convection Heat Transfer of a Nanofluid into a Cubical Enclosure: Lattice Boltzmann Investigation, *Arab. J. Sci. Eng.* 41 (2016) 1969–1980. doi:10.1007/s13369-016-2052-3.

- [88] J. Ravnik, L. Škerget, A numerical study of nanofluid natural convection in a cubic enclosure with a circular and an ellipsoidal cylinder, *Int. J. Heat Mass Transf.* 89 (2015) 596–605. doi:10.1016/j.ijheatmasstransfer.2015.05.089.
- [89] M.A. Sheremet, I. Pop, M.M. Rahman, Three-dimensional natural convection in a porous enclosure filled with a nanofluid using Buongiorno's mathematical model, *Int. J. Heat Mass Transf.* 82 (2015) 396–405. doi:10.1016/j.ijheatmasstransfer.2014.11.056.
- [90] M.Z. Saghir, A. Ahadi, A. Mohamad, S. Srinivasan, Water aluminum oxide nanofluid benchmark model, *Int. J. Therm. Sci.* 109 (2016) 148–158. doi:10.1016/j.ijthermalsci.2016.06.002.
- [91] C.J. Ho, W.K. Liu, Y.S. Chang, C.C. Lin, Natural convection heat transfer of alumina-water nanofluid in vertical square enclosures: An experimental study, *Int. J. Therm. Sci.* 49 (2010) 1345–1353. doi:10.1016/j.ijthermalsci.2010.02.013.
- [92] H.C. Birkman, The viscosity of concentrated suspensions and solution, *J. Chem. Phys.* 20 (1952) 571. doi:10.1063/1.1700493.
- [93] A. Purusothaman, N. Nithyadevi, H.F. Oztop, V. Divya, K. Al-Salem, Three dimensional numerical analysis of natural convection cooling with an array of discrete heaters embedded in nanofluid filled enclosure, *Adv. Powder Technol.* 27 (2016) 268–280. doi:10.1016/j.apt.2015.12.012.
- [94] L. Snoussi, R. Chouikh, N. Ouercif, A. Guizani, Numerical simulation of heat transfer enhancement for natural convection in a cubical enclosure filled with $\text{Al}_2\text{O}_3/\text{water}$ and Ag/water nanofluids, *Phys. Chem. Liq.* 54 (2016) 703–716. doi:10.1080/00319104.2016.1149173.
- [95] O. Mahian, H. Oztop, I. Pop, S. Mahmud, S. Wongwises, Entropy generation between two vertical cylinder in the presence of MHD flow subjected to constant wall temperature, *Int. Commun. Heat Mass Transf.* 44 (2013). doi:10.1016/j.icheatmasstransfer.2013.03.005.
- [96] O. Mahian, I. Pop, A.Z. Sahin, H.F. Oztop, S. Wongwises, Irreversibility analysis of a vertical annulus using $\text{TiO}_2/\text{water}$ nanofluid with MHD flow effects, *Int. J. Heat Mass Transf.* 64 (2013) 671–679. doi:10.1016/j.ijheatmasstransfer.2013.05.001.

- [97] O. Mahian, S. Mahmud, I. Pop, Analysis of first and second laws of thermodynamics between two isothermal cylinders with relative rotation in the presence of MHD flow, *Int. J. Heat Mass Transf.* 55 (2012). doi:10.1016/j.ijheatmasstransfer.2012.04.048.
- [98] O. Mahian, H.F. Oztop, I. Pop, S. Mahmud, S. Wongwises, Design of a vertical annulus with mhd flow using entropy generation analysis, *Therm. Sci.* 17 (2013) doi:10.2298/TSCI121017038M.
- [99] O. Mahian, S. Mahmud, S. Wongwises, Entropy generation between two rotating cylinders with magnetohydrodynamic flow using nanofluids, *J. Thermophys. Heat Transf.* 27 (2013). doi:10.2514/1.T3908.
- [100] M. Sheikholeslami, R. Ellahi, Three dimensional mesoscopic simulation of magnetic field effect on natural convection of nanofluid, *Int. J. Heat Mass Transf.* 89 (2015) 799–808. doi:10.1016/j.ijheatmasstransfer.2015.05.110.
- [101] A.A.A.A. Al-Rashed, K. Kalidasan, L. Kolsi, A. Ajdi, E.H. Malekshah, A.K. Hussein, P.R. Kanna, Three-dimensional investigation of the effects of external magnetic field inclination on laminar natural convection heat transfer in CNT–water nanofluid filled cavity, *J. Mol. Liq.* 252 (2018) 454–468. doi:https://doi.org/10.1016/j.molliq.2018.01.006.
- [102] Q.Z. Xue, Model for thermal conductivity of carbon nanotube-based composites, *Phys. B Condens. Matter.* 368 (2005) 302–307. doi:10.1016/j.physb.2005.07.024.
- [103] A.A.A.A. Al-Rashed, L. Kolsi, K. Kalidasan, C. Maatki, M.N. Borjini, M. Aichouni, P. Rajesh Kanna, Effect of magnetic field inclination on magnetoconvective induced irreversibilities in a CNT-water nanofluid filled cubic cavity, *Front. Heat Mass Transf.* 8 (2017). doi:10.5098/hmt.8.31.
- [104] L. Kolsi, A.A.A.A. Al-Rashed, K. Al-Salem, H.F. Oztop, M.N. Borjini, Control of natural convection via inclined plate of CNT-water nanofluid in an open sided cubical enclosure under magnetic field, *Int. J. Heat Mass Transf.* 111 (2017) 1007–1018. doi:10.1016/j.ijheatmasstransfer.2017.04.069.
- [105] L. Kolsi, H.F. Oztop, A. Alghamdi, N. Abu-Hamdeh, M.N. Borjini, H. Ben Aissia, A

- computational work on a three dimensional analysis of natural convection and entropy generation in nanofluid filled enclosures with triangular solid insert at the corners. *J. Mol. Liq.* 218 (2016) 260–274. doi:10.1016/j.molliq.2016.02.083.
- [106] L. Kolsi, A.K. Hussein, M.N. Borjini, H.A. Mohammed, H. Ben Aïssia, Computational Analysis of Three-Dimensional Unsteady Natural Convection and Entropy Generation in a Cubical Enclosure Filled with Water- Al_2O_3 Nanofluid, *Arab. J. Sci. Eng.* 39 (2014) 7483–7493. doi:10.1007/s13369-014-1341-y.
- [107] X. Meng, Y. Li, Numerical study of natural convection in a horizontal cylinder filled with water-based alumina nanofluid, *Nanoscale Res. Lett.* 10 (2015). doi:10.1186/s11671-015-0847-x.
- [108] P. Ternik, R. Rudolf, Heat-Transfer Characteristics of a non-Newtonian Au nanofluid in a cubical enclosure with differentially heated side walls, *Mater. Tehnol.* 49 (2015) 87–93.
- [109] L. Kolsi, K. Kalidasan, A. Alghamdi, M.N. Borjini, P.R. Kanna, Natural convection and entropy generation in a cubical cavity with twin adiabatic blocks filled by aluminum oxide–water nanofluid, *Numer. Heat Transf. Part A Appl.* 70 (2016) 242–259. doi:10.1080/10407782.2016.1173475.
- [110] M. Salari, E. Hasani Malekshah, M. Hasani Malekshah, M. Alavi, R. Hajihashemi, 3D numerical analysis of natural convection and entropy generation within tilted rectangular enclosures filled with stratified fluids of MWCNTs/water nanofluid and air, *J. Taiwan Inst. Chem. Eng.* 80 (2017) 62–73. doi:10.1016/j.jtice.2017.08.041.
- [111] A. A.A.A. Al-Rashed, K. Kalidasan, L. Kolsi, M.N. Borjini, P.R. Kanna, Three-dimensional natural convection of CNT-water nanofluid confined in an inclined enclosure with Ahmed body, *J. Therm. Sci. Technol.* 12 (2017) JTST0002–JTST0002. doi:10.1299/jtst.2017jtst0002.
- [112] A.A.A.A. Al-Rashed, L. Kolsi, K. Kalidasan, E.H. Malekshah, M.N. Borjini, P.R. Kanna, Second law analysis of natural convection in a CNT-water nanofluid filled inclined 3D cavity with incorporated Ahmed body, *Int. J. Mech. Sci.* 130 (2017) 399–415. doi:10.1016/j.ijmecsci.2017.06.028.

- [113] L. Kolsi, O. Mahian, H.F. Öztop, W. Aich, M.N. Borjini, N. Abu-Hamdeh, H. Ben Aissia, 3D buoyancy-induced flow and entropy generation of nanofluid-filled open cavities having adiabatic diamond shaped obstacles, *Entropy*. 18 (2016). doi:10.3390/e18060332.
- [114] L. Kolsi, E. Lajnef, W. Aich, A. Alghamdi, M. Ahmed Aichouni, M.N. Borjini, H. Ben Aissia, Numerical investigation of combined buoyancy-thermocapillary convection and entropy generation in 3D cavity filled with Al₂O₃nanofluid, *Alexandria Eng. J.* 56 (2017) 71–79. doi:10.1016/j.aej.2016.09.005.
- [115] A.A.A.A. Al-Rashed, L. Kolsi, H.F. Oztop, A. Aydi, E.H. Malekshah, N. Abu-Hamdeh, M.N. Borjini, 3D magneto-convective heat transfer in CNT-nanofluid filled cavity under partially active magnetic field, *Phys. E Low-Dimensional Syst. Nanostructures*. (2018). doi:https://doi.org/10.1016/j.physe.2018.02.011.
- [116] A. Rahimi, A. Kasaeipoor, A. Amiri, M.H. Doranehgar, E.H. Malekshah, L. Kolsi, Lattice Boltzmann method based on Dual-MRT model for three-dimensional natural convection and entropy generation in CuO-water nanofluid filled enclosure included with discrete active walls, *Comput. Math. with Appl.* (2017). doi:10.1016/j.camwa.2017.11.037.
- [117] A. Rahimi, A. Kasaeipoor, E.H. Malekshah, L. Kolsi, Experimental and numerical study on heat transfer performance of three-dimensional natural convection in an enclosure filled with DWCNTs-water nanofluid, *Powder Technol.* 322 (2017) 340–352. doi:10.1016/j.powtec.2017.06.008.
- [118] M.A. Sheremet, T. Grosan, I. Pop, Free Convection in a Square Cavity Filled with a Porous Medium Saturated by Nanofluid Using Tiwari and Das' Nanofluid Model, *Transp. Porous Media*. 106 (2015) 575–590. doi:10.1007/s11242-014-0415-3.
- [119] H. Khosravi-Bizhaem, A. Abbassi, Effects of curvature ratio on forced convection and entropy generation of nanofluid in helical coil using two-phase approach, *Adv. Powder Technol.* (2018). doi:10.1016/j.apt.2018.01.005.
- [120] S.M. Hassanien, M.A. Akhavan-Behabadi, An empirical study on heat transfer and pressure drop characteristics of CuO-base oil nanofluid flow in a horizontal helically coiled tube under constant heat flux, *Int. Commun. Heat Mass Transf.* 39 (2012) 144–151.

doi:10.1016/j.icheatmasstransfer.2011.09.002.

- [121] C. Yang, X. Wu, Y. Zheng, T. Qiu, Heat transfer performance assessment of hybrid nanofluids in a parallel channel under identical pumping power, *Chem. Eng. Sci.* 163 (2017) 67–77. doi:10.1016/j.ces.2017.04.045.
- [122] F.I. Doshmanziari, M.R. Kadivar, M. Yaghoubi, D. Jalali-Vahid, M. A. Arvinfar, Experimental and Numerical Study of Turbulent Fluid Flow and Heat Transfer of Al_2O_3 /Water Nanofluid in a Spiral-Coil Tube, *Heat Transf. Eng.* 38 (2017) 611–626. doi:10.1080/01457632.2016.1200380.
- [123] A. Tohidi, H. Ghaffari, H. Nasibi, A.S. Mujumdar, Heat transfer enhancement by combination of chaotic advection and nanofluids flow in helically coiled tube, *Appl. Therm. Eng.* 86 (2015) 91–105. doi:10.1016/j.applthermaleng.2015.04.043.
- [124] H. Togun, H.I. Abu-Mulaweh, S.N. Kazi, A. Badarudin, Numerical simulation of heat transfer and separation Al_2O_3 /nanofluid flow in concentric annular pipe, *Int. Commun. Heat Mass Transf.* 71 (2016) 108–117. doi:10.1016/j.icheatmasstransfer.2015.12.014.
- [125] A. Shalchi-Tabrizi, H.R. Seyf, Analysis of entropy generation and convective heat transfer of Al_2O_3 nanofluid flow in a tangential micro heat sink, *Int. J. Heat Mass Transf.* 55 (2012) 4366–4375. doi:10.1016/j.ijheatmasstransfer.2012.04.005.
- [126] H.R. Seyf, Z. Zhou, H.B. Ma, Y. Zhang, Three dimensional numerical study of heat-transfer enhancement by nano-encapsulated phase change material slurry in microtube heat sinks with tangential impingement, *Int. J. Heat Mass Transf.* 56 (2013) 561–573. doi:10.1016/j.ijheatmasstransfer.2012.08.052.
- [127] N. Yavuz, M.M.; Tokgöç, Investigation of nanofluid flow characteristics and wall stress concentration locations in a 3D channel with sharp edge rectangular ribs, *J. Therm. Eng.* 2 (2016) 715–734.
- [128] P. Gunnasegaran, N.H. Shuaib, M.F. Abdul Jalal, E. Sandhita, Numerical Study of Fluid Dynamic and Heat Transfer in a Compact Heat Exchanger Using Nanofluids, *ISRN Mech. Eng.* 2012 (2012) 1–11. doi:10.5402/2012/585496.
- [129] H.R. Seyf, M. Feizbakhshi, Computational analysis of nanofluid effects on convective heat

- transfer enhancement of micro-pin-fin heat sinks, *Int. J. Therm. Sci.* 58 (2012) 168–179. doi:10.1016/j.ijthermalsci.2012.02.018.
- [130] S. Dugaria, M. Bortolato, D. Del Col, Modelling of a direct absorption solar receiver using carbon based nanofluids under concentrated solar radiation, *Renew. Energy*. (2017). doi:10.1016/j.renene.2017.06.029.
- [131] A.K. Tiwari, P. Ghosh, J. Sarkar, H. Dahiya, J. Parekh, Numerical investigation of heat transfer and fluid flow in plate heat exchanger using nanofluids, *Int. J. Therm. Sci.* 85 (2014) 93–103. doi:10.1016/j.ijthermalsci.2014.06.015.
- [132] A. Mukherjee, S. Rout, A.K. Barik, Heat transfer and entropy generation analysis of a protruded surface in presence of a cross-flow jet using Al_2O_3 -water nanofluid, *Therm. Sci. Eng. Prog.* 5 (2018) 327–338. doi:https://doi.org/10.1016/j.tsep.2018.09.001.
- [133] A.M. Hussein, H.K. Dawood, R.A. Bakara, K. Madi Gamaa, Numerical study on turbulent forced convective heat transfer using nanofluids TiO_2 in an automotive cooling system, *Case Stud. Therm. Eng.* 9 (2017). doi:10.1016/j.csite.2016.11.005.
- [134] M. Sheikholeslami, T. Hayat, A. Alsaedi, Numerical simulation of nanofluid forced convection heat transfer improvement in existence of magnetic field using lattice Boltzmann method, *Int. J. Heat Mass Transf.* 108 (2017) 1870–1883. doi:10.1016/j.ijheatmasstransfer.2017.01.044.
- [135] A.A. Hawwash, A.K. Abdel Rahman, S.A. Nada, S. Ookawara, Numerical Investigation and Experimental Verification of Performance Enhancement of Flat Plate Solar Collector Using Nanofluids, *Appl. Therm. Eng.* 130 (2018) 363–374. doi:10.1016/J.APPLTHERMENG.2017.11.027.
- [136] H.R. Seyf, B. Nikseini, Analysis of Brownian motion and particle size effects on the thermal behavior and cooling performance of microchannel heat sinks, *Int. J. Therm. Sci.* 58 (2012) 36–44. doi:10.1016/j.ijthermalsci.2012.02.022.
- [137] H. Hadi Najafabadi, M. Keshavarz Moraveji, CFD investigation of local properties of Al_2O_3 /water nanofluid in a converging microchannel under imposed pressure difference, *Adv. Powder Technol.* 28 (2017) 763–774. doi:10.1016/j.apt.2016.11.024.

- [138] W.C. Chen, W.T. Cheng, Numerical simulation on forced convective heat transfer of titanium dioxide/water nanofluid in the cooling stage of blast furnace, *Int. Commun. Heat Mass Transf.* 71 (2016) 208–215. doi:10.1016/j.icheatmasstransfer.2015.12.020.
- [139] G. Huminic, A. Huminic, The heat transfer performances and entropy generation analysis of hybrid nanofluids in a flattened tube, *Int. J. Heat Mass Transf.* 119 (2017) 813–827. doi:10.1016/j.ijheatmasstransfer.2017.11.155.
- [140] A.A. Alfaryjat, H.A. Mohammed, N.M. Adam, D. Stanciu, A. L. Dobrovicescu, Numerical investigation of heat transfer enhancement using various nanofluids in hexagonal microchannel heat sink, *Therm. Sci. Eng. Prog.* (2017). doi:https://doi.org/10.1016/j.tsep.2017.12.003.
- [141] S.E. Ghasemi, A.A. Ranjbar, Thermal performance analysis of solar parabolic trough collector using nanofluid as working fluid: A CFD modelling study, *J. Mol. Liq.* 222 (2016) 159–166. doi:10.1016/j.molliq.2016.06.091.
- [142] A. Karimi, M. Afrand, Numerical study on thermal performance of an air-cooled heat exchanger: Effects of hybrid nanofluid, pipe arrangement and cross section, *Energy Convers. Manag.* 164 (2018) 615–628. doi:https://doi.org/10.1016/j.enconman.2018.03.038.
- [143] M. Hosseinzadeh, A. Salari, M. Sarfarabad, M. Passandideh-Fard, Optimization and parametric analysis of a nanofluid based photovoltaic thermal system: 3D numerical model with experimental validation, *Energy Convers. Manag.* 160 (2018) 93–108. doi:https://doi.org/10.1016/j.enconman.2018.01.006.
- [144] D. Toghraie, M. Mahmoudi, J.A. Akbari, F. Pourfattah, M. Heydari, The effect of using water/CuO nanofluid and L-shaped porous ribs on the performance evaluation criterion of microchannels, *J. Therm. Anal. Calorim.* (2018). doi:10.1007/s10973-018-7254-3.
- [145] M. Khoshvaght-Aliabad, Z. Arani, F. Rahimpour, Influence of Al_2O_3 - H_2O nanofluid on performance of twisted minichannels, *Adv. Powder Technol.* 27 (2016) 1514–1525. doi:10.1016/j.apt.2016.05.013.
- [146] X.W. Zhu, Y.H. Fu, J.Q. Zhao, L. Zhu, Three-dimensional numerical study of the laminar flow and heat transfer in a wavy-finned heat sink filled with Al_2O_3 /ethylene glycol-water nanofluid,

- Numer. Heat Transf. Part A Appl. 69 (2016) 195–208. doi:10.1080/10407782.2015.1052323.
- [147] H.K. Bizhaem, A. Abbassi, Numerical study on heat transfer and entropy generation of developing laminar nanofluid flow in helical tube using two-phase mixture model, Adv. Powder Technol. 28 (2017) 2110–2125. doi:10.1016/j.appt.2017.05.018.
- [148] V. Delavari, S.H. Hashemabadi, CFD simulation of heat transfer enhancement of Al₂O₃/water and Al₂O₃/ethylene glycol nanofluids in a car radiator, Appl. Therm. Eng. 73 (2014) 378–388. doi:10.1016/j.applthermaleng.2014.07.061.
- [149] S.M. Peyghambarzadeh, S.H. Hashemabadi, S.M. Hoseini, M. Seifi, Jamnani, Experimental study of heat transfer enhancement using water/ethylene glycol based nanofluids as a new coolant for car radiators, Int. Commun. Heat Mass Transf. 38 (2011) 1283–1290. doi:10.1016/j.icheatmasstransfer.2011.07.001.
- [150] H.E. Ahmed, M.Z. Yusoff, M.N.A. Hawlader, M.A. Ahmed, B.H. Salman, A.S. Kerbeet, Turbulent heat transfer and nanofluid flow in a triangular duct with vortex generators, Int. J. Heat Mass Transf. 105 (2017) 495–504. doi:10.1016/j.ijheatmasstransfer.2016.10.009.
- [151] Z.Y. Ghale, M. Haghshenasfard, M.N. Esfahany, Investigation of nanofluids heat transfer in a ribbed microchannel heat sink using single-phase and multiphase CFD models, Int. Commun. Heat Mass Transf. 68 (2015) 122–129. doi:10.1016/j.icheatmasstransfer.2015.08.012.
- [152] V. Kumar, J. Sarkar, Two-phase numerical simulation of hybrid nanofluid heat transfer in minichannel heat sink and experimental validation, Int. Commun. Heat Mass Transf. 91 (2018) 239–247. doi:https://doi.org/10.1016/j.icheatmasstransfer.2017.12.019.
- [153] Y.T. Yang, H.W. Tang, J.Y. Zeng, M.H. Jian, Numerical simulation and optimization of turbulent nanofluids in a three-dimensional arc rib-grooved channel, Numer. Heat Transf. Part A Appl. 70 (2016) 831–846. doi:10.1080/10407782.2016.1214513.
- [154] Y.T. Yang, H.W. Tang, S.J. Jian, Numerical simulation and optimization of turbulent nanofluids in a three-dimensional wavy channel, Numer. Heat Transf. Part A Appl. 69 (2016) 1169–1185. doi:10.1080/10407782.2015.1125729.
- [155] M. Bahiraei, M. Hangi, M. Saeedan, A novel application for energy efficiency improvement

- using nanofluid in shell and tube heat exchanger equipped with helical baffles. *Energy*. 93 (2015) 2229–2240. doi:10.1016/j.energy.2015.10.120.
- [156] S.E. Ghasemi, A.A. Ranjbar, M.J. Hosseini, Numerical study on effect of CuO-water nanofluid on cooling performance of two different cross-sectional heat sinks, *Adv. Powder Technol.* 28 (2017) 1495–1504. doi:10.1016/j.appt.2017.03.019.
- [157] R. Mashayekhi, E. Khodabandeh, M. Bahiraei, L. Bahrami, D. Toghraie, O.A. Akbari, Application of a novel conical strip insert to improve the efficacy of water–Ag nanofluid for utilization in thermal systems: A two-phase simulation, *Energy Convers. Manag.* 151 (2017) 573–586. doi:https://doi.org/10.1016/j.enconman.2017.05.025.
- [158] P. Naphon, L. Nakharintr, Turbulent two phase approach model for the nanofluids heat transfer analysis flowing through the minichannel heat sinks, *Int. J. Heat Mass Transf.* 82 (2015) 388–395. doi:10.1016/j.ijheatmasstransfer.2014.11.024.
- [159] J. Rostami, A. Abbassi, Conjugate heat transfer in a wavy microchannel using nanofluid by two-phase Eulerian-Lagrangian method, *Adv. Powder Technol.* 27 (2016) 9–18. doi:10.1016/j.appt.2015.10.003.
- [160] X. Shi, S. Li, Y. Wei, J. Gao, Numerical investigation of laminar convective heat transfer and pressure drop of water-based Al₂O₃ nanofluids in a microchannel, *Int. Commun. Heat Mass Transf.* 90 (2018) 111–120. doi:10.1016/j.icheatmasstransfer.2017.11.007.
- [161] A. Bhattad, J. Sarkar, P. Ghosh, Discrete phase numerical model and experimental study of hybrid nanofluid heat transfer and pressure drop in plate heat exchanger, *Int. Commun. Heat Mass Transf.* 91 (2018) 262–273. doi:https://doi.org/10.1016/j.icheatmasstransfer.2017.12.020.
- [162] T. Sheikhalipour, A. Abbassi, Numerical analysis of nanofluid flow inside a trapezoidal microchannel using different approaches, *Adv. Powder Technol.* (2018). doi:https://doi.org/10.1016/j.appt.2018.04.010.
- [163] M. Keshavarz Moaiveji, R. Barzegarian, M. Bahiraei, M. Barzegarian, A. Aloueyan, S. Wongwises, Numerical evaluation on thermal--hydraulic characteristics of dilute heat-dissipating nanofluids flow in microchannels, *J. Therm. Anal. Calorim.* (2018).

doi:10.1007/s10973-018-7181-3.

- [164] S.M. Mousavizadeh, G.R. Ansarifar, M. Talebi, Assessment of the TiO₂/water nanofluid effects on heat transfer characteristics in VVER-1000 nuclear reactor using CFD modeling, *Nucl. Eng. Technol.* 47 (2015) 814–826. doi:10.1016/j.net.2015.07.001.
- [165] H.K. Dawood, H.A. Mohammed, K.M. Munisamy, Heat transfer augmentation using nanofluids in an elliptic annulus with constant heat flux boundary condition, *Case Stud. Therm. Eng.* 4 (2014) 32–41. doi:10.1016/j.csite.2014.06.001.
- [166] A.S. Kherbeet, H.A. Mohammed, K.M. Munisamy, B.H. Salman, Combined convection nanofluid flow and heat transfer over microscale forward-facing step, *Int. J. Nanoparticles.* 7 (2014). doi:10.1504/IJNP.2014.062008.
- [167] A.A.A.A. Al-Rashed, W. Aich, L. Kolsi, O. Mahian, A.K. Hussein, M.N. Borjini, Effects of movable-baffle on heat transfer and entropy generation in a cavity saturated by CNT suspensions: Three-dimensional modeling, *Entropy* 19 (2017). doi:10.3390/e19050200.
- [168] A.A.A.A. Al-Rashed, K. Kalidasan, L. Kolsi, R. Velkennedy, A. Aydi, A.K. Hussein, E.H. Malekshah, Mixed convection and entropy generation in a nanofluid filled cubical open cavity with a central isothermal block, *Int. J. Mech. Sci.* 135 (2018) 362–375. doi:10.1016/j.ijmecsci.2017.11.013.
- [169] B. Karbasifar, M. Akbari, D. Moghrami, Mixed convection of Water-Aluminum oxide nanofluid in an inclined lid-driven cavity containing a hot elliptical centric cylinder, *Int. J. Heat Mass Transf.* 116 (2018) 1237–1249. doi:10.1016/j.ijheatmasstransfer.2017.09.110.
- [170] H.A. Mohammed, N.I. Cui, N.H. Shuaib, R. Saidur, Heat transfer enhancement for combined convection flow of nanofluids in a vertical rectangular duct considering radiation effects, *Heat Transf. - Asian Res.* 40 (2011) 448–463. doi:10.1002/htj.20354.
- [171] F. Selimeferdigil, F. Öztop, Mixed convection of nanofluids in a three dimensional cavity with two adiabatic inner rotating cylinders, *Int. J. Heat Mass Transf.* 117 (2018) 331–343. doi:10.1016/j.ijheatmasstransfer.2017.09.116.
- [172] M. Akbari, A. Behzadmehr, Developing mixed convection of a nanofluid in a horizontal tube

- with uniform heat flux, *Int. J. Numer. Methods Heat Fluid Flow*. 17 (2007) 566–586.
doi:10.1108/09615530710761216.
- [173] H. Safikhani, A. Abbassi, A. Khalkhali, M. Kalteh, Multi-objective optimization of nanofluid flow in flat tubes using CFD, Artificial Neural Networks and genetic algorithms, *Adv. Powder Technol.* 25 (2014) 1608–1617. doi:10.1016/j.appt.2014.05.014.
- [174] H.I. Mohammed, D. Giddings, G.S. Walker, H. Power, CFD assessment of the effect of nanoparticles on the heat transfer properties of acetone/ZnBr₂ solution, *Appl. Therm. Eng.* 128 (2018) 264–273. doi:10.1016/j.applthermaleng.2017.08.169.
- [175] M. Shariat, A. Akbarinia, A.H. Nezhad, A. Behzadmehr, R. Pour, Numerical study of two phase laminar mixed convection nanofluid in elliptic ducts, *Appl. Therm. Eng.* 31 (2011) 2348–2359. doi:10.1016/j.applthermaleng.2011.03.035.
- [176] S.A. Zonouzi, H. Aminfar, M. Mohammadpourfar, 3D numerical investigation of thermal characteristics of nanofluid flow through helical tubes using two-phase mixture model, *Int. J. Comput. Methods Eng. Sci. Mech.* 15 (2014) 512–521. doi:10.1080/15502287.2014.952847.
- [177] S. Sivasankaran, K. Narrein, Numerical investigation of two-phase laminar pulsating nanofluid flow in helical microchannel filled with a porous medium, *Int. Commun. Heat Mass Transf.* 75 (2016) 86–91. doi:10.1016/j.icheatmasstransfer.2016.03.016.
- [178] A.K. Kareem, S. Gao, Computational study of unsteady mixed convection heat transfer of nanofluids in a 3D closed lid-driven cavity, *Int. Commun. Heat Mass Transf.* 82 (2017) 125–138. doi:10.1016/j.icheatmasstransfer.2017.02.009.
- [179] H. Aminfar, M. Mohammadpourfar, Y.N. Kahnamouei, Numerical study of magnetic field effects on the mixed convection of a magnetic nanofluid in a curved tube, *Int. J. Mech. Sci.* 78 (2014) 81–90. doi:10.1016/j.ijmecsci.2013.10.014.
- [180] A. Shahmoradian, A. Jafari, Application of different CFD multiphase models to investigate effects of baffles and nanoparticles on heat transfer enhancement, *Front. Chem. Sci. Eng.* 8 (2014) 326–329. doi:10.1007/s11705-014-1437-7.
- [181] R. Yousofvand, S. Derakhshan, K. Ghasemi, M. Siavashi, MHD transverse mixed convection

and entropy generation study of electromagnetic pump including a nanofluid using 3D LBM simulation, *Int. J. Mech. Sci.* 133 (2017) 73–90. doi:10.1016/j.ijmecsci.2017.08.034.

[182] O. Mahian, A. Kianifar, S.Z. Heris, D. Wen, A.Z. Sahin, S. Wongwises, Nanofluids effects on the evaporation rate in a solar still equipped with a heat exchanger, *Nano Energy*. 36 (2017) 134–155. doi:10.1016/j.nanoen.2017.04.025.

[183] S.N. Shoghl, M. Bahrami, M.K. Moraveji, Experimental investigation and CFD modeling of the dynamics of bubbles in nanofluid pool boiling, *Int. Commun. Heat Mass Transf.* 58 (2014) 12–24. doi:10.1016/j.icheatmasstransfer.2014.07.027.