

Identification of material properties with indentation test and shape manifold learning approach

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Résumé — Limiting oneself to the use of solely the indentation curve prevents one from obtaining valuable information about anisotropic materials through the indentation test. Mapping the residual deformation (imprint shape) can help us identify a larger number of material properties. In the present work, we propose an approach that automatically determines the minimum number of parameters needed in order to characterize the imprint shape and link them with the material properties in the constitutive law.
Mots clés — indentation test, shape manifold, material properties.

1 Introduction

Nondestructive indentation testing applied to the identification of material properties usually relies on using the force-displacement curve. However, the number of parameters that can be identified this way is limited. This prevents the engineer from obtaining valuable information about possible anisotropy, resulting in non-unicity of the solution in the inverse analysis.

To identify material properties in a more accurate fashion, indentation test may be combined with the mapping of the residual deformation [1]. This method was also used for the identification of material properties in film-substrate systems [2]. [3] also stressed that the residual deformed configuration enriches the information for the identification of material parameters of the elastic-plastic material model, and examined the sensitivity with respect to the desired parameters. A comparative work [4] focused on analyzing the effect of model dimensions and their influence on numerical results.

Following [5, 6], originally applied to the numerical assessment of spring back for the deep drawing process, we apply the concept of the shape manifold as a supplement to the traditional indentation curve. The shape manifold is constructed by a series of simulated shape imprints using proper orthogonal decomposition. A manifold walking algorithm is then proposed to determine the search direction in an inverse analysis problem. Finally, we apply the approach to an indentation test using a spherical tip indenter on C100 steel.

2 Overall concept and formulation

We apply the method of POD to the collection of imprint shapes obtained by an indentation test. The finite element (FE) model and the residual deformation on the specimen are shown in Figure 1.

2.1 Proper orthogonal decomposition applied to imprint shapes

We start with M numerical experiments defined by an appropriate DOE for the varying material parameters. The different imprint shapes $h_i(x, y), i = 1, 2, \dots, M$ are next extracted from the simulation results and each of them is considered a snapshot. The snapshot matrix is generated from the centered snapshots \mathbf{S}

$$\mathbf{S} = [\mathbf{s}_1 - \bar{\mathbf{s}}, \mathbf{s}_2 - \bar{\mathbf{s}}, \dots, \mathbf{s}_M - \bar{\mathbf{s}}], \quad (1)$$

where $\bar{\mathbf{s}}$ is the mean snapshot

$$\bar{\mathbf{s}} = \frac{1}{M} \sum_{i=1}^M \mathbf{s}_i. \quad (2)$$

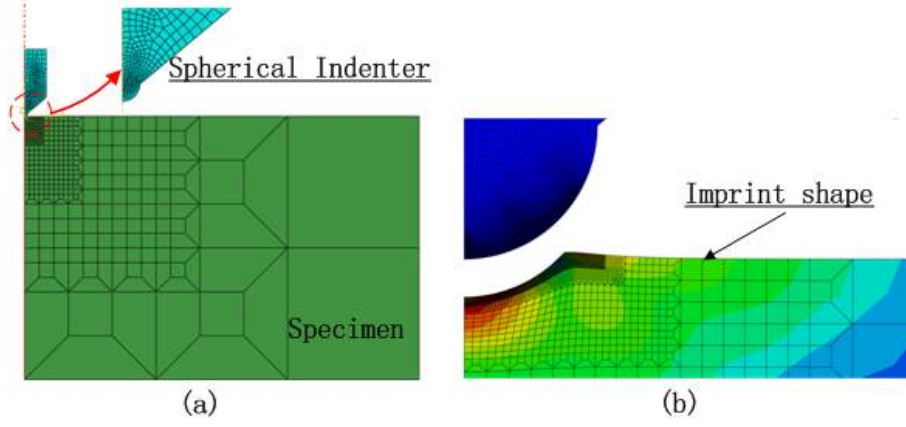


FIGURE 1 – FE simulation of indentation test and the residual displacement on the specimen

We use proper orthogonal decomposition (POD) to determine the minimum number of parameters that describe the imprint shape. Singular value decomposition of \mathbf{S} yields

$$\mathbf{S} = \Phi \mathbf{D} \mathbf{V}^T, \quad (3)$$

where \mathbf{D} contains the singular values d_i ; each column of Φ is an eigenvector of the covariance matrix $\mathbf{C} = \mathbf{S} \mathbf{S}^T$, and $\lambda_i = d_i^2$ are the corresponding eigenvalues. The eigenvectors ϕ_i are generally called the POD modes. Each snapshot \mathbf{s}_i can then be accurately reconstructed by a projection basis $\Phi = [\phi_1, \phi_2 \cdots \phi_M]$

$$\mathbf{s}_i = \bar{\mathbf{s}} + \Phi \alpha_i = \bar{\mathbf{s}} + \sum_{j=1}^M \alpha_i^j \phi_j, \quad (4)$$

where α_i^j is the projection coefficient for the i^{th} snapshot on the j^{th} mode, and is calculated by

$$\alpha_i^j = \phi_j^T \mathbf{s}_i, j = 1, 2 \cdots M. \quad (5)$$

Truncation of the basis to $m \ll M$ modes is made after considering the projection error η

$$\tilde{\mathbf{s}}_i = \bar{\mathbf{s}} + \sum_{j=1}^m \alpha_i^j \phi_j; \quad \eta = 1 - \frac{\sum_{i=1}^m \lambda_i}{\sum_{j=1}^M \lambda_j}. \quad (6)$$

2.2 Generating local manifold with Diffuse Approximation

By referring to Eq. (5), we already have a set of M snapshots represented by their coordinates in α -space, i.e. $\alpha_1, \alpha_2, \cdots, \alpha_M$. The transition is then made from these discrete snapshots to a smooth manifold to connect all the imprint shapes :

$$\mu(\alpha) = 0 \quad (7)$$

By using the Diffuse approximation, we may interpolate between these scattered snapshots and obtain a functional description of the manifold :

$$\mathbf{p}^T \mathbf{a} = 0 \quad (8)$$

where \mathbf{p} is composed of the basis functions of interpolation; $\mathbf{a} = (a_0, a_1, a_2, \cdots)^T$ is the coefficient vector. At this point, each point on the shape manifold corresponds to a particular imprint shape of the indentation test. The smooth manifold then approaches all the admissible imprint shapes attainable using the given constitutive law (used for performing the simulations), and it's intrinsic dimensionality gives us an idea of how many parameters we can actually identify. For example, if the manifold is a 2D hyper surface, then only two coefficients are needed to describe the manifold, which in turn implies that the manifold is governed by two material parameters. Figure 2 shows a 2D manifold fitted over 50 snapshots.

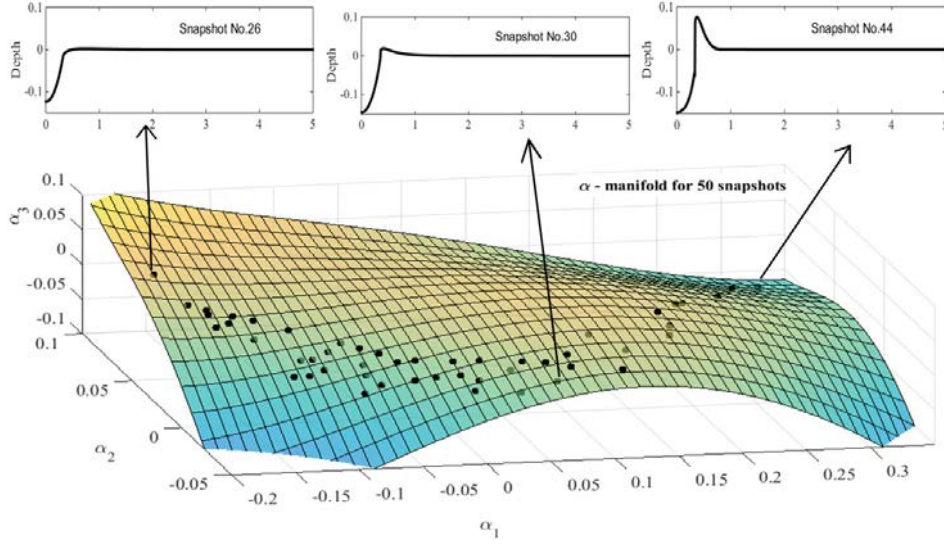


FIGURE 2 – A manifold connecting a series of imprint shapes of an indentation test.

2.3 Manifold walking algorithm

While simply projecting the experimental imprint shape onto the *global* shape manifold leads to a direct estimation of the parameters set to be identified, it is unrealistic since in most cases a massive number of simulations would be demanded which is time consuming and requires a prohibitively large amount of storage. Hence, we construct local α -manifolds "piece by piece" instead. A predictor-corrector manifold walking algorithm is proposed to iteratively locate the local manifold that contains the projection of the experimental result.

3 Numerical example

In the present work, Hollomon's law is employed to describe a power relationship between the equivalent stress and the plastic strain, while the elastic portion follows Hooke's law. The continuity at the point where the material starts to deform plastically is considered, resulting in[4]

$$\sigma = \sigma_y \left(\frac{E}{\sigma_y} \right)^n \epsilon^n \quad (9)$$

where σ is the equivalent stress, ϵ refers to the total strain. The behavior law of material is thus described by three parameters : yield stress σ_y , strain hardening exponent n and Young's modulus E .

The methodology proposed is verified by an axisymmetric indentation test on C100 steel using a spherical tip indenter. The linear portion of the material behavior curve is governed by elastic modulus E , while two other parameters governing the plastic properties need to be identified. The contact interface between the indenter and the specimen is characterized by Coulomb friction coefficient of 0.2. A series of imprints are provided in Figure 2 along with their projections on the manifold.

For the simulation procedure, the feasible regions for the two desired parameters are chosen as $\sigma_y \in [50, 900]$ and $n \in [0, 0.5]$. For each predictor-corrector step a local manifold is built with a small variation of $\pm 50 \text{ MPa}$ for σ_y and ± 0.05 for n . We start the inverse analysis from the initial estimation : $\sigma_y = 250, n = 0.25$.

Figure. 3 gives the results of successive iteration of the manifold walking algorithm, where the points in black refer to the snapshots of each DOE step, while those in red represent the actual experimentally obtained imprint shape projected on the local manifold. The material parameters that can, by simulation, accurately obtain the red point in the figure are the ones identified, thus it is called the target shape. The distance between the target on the current local basis Φ and its projection on the manifold is progressively minimized as the DOE pattern advances with successive iterations. Convergence is achieved when the target is located as accurately as possible on the local manifold. The material parameters are then

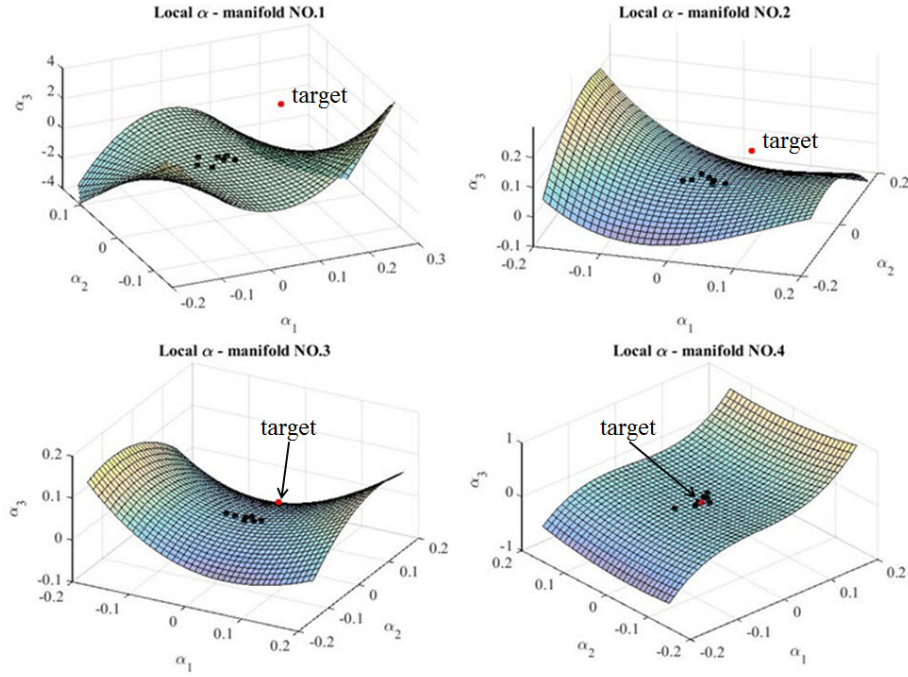


FIGURE 3 – Intermediate stage of manifold walking (red points refer to projections of actual imprint shape on local manifolds).

identified from the relationship between local coordinates in α -space and the numerical values of the material parameters.

4 Conclusion and prospective

The proposed methodology clearly relies on the hypothesis of the existence of a smooth manifold in α -space that connects all admissible shapes. The efficiency of the proposed manifold walking method is validated by a simple 2D numerical example. A more complex indentation test needs to be studied in order to verify the robustness of the methodology for the identification of anisotropic material properties.

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