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# Millimeter-Wave Digital Beamsteering in Highly Line-of-Sight Environments for Massive MIMO Systems

A. Rozé<sup>1,2</sup>, M. H elard<sup>1,2</sup>, M. Cruss iere<sup>1,2</sup> and C. Langlais<sup>1,3</sup>

<sup>1</sup>INSA, IETR, CNRS UMR 6164, Rennes, France

<sup>2</sup>IRT b<>com, Rennes, France

<sup>3</sup>Institut Mines-Telecom, Telecom Bretagne, CNRS UMR 6285 Lab-STICC, Brest, France

**Abstract**—Massive MIMO systems are now well known for their energy efficiency. The ability to focus the energy to a targeted user not only increases the gain of useful signal but also reduces inter-user interference to the point where Spatial Division Multiple Access becomes possible. Using such systems at high frequencies, thus greatly reducing the array form factor, seems a smart mix and we give an insight into how well an angle based precoder fares compared to well known massive MIMO linear precoders in a highly Line-of-Sight (LOS) environment.

In this paper we prove the optimality of this angle based precoder, called Digital Beamsteering (DBS), in an ideal pure LOS configuration and then we show how it performs in a more realistic environment. Furthermore, its robustness against angle estimation error is illustrated for a specific scenario and serves to back up the importance such a solution represents for future dense 5G networks, angular information being easier to get and especially to keep track of.

**Index Terms**—Massive MIMO, mmWave, Zero-Forcing, Conjugate, LOS, Beamforming, Beamsteering, DoA

## I. INTRODUCTION

As the demand in data rate constantly increases, carrier frequencies are raised to use the parts of the spectrum that offer a high bandwidth. This allows for a reduction of the cell sizes as well as a densification of the network.

At the same time, the studies about base stations with a large number of antennas, called Massive MIMO base stations [1], have demonstrated many great features linked to the ability to form beams toward a User Equipment (UE). This technique is called beamforming [2] and makes possible multi-user transmissions that benefit from very high gains without an excess of complexity while being very energy efficient [3].

Implementing such a system at high frequencies means:

- The size of the antenna array is drastically reduced which makes the system appropriate for small cell scenarios;
- The high path losses induced by millimeter-Wave (mmWave) transmissions can be compensated by the high beamforming gains [4];
- Transmissions are mostly Line-of-Sight (LOS) which in turn allows Massive MIMO systems to adapt to Direction-of-Arrival (DoA) based precoding [5].

Nowadays only Analog Beamsteering (ABS) is used at such high frequencies, using analog phase shifters to steer a beam

toward a specific direction [6]. Although it is effective and with relatively low implementation complexity, the transmission is limited to one user at a time and thus multiple UEs can only be handled with Time Division Multiple Access (TDMA).

Digital Beamforming (DBF), which relies on baseband processing and is widely called precoding, overcomes this limitation [1], separating the users spatially so that each of them can benefit from the entire bandwidth at all times. A Channel State Information (CSI) estimation is needed as well as a Radio Frequency (RF) chain behind each antenna to enforce the Spatial Division Multiple Access (SDMA).

Studies about mmWave beamforming tend to assume the only way to build a working Massive MIMO solution is to mix Analog and Digital Beamforming to form a hybrid system. In fact, having as many RF chains as there are antennas seems to produce many undesired effects and thus reducing the size of the digital stage is compulsory. An analog stage (which we call ABS since it steers a beam with analog phase shifters) can however be added to compensate the loss of gain so that the resulting beamforming performance remains unchanged. Such a solution allows multi-stream single user beamforming [6], [7] and manages to reach high capacities. Multi-user transmission is also possible with a hybrid system [8] but the part of the cell covered by the DoA estimation will directly depend on the number of antennas of the analog phased array stage. The more antennas in the ABS section, the higher the gain but also the longer the estimation process to cover the cell. In the end an optimum number of ABS antennas can surely be found for each applications that will benefit from such a solution.

In this paper we choose to study how a fully digital beamsteering (DBS), with no hybrid structure, performs when compared to two well known Massive MIMO precoders, the Zero-Forcing (ZF) and the Conjugate Beamforming (CB), in a highly LOS environment to prove how efficient such a simple solution can be. Some architectures, as the Code modulated Path sharing Multi-Antenna (CPMA) cluster described in [5], might actually make possible the use of that many RF chains and that motivates us to continue on this path.

First, section II sheds light on the difference between Analog and Digital Beamforming, completed with a comparison of

the chosen multi-user Massive MIMO precoders. Then section III describes how close to optimum the DBS is in a highly LOS environment due to the nature of the channel response and precoders are analytically derived for the single user case. Finally section IV shows how high the DBS sum capacity can go and a new insight about its robustness against angle estimation error is given. Conclusions are drawn in section V.

As partly illustrated on Fig 1, we use the following notations:

$M$  is the number of antennas at the base station with  $M_C$  (resp.  $M_R$ ) the number of columns (resp. rows) of the array,  $K$  the total number of users,  $d$  is the distance between two antennas (assuming the rows and columns are spaced equally), and  $\theta$  and  $\varphi$  are the azimuth and elevation angles respectively.  $\mathbf{X}$  is a matrix,  $\mathbf{x}$  a vector,  $x$  a scalar and  $(\cdot)^H$  is the Hermitian transpose.

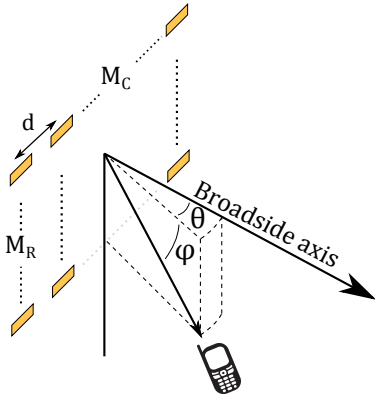


Figure 1. Antenna array structure and angle illustration.

## II. STEERING OR PRECODING?

In this section we first describe the differences between steering a beam analogically or forming one digitally. Then a comparison of digital multi-user Massive MIMO precoders is given, among which the angle based one called Digital Beamsteering (DBS).

### A. Analog and Digital: What's the difference?

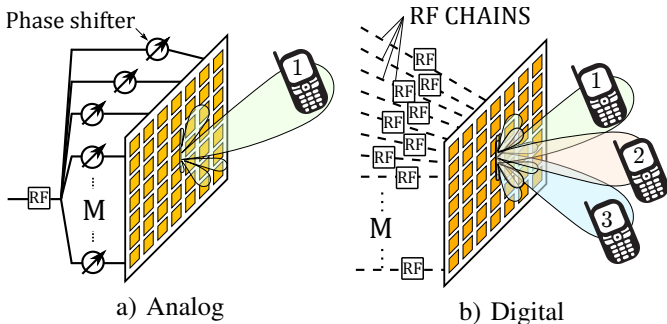


Figure 2. Analog and Digital beamforming representation.

Analog Beamsteering (ABS), illustrated on Fig. 2 a), is a technique that has been known and used for a long time. It has several names such as beamforming, reconfigurable array

or smart antennas. This system uses phase shifters to apply a delay to the incoming signal at each antenna therefore tilting the beam in a specific direction. One single RF chain is used and it bounds the system to a single user transmission with no spatial multiplexing. A LOS channel is highly recommended to implement the ABS.

Digital Beamforming (DBF), illustrated on Fig. 2 b), does not steer a beam but creates it. Indeed having as many RF chains as there are antennas allows the system to accurately control the amplitude and phase that the incoming signal will have at each antenna, therefore allowing complex radiation patterns to be shaped. Also, SDMA communications become possible and optimum numbers of users can be found for a fixed number of antennas to maximize any performance criteria, e.g. the energy efficiency in [3].

In other words the former can change the direction of an already formed beam (ABS) whereas the latter builds it (DBF).

Digital Beamsteering (DBS) is the multi-user version of the ABS and uses the DoA of each user to form and steer a beam in their direction. The hardware needed for its implementation is the same as the DBF one but what changes radically is the information needed to precode the signal. Indeed, we consider a 3D environment where the DoA of one user is described with two angles,  $\theta$  and  $\varphi$  which means that the number of coefficients for the ABS is 2.

This is compared to the other precoders in table 3 along with the channel to which these techniques apply, the number of RF chains needed the number of users the system can serve simultaneously as well as the number of coefficients that the system needs to form the precoder.

We can see that the same applies to the DBS with the difference that the DoA of all  $K$  users are needed simultaneously for multi-user transmission, hence the  $2K$  coefficients. The number of coefficients needed for the DBF is developed in II-B and we consider an OFDM system with  $N_{FFT}$  the number of subcarriers and  $N_{Bc}$  the number of subcarriers in the coherence bandwidth.

Technique	Channel	Nb RF	Nb UE	Nb coefficients
ABS	LOS	1	1	2
DBF	LOS, NLOS	$M$	$K$	$\frac{K \times M \times N_{FFT}}{N_{Bc}}$
DBS	LOS	$M$	$K$	$2K$

Figure 3. Summary of ABS, DBF and DBS differences.

### B. MU-MIMO precoder comparison

Assuming the same path losses apply between a UE and all antennas of the array, we can write the generalized downlink channel response as:

$$\mathbf{G}_n = \mathbf{D}_\beta^{1/2} \mathbf{H}_n \quad (1)$$

where  $\mathbf{H}_n \in \mathbb{C}^{K \times M}$  is the normalized downlink channel matrix of the subcarrier  $n$  taking only small scale fading into account and  $\mathbf{D}_\beta = \text{diag}(\beta_1, \dots, \beta_K)$  is the the path loss ( $\beta$ ) matrix.

We note  $\mathbf{r}_n = [r_{1,n}, \dots, r_{K,n}]^T$  the received signal vector with  $r_{k,n}$  the signal user  $k$  receives on subcarrier  $n$ ,  $\mathbf{W}_n \in \mathbb{C}^{M \times K}$  the precoding matrix,  $\mathbf{s}_n = [s_{1,n}, \dots, s_{K,n}]^T$  the useful signal vector intended to the  $K$  users and  $\eta_n$  the power normalization factor ensuring the precoding process leaves the transmitted power unchanged:

$$\eta_n = \frac{1}{\mathbb{E}[\text{tr}(\mathbf{W}_n^H \mathbf{W}_n)]} \quad (2)$$

Now we can write the propagation equation:

$$\mathbf{r}_n = \sqrt{\eta_n} \mathbf{G}_n \mathbf{W}_n \mathbf{s}_n \quad (3)$$

Let us introduce the precoders of interest whose expressions are given in Table 4:

- 1) The Conjugate Beamforming (CB), also often referred to as Maximum Ratio Transmission (MRT), applies the conjugate value of the channel response to the signal to maximize the Signal to Noise Ratio (SNR). The channel needs to be flat therefore one coefficient per subcarrier is needed, or at least one per coherence bandwidth, (see DBF in Table 3) which means there are  $M \times K \times N_{FFT} \div N_{Bc}$  coefficients per channel estimation.
- 2) The Zero-Forcing (ZF) precoder, needing the same channel estimation as the CB, cancels inter-user interference therefore maximizing the Signal to Interference Ratio (SIR). It has already been proven a lot more effective in complex environments [9], hence the many occurrences in the literature [10]–[13], but it is also common knowledge that it outperforms the CB only below a certain number of users [14], [15].
- 3) The DBS: the main purpose of an angle based precoder is to drastically reduce the amount of CSI information that the base station needs to form the beams ( $2K$  angles against  $K \times M \times N_{FFT} \div N_{Bc}$  channel coefficients). Furthermore angular updates can be predicted based on terminal trajectories therefore increasing the time lapse between two angle estimations and thus increasing the overall capacity.

Precoding	Complexity	$\mathbf{W}_n$	Maximize
CB	++	$\mathbf{H}_n^H$	SNR
ZF	+++	$\mathbf{H}_n^H (\mathbf{H}_n \mathbf{H}_n^H)^{-1}$	SIR
DBS	+	$\mathbf{A}^H$	SNR

Figure 4. Precoder comparison table.

In Table 4 we sum up the precoder expressions with a few characteristics such as the complexity of hardware implementation and what the beamformers maximize.  $\mathbf{A} = [\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_K]^T$  is the steering matrix composed of all steering vectors  $\mathbf{a}_k \in \mathbb{C}^{M \times 1}$ ,  $k \in [1, K]$ :

$$\mathbf{a}_k = [1, \dots, e^{j(M_C - 1) \frac{2\pi}{\lambda} d \cos(\varphi_k) \sin(\theta_k)}, \dots, e^{j \frac{2\pi}{\lambda} d ((M_C - 1) \cos(\varphi_k) \sin(\theta_k) + (M_R - 1) \sin(\varphi_k))}]^T \quad (4)$$

As we can see from the precoder expressions, the DBS takes a lot more from the CB than the ZF. Indeed, steering a beam is an other way to maximize the useful signal power regardless of the amount of interference it creates while Zero-Forcing cancels inter-user interference and is more complex to implement due to its matrix inversion. We have shown in [15] that such an operation limits the number of users the cell can serve in a pure LOS environment, and that the antenna array structure is the principal reason. This comes from the correlation between the columns and rows of the array which results in bringing the rank of the channel matrix down.

Also a multi-user Massive MIMO system only supports Time Division Duplex (TDD) transmissions because of the need for channel reciprocity so that precoding for downlink transmission can immediately follow an uplink channel estimation [1]. Although the channel is reciprocal, the hardware part is not (e.g. [14]) therefore, for such an assumption to be made and used, a calibration process is compulsory to compensate hardware imperfections and restore channel reciprocity. Using an angular based precoder cancels the need for system calibration. More than that, it is not limited to TDD communications anymore since  $\theta$  and  $\varphi$  do not depend on the frequency.

### III. ANALYTICAL IMPACT OF A LOS CHANNEL

Obviously a pure LOS channel without any reflections is not realistic but the fact is that the more we raise the carrier frequency, the closer we get to such a channel. So that performance evaluation can be made in section IV, we first describe the channel so that the DoA appear analytically, assuming such a channel gives clarity in how the different precoders perform in a realistic mmWave environment. Then we point out the similarity of each precoder in a single user scenario (MISO), such information helping the validation of simulation results.

#### A. Angular channel model

The channel between a user and the base station is the sum of  $L$  paths,  $L$  being the number of reflections that can occur in a specific environment. Each path  $l \in [1, L]$  has an attenuation ( $\beta_l$ ), a phase ( $e^{j\phi_l}$ ) and a DoA ( $\theta_l, \varphi_l$ ) that forms the steering vector  $\mathbf{a}_l$ .

The channel on subcarrier  $n$  can therefore be written as follows:

$$\mathbf{G}_n = \sum_{l=1}^L \mathbf{D}_{\beta_l, n}^{1/2} \mathbf{A}_l \quad (5)$$

with  $\mathbf{D}_{\beta_l} = \text{diag}(\sqrt{\beta_{1,l}} e^{j\phi_{1,l,n}}, \dots, \sqrt{\beta_{K,l}} e^{j\phi_{K,l,n}})$  the matrix including path loss and phase of the signal and  $\mathbf{A}_l$  the steering matrix of path  $l$ .

In a pure LOS scenario,  $L = 1$  and thus:

$$\mathbf{G}_n = \mathbf{D}_{\beta_n}^{1/2} \mathbf{A} \quad (6)$$

therefore  $\mathbf{H}_n = \mathbf{H} = \mathbf{A}$ ,  $\forall n \in [1, N_{FFT}]$ . This means that  $\mathbf{W}_{CB} = \mathbf{W}_{DBS}$ , which in itself proves that the closer to pure LOS the channel is, the more effective can a simple solution such as Digital Beamsteering be.

### B. Analysis for single user MIMO case

Let us have a look at the single user case so that an analytic insurance can be made concerning the trustworthiness of simulation results when increasing the number of users in the cell. We note  $\overline{\mathbf{W}}_n$  the normalized precoder of subcarrier  $n$  such that  $\overline{\mathbf{W}}_n = \sqrt{\eta_n} \mathbf{W}_n$ .

- *Conjugate Beamforming:*

$$\mathbf{W}_n^{(CB)} = \mathbf{H}_n^H \quad (7)$$

$$\eta_n = \frac{1}{\mathbb{E}[\mathbf{H}_n \mathbf{H}_n^H]} = \frac{1}{\mathbb{E}[\sum_{m=1}^M |h_{m,n}|^2]} = \frac{1}{M} \quad (8)$$

$$\overline{\mathbf{W}}_n^{(CB)} = \frac{\mathbf{H}_n^H}{\sqrt{M}} \quad (9)$$

- *Zero Forcing:*

$$\mathbf{W}_n^{(ZF)} = \mathbf{H}_n^H (\mathbf{H}_n \mathbf{H}_n^H)^{-1} \quad (10)$$

$$\eta_n = \frac{1}{\mathbb{E}[\text{tr}((\mathbf{H}_n^H \mathbf{H}_n)^{-1} \mathbf{H}_n \mathbf{H}_n^H (\mathbf{H}_n \mathbf{H}_n^H)^{-1})]} \quad (11)$$

$$\eta_n = \mathbb{E}[\text{tr}(\mathbf{H}_n^H \mathbf{H}_n)] = M$$

$$\overline{\mathbf{W}}_n^{(ZF)} = \mathbf{H}_n^H \left( \sqrt{M} (\mathbf{H}_n \mathbf{H}_n^H)^{-1} \right)$$

$$= \mathbf{H}_n^H \frac{\sqrt{M}}{\sum_{m=1}^M |h_{m,n}|^2} \stackrel{M \gg 1}{\approx} \frac{\mathbf{H}_n^H}{\sqrt{M}} \quad (12)$$

- *Digital Beamsteering:*

$$\mathbf{W}_n^{(BS)} = \mathbf{A}^H \quad (13)$$

$$\eta_n = \frac{1}{\mathbb{E}[\mathbf{A} \mathbf{A}^H]} = \frac{1}{M} \quad (14)$$

$$\overline{\mathbf{W}}_n^{(BS)} = \frac{\mathbf{A}^H}{\sqrt{M}} \quad (15)$$

The fact that the CB and the ZF are identical with many antennas for a cell with a single user can be easily predicted without any calculation. Indeed the drop in beamforming gain that the ZF suffers comes from the interference cancellation process and since there are no inter-user interference with only one user, the radiation diagram will purely be the same as the one with the CB. Also if the channel is purely LOS then  $\mathbf{H}_n = \mathbf{A}$  and all precoders become identical.

## IV. SIMULATION RESULTS

For performance analysis purposes we first introduce the channel scenario and the cell characteristics. We then illustrate how good the DBS is for multi-user transmissions, if not better than the CB, which is enough to evaluate the opportunities such an angle based precoder can provide. We also show how the sum capacity changes with angle estimation error.

### A. Channel scenarios

A deterministic channel model has been chosen so that all effects of correlation between antennas can be taken into account and therefore reflect on the system performance (see [15] for a more detailed description). As illustrated on Fig. 5, we assume a street of size  $30 \times 15$  m, the array is located 10 meters above ground, the frequency is set at 60 GHz, the antenna spacing is  $d = \frac{\lambda}{2}$  and the array plane is tilted so that the broadside axis targets the center of all possible user locations.

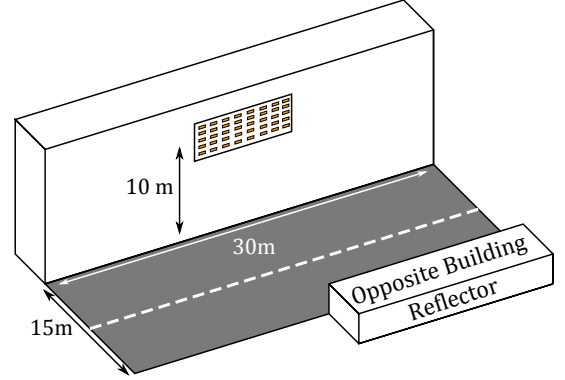


Figure 5. Cell structure for deterministic channel model.

The *1 Ray* scenario represents the pure LOS case where only the direct path exists. *3 Rays* takes two other reflections into account so that the effect of channel diversity can be observed (one reflection is on the ground and the other on the building on the opposite side of the street). Ground and building are considered perfect reflectors so that we can assume that real world performance are somewhere between these two bounds. Users are spread randomly in the cell and the distance between two individuals is at least 1 m.

### B. Multi-user sum capacity

We assume that the CSI is perfectly known when precoding with the CB and the ZF. We also consider the DoA perfectly known when using the DBS.

Fig. 6 shows how the sum capacity of each precoder evolves when increasing the number of users served simultaneously. Both scenarios are represented (*1 Ray* and *3 Rays*) so that we can observe the impact of multi-paths components on the performance. As pointed out in III-B, the performance of all precoders is identical for the single user case. If we look at the starting point precisely we can see that the DBS capacity is slightly lower than the others in the *3 Rays* scenario. This is because it is not able to compensate the different phase shifts that each path suffers. Therefore instead of all paths adding up constructively, as is the case with the CB and the ZF, channel diversity will slightly bring down the received power. However, this effect seems to be barely noticeable at such high frequencies.

If we focus on the ZF precoder in pure LOS (plain black curve), its limitation explained in [15] is obvious for such a dense urban scenario. Also, the fact that it performs better in

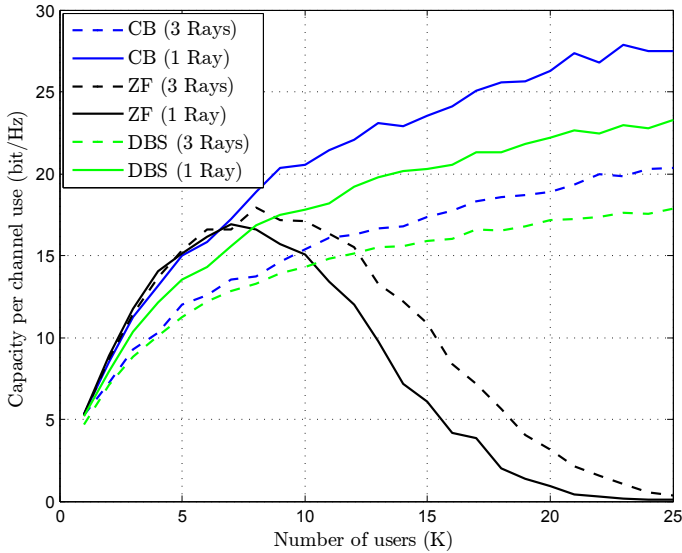


Figure 6. Sum capacity of the Conjugate (blue) and Zero-Forcing (black) Beamforming opposed to Digital Beamsteering (green) in the 1 Ray (plain lines) and the 3 Rays (dotted lines) environment with  $M = 36$  ( $M_C = M_V = 6$ ) antennas. Transmitted power  $P_e$  is constant.

the 3 Rays environment (dotted lines) highlights the fact that ZF precoding needs high diversity channels to reach its full potential.

Now we compare the CB (blue) with the DBS (green). The CB is supposed to handle channel diversity better than the DBS and therefore grants a higher capacity in the 3 Rays environment. However, if  $\mathbf{H}_n = \mathbf{A}$  (1 Ray), both precoder are identical and their performance should not be that far off.

This comes from the fact that, instead of using the normalized matrix  $\mathbf{H}_n$  for the CB and the ZF precoder, we have used  $\mathbf{G}_n$ . It means that each user gets an implicit power allocation that is linked to their free space attenuation along with the beamforming effect. In other words the further from the base station the users are, the less power they are allocated.

Since we are looking at the sum capacity we chose this use of  $\mathbf{G}_n$  instead of  $\mathbf{H}_n$  to boost the cell throughput. However we need not forget that this sort of precoding does not give any fairness of experience to the users and this should be considered in future works on the subject.

Fig. 7 shows how the use of  $\mathbf{G}_n$  instead of  $\mathbf{H}_n$  impacts both linear precoder performance. We can see that while the CB benefits from such a use of implicit power allocation, the ZF does not.

### C. Angle estimation error effect on capacity

Depending on the number of antennas (thickness of the beams) an error on the the DoA might not be critical. Indeed the precision of the angular estimation directly depends on  $M$ , be the estimation using an iterative scan of the cell [16] or a simultaneous subcarrier powered one [17].

Fig. 8 illustrates how the sum capacity of a cell behaves with a base station serving 15 users in a pure LOS environment (1 Ray). The error is implemented with a normal distribution  $\mathcal{N}(0, \Delta_a)$  (with  $\Delta_a$  the angular deviation in azimuth or

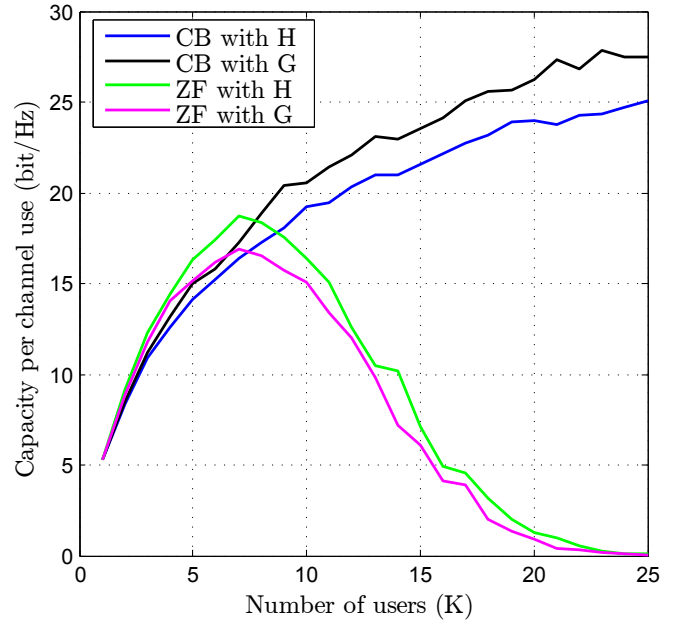


Figure 7. Conjugate and Zero-Forcing Beamforming using the normalized ( $\mathbf{H}_n$ ) and the generalized ( $\mathbf{G}_n$ ) channel matrix.

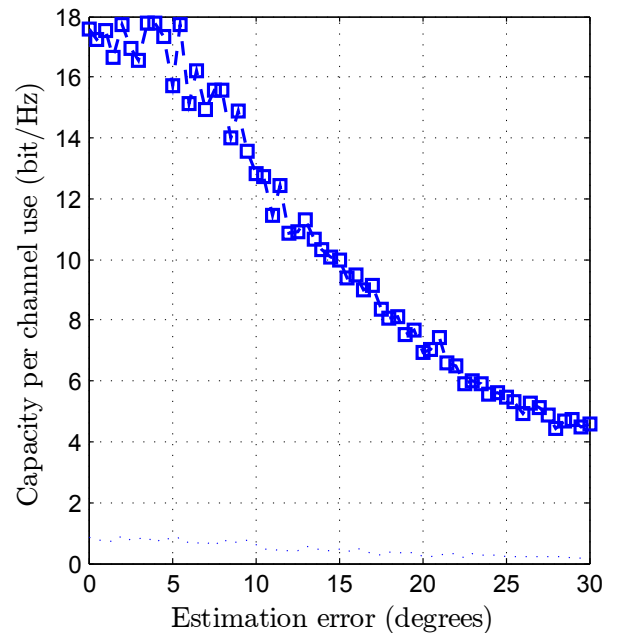


Figure 8. Angle estimation error effect on DBS sum capacity in pure LOS with the number of users set at  $K = 15$  and with  $M = 36$  antennas at the base station.

elevation). As an example, if we look at the error that produces 80% of the maximum capacity we find that it comes from an angle estimation error of about  $8^\circ$  ( $\sim 0.8 \times 18 = 14.4$  bits/Hz which corresponds to an error of  $8^\circ$  on Fig. 8). If we consider a user located at the center of the cell, this error translates into approximately 2,5 – 3m localization error. Of course if  $M$  increases the localization error is reduced since the precision depends on the number of antennas (beam width)

but an estimation error would have a much greater impact.

## V. CONCLUSION

Although many solutions are being studied for small cell base stations, mmWave Massive MIMO seems quite a candidate for 5G systems. From energy efficiency to system flexibility it covers many features that are expected to be part of future networks.

In this paper we have shown how promising steering beams digitally is to handle multi-user transmissions for highly LOS environments:

- 1) Close performance to the Conjugate Beamforming;
- 2) Less sensitive than the Zero-Forcing to the number of users;
- 3) Low implementation complexity;
- 4) Robust against angle estimation error.

On the one hand, if implemented on the same hardware structure than CSI based linear Massive MIMO precoding (Conjugate, Zero-Forcing and others), the system could switch between different techniques, depending on the user locations.

On the other hand a standalone Digital Beamsteering structure makes FDD communications possible because the angular information doesn't change with the frequency and therefore the system doesn't need the calibration process compulsory to the use of CSI based precoding (channel reciprocity needed for TDD transmission). It means that LOS applications can greatly benefit from DBS and with very low implementation complexity.

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