Accuracy of Singularity Expansion Method in Time and Frequency Domains to Characterize Antennas in Presence of Noise

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***Abstract*—In this paper, the accuracy of the Singularity Expansion Method (SEM) used for antenna characterization is investigated. A well-known limitation of the SEM is that pole extraction is very sensitive to noise. A comparison between two main methods of pole extraction is presented. The Matrix Pencil (MP) method and the Cauchy's method are used to extract poles from the radiated fields of a dipole antenna and two bowtie antennas. Results are presented for simulated fields and the robustness to a white Gaussian noise is also analyzed. We show that the MP method allows working with lower SNR than Cauchy's method and is more accurate for field reconstruction.**

***Index Terms*—Antenna characterization, Cauchy's method, complex natural resonance, matrix pencil method, poles, residues, singularity expansion method.**

I. Introduction

FOR many years, the Singularity Expansion Method (SEM) [1], introduced by C. E. Baum in 1971, has been widely used in the radar domain. The SEM represents a solution of an electromagnetic problem in terms of singularities (poles) in the complex frequency plane. Since singularities are independent of the direction of the incoming wave, the SEM has been widely studied for target identification [2][3]. The information on poles can give some indications on the general shape and the constitution of the illuminated target. Moreover, the SEM has been used in the antenna domain such as in [4] where the SEM formalism has been applied for modeling the time response of the current of a thin wire antenna. In [5], Barnes analyses the dipole antenna response to an electromagnetic pulse using the SEM. More recently, this method has been applied in both time and frequency domains to model antenna effective length, in order to fully describe the antenna radiation patterns, directivity and gain using only a few sets of parameters (poles and residues) [6][7]. In [6][7], the authors extract poles from the whole antenna response. But, according to the theory of C.E. Baum [1], physics phenomena can be observed only in the late time antenna response. In this paper, we focus on the physical approach of the SEM and only the late time response is considered.

The well-known limitation of the SEM is that pole extraction is very sensitive to noise. In transient domain, there are two main methods to extract poles from the antenna electromagnetic response. The first one is the well-known Prony's method introduced by Baron de Prony in 1795 [8]. This method has been modified to be used on noisy data using a Least Square (LS) approach in 1950 [9] and a Total Least Square (TLS) approach in 1987 [10]. In 1990, Hua and Sarkar suggested the Matrix Pencil (MP) method [11] also based on a TLS approach. They have compared Prony and MP methods and have shown that the MP method is more robust to noise than Prony's algorithm [12]. This has been verified on noisy antenna responses [13]. Moreover, the MP method is computationally more efficient [14]. More details on MP are given in appendix I. In the frequency domain, the main way to extract poles of a transfer function is the Cauchy's method developed in 1821 by Cauchy [15]. A TLS approach has also been used to improve its robustness to noise [16]. More details on Cauchy's method are given in appendix II. MP and Cauchy methods are the two main efficient methods of pole extraction, in transient and frequency domains, respectively, but there are few works dealing with their comparison [17].

The objective of this paper is to determine which method is the most appropriate to extract poles for antenna characterization by using either the Total Least Square Matrix Pencil (TLS MP) method in the time domain or the Total Least Square Cauchy (TLS Cauchy) method in the frequency domain. In section II, the SEM is presented. Next, in section III, the two methods are applied to noiseless fields radiated by three different antennas: a narrow band dipole antenna and two Ultra Wide Band (UWB) bowtie antennas with different flare angles. To study the robustness of these methods in the presence of noise, two different kinds of noise are added to the simulated fields and results are compared in section IV.

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II. SEM Theory

The SEM was developed to describe the global behavior of an object's response excited by an electromagnetic wave. In the time domain, this response is composed of two successive parts. The first one is called the early time response and is mainly due to the excitation impulse. The duration T_E of this early time response depends on the pulse duration T_P and the greatest dimension D of the antenna as $T_E = D/c + T_P$ where c is the speed of light. The second part, called the late time response, occurs after the early time response and is only due to the radiation of the induced current propagated on the antenna after its illumination. The SEM allows modeling the late time response of an object as a decaying exponential sum as:

$$y(t) \approx \sum_{n=1}^{M} R_n e^{s_n t}, \tag{1}$$

where $y(t)$ is the response, s_n is the nth pole, R_n is the residue associated to the nth pole and M is the number of poles. In frequency domain, SEM allows modeling the transfer function $H(s)$ of the antenna as:

$$H(s) \approx \sum_{n=1}^{M} \frac{R_n}{s - s_n}, \tag{2}$$

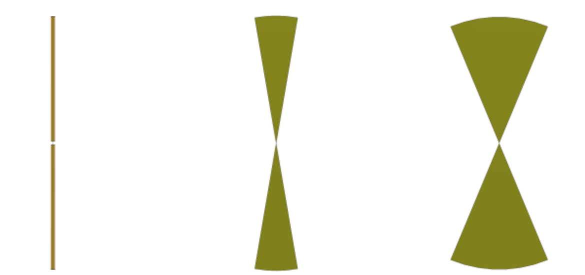
where s is the Laplace complex variable. Each pole is defined as:

$$s_n = \alpha_n \pm j\omega_n \tag{3}$$

where α_n is the negative damping coefficient of the nth pole and ω_n is the resonant pulsation of the nth pole.

III. Pole Extraction from Noiseless Radiated Antenna Responses

This section presents a comparison between the two methods applied to fields radiated by three antennas: a dipole and two bowtie antennas shown in Fig. 1. Their lengths are L = 33.75 mm. The diameter of the dipole is D = 1.12 mm, so the ratio L/D = 30. Its gap length is 1.12 mm and the impedance of its lumped port is 73 Ω. Bowtie antennas have



a. Dipole b. 20° Bowtie c. 45° Bowtie

Fig. 1. Geometry of the three considered antennas.

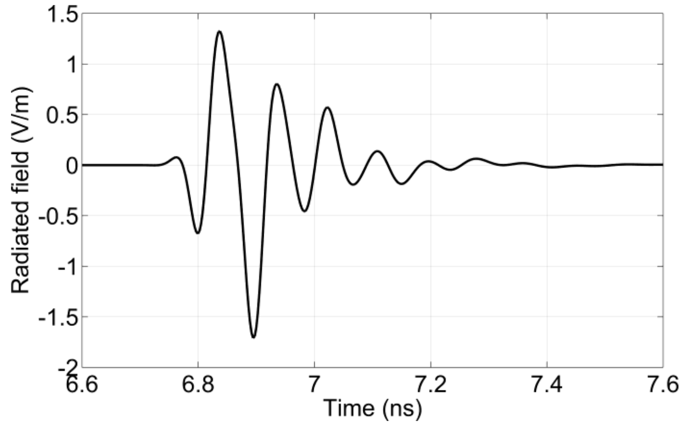


Fig. 2. Simulated transient electric far field of the dipole antenna in the boresight direction.

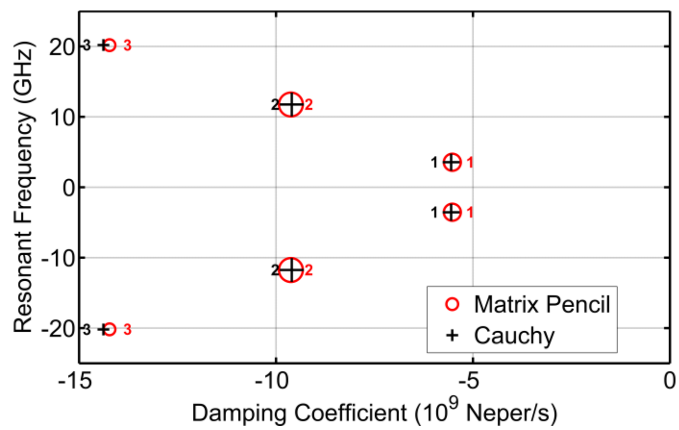


Fig. 3. Poles extracted from transient simulated electric far field of the dipole antenna with MP and Cauchy methods in a complex plane.

two different flare angles: 20° and 45° and the impedance of their lumped port is 200 Ω. These antennas are simulated using CST Microwave Studio [18] with transient solver. A power source is used and the excitation signal is a Gaussian pulse. A far field probe is used to measure the electric field in the boresight direction at a distance R = 2 m.

A. Dipole Antenna

The field radiated by the dipole antenna is shown in Fig. 2. The MP algorithm is directly applied on the late time transient radiated field of the dipole antenna whereas the Cauchy's

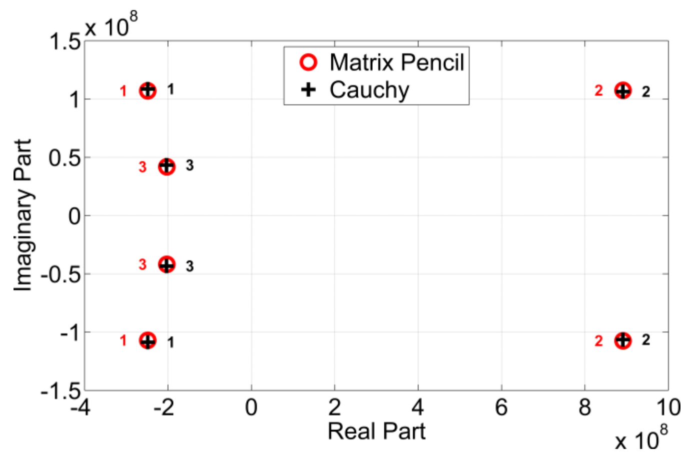


Fig. 4. Residues extracted from transient simulated electric far field of the dipole antenna with MP and Cauchy methods in a complex plane.

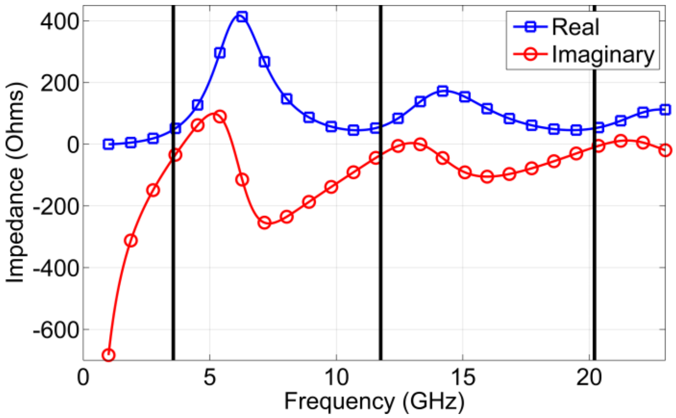


Fig. 5. Complex impedance of the dipole antenna. Black lines represent resonant frequencies of extracted poles.

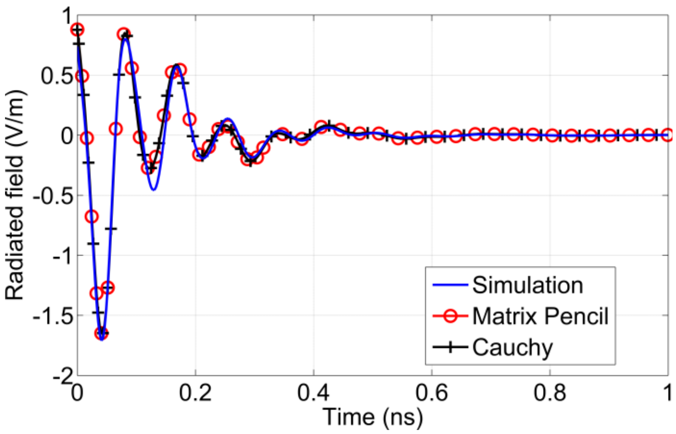


Fig. 6. Radiated fields of the dipole antenna, simulated and reconstructed using Cauchy poles and Matrix Pencil poles.

algorithm is used on the frequency radiated field obtained by a Fast Fourier Transform (FFT). Results of both extractions are shown in Fig. 3 and Fig. 4 for poles and residues, respectively. Three pairs of poles and residues are extracted by the two methods and their agreement is very good (relative difference less than 1 %). Resonant frequencies of the poles are represented using black lines on its input impedance in Fig. 5. We can see that these poles correspond almost to resonances of the dipole antenna at $\lambda/2$, $3\lambda/2$ and $5\lambda/2$, where λ is the wavelength in free space. So all extracted poles have physical meanings. Late time radiated fields are reconstructed using these two different sets of poles and residues, extracted with MP and Cauchy methods, and (1). Results are presented in Fig. 6. The three curves are close; the Normalized Mean Square Error (NMSE) is less than 3 % for both methods. In Fig. 6, only the late time response of the dipole antenna is presented whereas in Fig. 2 the entire radiated field is shown. The early time duration is 0.2 ns. In Fig. 3, the marker size depends on the weight of each pole. The weight is computed as the ratio between residue and damping coefficient of the pole and is normalized by the maximum weight. This

TABLE I
NMSE OF THE RECONSTRUCTED DIPOLE RESPONSE

Poles	1-2-3	1-2	1-3	2-3	2
NMSE (%)	2.7	6.2	92	19.5	24.5

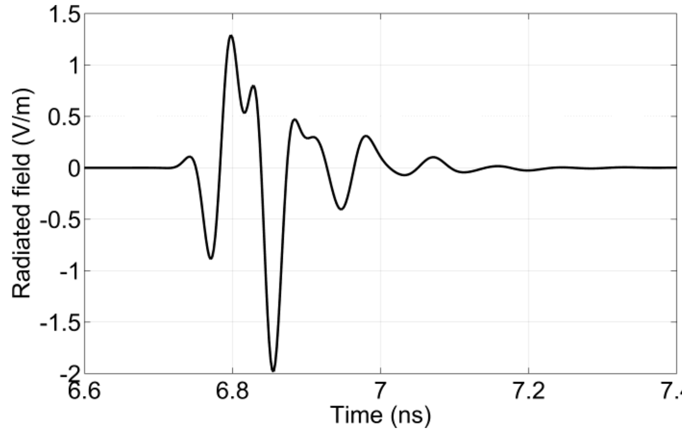


Fig. 7. Simulated transient electric far field of the bowtie antenna in the boresight direction.

representation is a good way to estimate the contribution of each pole [3]. In Table I, NMSE of the dipole responses reconstructed using only some of the physical poles extracted are presented. It shows that the pole 2 is the most important contribution to the late time dipole response, then the pole 1 and finally the pole 3. MP and Cauchy methods allow extracting the same physical poles from the noiseless field radiated by a dipole antenna. Moreover, it is possible to reconstruct the radiated field with a very small error. It means that a 6-poles set is enough to accurately model the late time response of a dipole antenna.

B. *Bowtie Antenna with 20° flare angle°*

Fig. 7 shows the field radiated by the bowtie antenna with flare angle equal to 20°. This antenna has a wider band than the dipole antenna. In fact, larger flare angles correspond to wider bandwidths. Since the antenna is less resonant, it is more difficult to define which poles have physical meanings when extracted by the two methods. In order to select poles extracted in the time domain, we use the Window Moving Technique (WMT) also known as Time-Frequency analysis [19][20]. The idea of the WMT is to move a time window with a given duration through the entire signal by small time steps. The minimum time step is equal to the sample period. MP method is applied on each time window. The assumption is that, depending on the window, the position of the

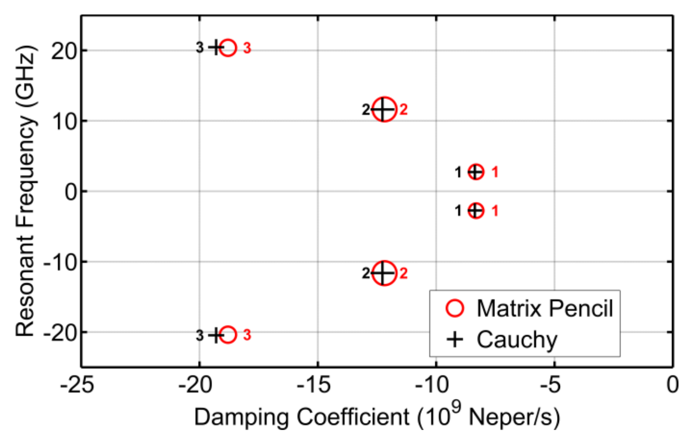


Fig. 8. Poles extracted from simulated transient electric far field of the bowtie 20° antenna with MP and Cauchy methods in a complex plane.

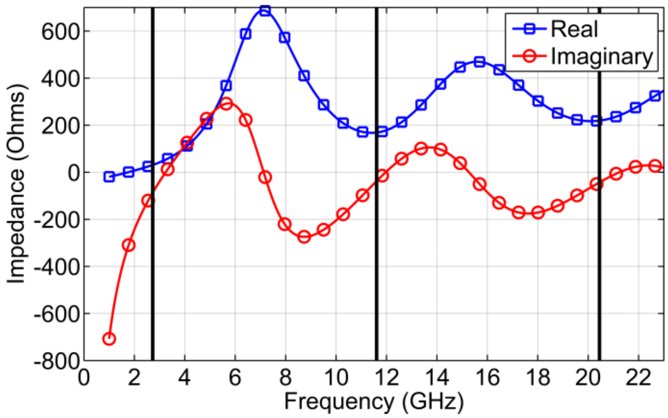


Fig. 9. Complex impedance of the bowtie antenna. Black lines represent resonant frequencies of extracted poles.

mathematical poles will change from window to window, whereas the physical poles will remain essentially unchanged. To improve pole extraction in the frequency domain, the frequency radiated field is split into two responses (from 1 to 7 GHz and from 7 to 23 GHz). In this paper, subbands have been selected empirically after several tests. However, a systematic approach relating the resonant behavior of the antenna to the needed number and size of the subbands is under investigation. The Cauchy's algorithm is applied separately on these two subbands. Each subband contains only

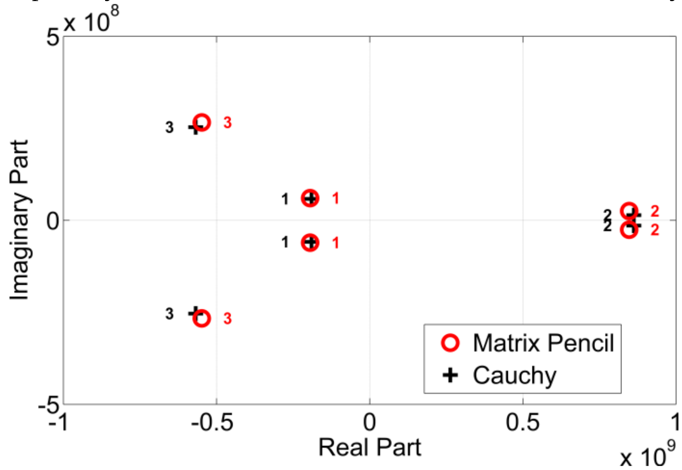


Fig. 10. Residues extracted from simulated transient electric far field of the bowtie antenna with MP and Cauchy methods in a complex plane.

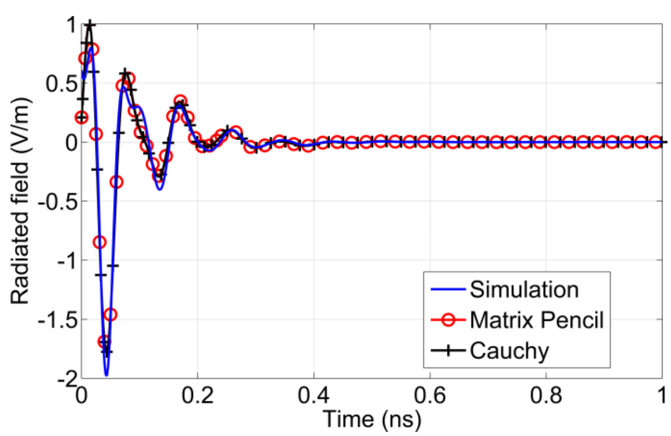


Fig. 11. Radiated fields of the bowtie antenna, simulated and reconstructed using Cauchy poles and Matrix Pencil poles.

TABLE II

NMSE OF THE RECONSTRUCTED BOWTIE 20° RESPONSE

Poles	1-2-3	1-2	1-3	2-3	2
NMSE (%)	4	26	80	12	33

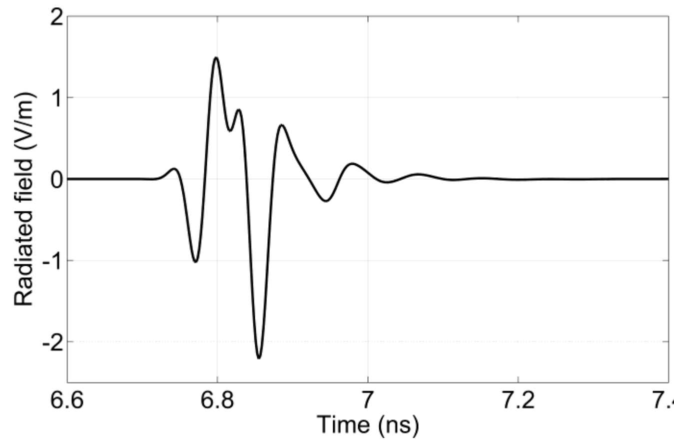


Fig. 12. Simulated transient electric far field of the 45° bowtie antenna in the boresight direction.

a few poles so it is easier to extract them accurately. Residues are then computed again using the extracted poles in each subband and the entire response in order to accurately model the complete response. Poles extracted with the two methods are shown in Fig. 8. Three pairs of poles are extracted by the two methods and they are in very good agreement. A maximum relative error of 1 % is found for the resonant frequencies of these poles whereas a relative error varying from 1 to 3 % is obtained for the damping coefficients. Resonant frequencies of the poles extracted with the MP algorithm are represented with a black line on the antenna's input impedance in Fig. 9. We can see that these frequencies mainly correspond to the resonances of the antenna input impedance, i.e. when its imaginary part is close to zero. Associated residues are presented in Fig. 10. Late time radiated field is reconstructed using poles and residues extracted with MP and Cauchy methods and results are shown in Fig. 11. The three curves are overlapped. NMSE with the two reconstructed fields are less than 4 %. Therefore, it is possible to model the late time response of this wideband antenna using only a set of 6 poles and residues. NMSE of the reconstructed fields using only some poles are presented in the Table II. It shows that the pole 2 is still the dominant one followed by the pole 3. Note that even if the pole 1 has the strongest damping coefficient, it provides the smallest contribution to the radiated field.

C. Bowtie Antenna with 45° flare angle

The field radiated by the bowtie antenna with flare angle equal to 45° is shown in Fig. 12. This UWB bowtie antenna has a much wider band than the previous one. Therefore, we need to use the WMT in the transient domain and split the frequency

TABLE III

NMSE OF THE RECONSTRUCTED BOWTIE 45° RESPONSE

Poles	1-2-3	1-2	1-3	2-3	2
NMSE (%)	2.5	54	98	4.5	60

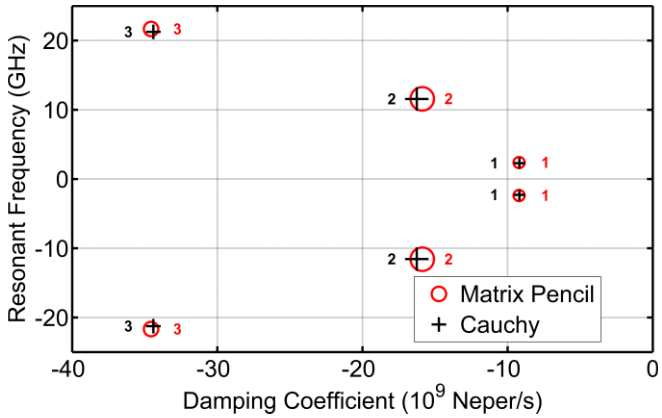


Fig. 13. Poles extracted from simulated transient electric far field of the bowtie 45° antenna with MP and Cauchy methods in a complex plane.

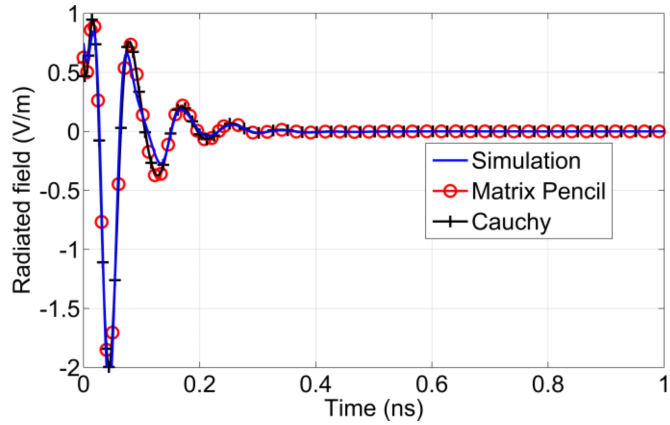


Fig. 16. Radiated fields of the bowtie antenna, simulated and reconstructed using Cauchy poles and Matrix Pencil poles.

response into three subbands in order to extract the physical poles presented in Fig. 13. The number of extracted poles is unchanged but damping coefficients are much larger in absolute value than for the previous bowtie antenna, especially for the third pole (around -35.10^9 Neper/s). The resonant frequencies, extracted with the MP method, are represented with a black line on the impedance of the antenna in Fig. 14. Associated residues are presented in Fig. 15 and they are in very good agreement. Late time responses reconstructed using the whole poles are presented in Fig. 16. The NMSE is less than 3% for both methods. NMSE of the reconstructed fields using only a few poles are presented in Table III. They confirm the weight of each pole presented in Fig. 13, i.e. the pole 2 is the dominant one, followed by the poles 3 and 1.

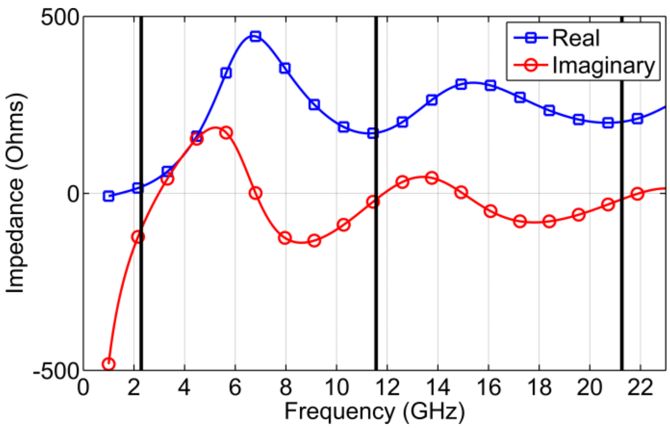


Fig. 14. Complex impedance of the bowtie antenna. Black lines represent resonant frequencies of extracted poles.

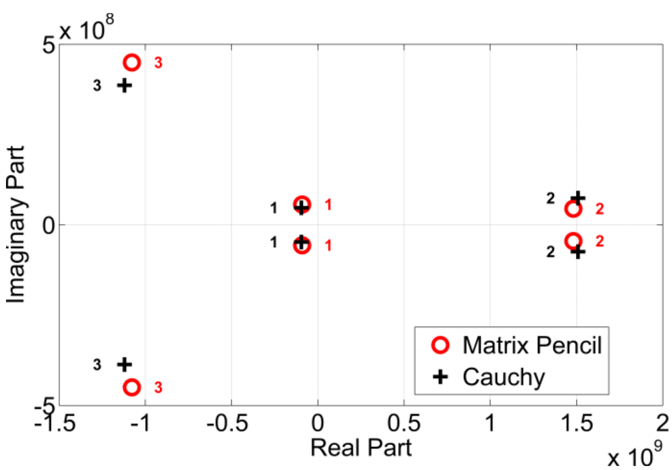


Fig. 15. Residues extracted from simulated transient electric far field of the bowtie antenna with MP and Cauchy in a complex plane.

IV. Pole Extraction from Noisy Radiated Antenna Responses

Behaviors of MP and Cauchy methods in presence of noise are now compared. We consider two different approaches as presented in the flowcharts in Fig. 17. The first one considers that the original data is in the transient domain and a noise is added to the transient simulated field to obtain a noisy response with a desired Signal to Noise Ratio (SNR). Then, poles are extracted directly with MP and using a FFT. In the second one, it is considered that the original data is in the frequency domain, so the noise is added to the frequency simulated field. The Cauchy's method is then directly applied and an IFFT is performed to use MP. Two different kinds of noise are considered: a White Gaussian Noise (WGN) and a mixed noise composed of WGN, impulse noise and single carrier noise.

Due to the noise and the overestimation of the number of poles to be extracted, some poles can be very different of those extracted from the noiseless case. These poles are called

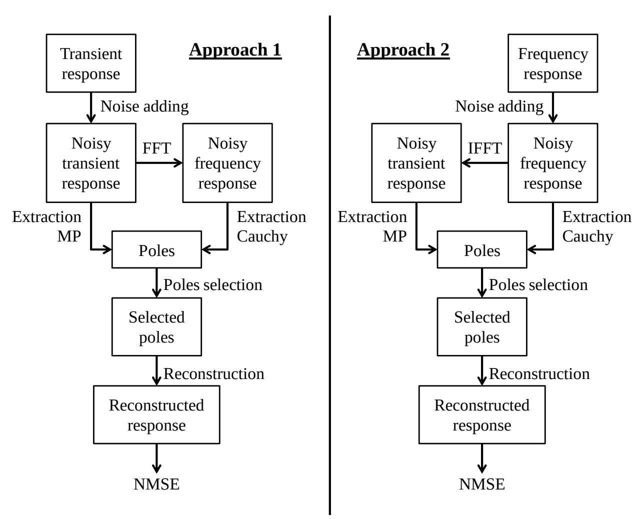
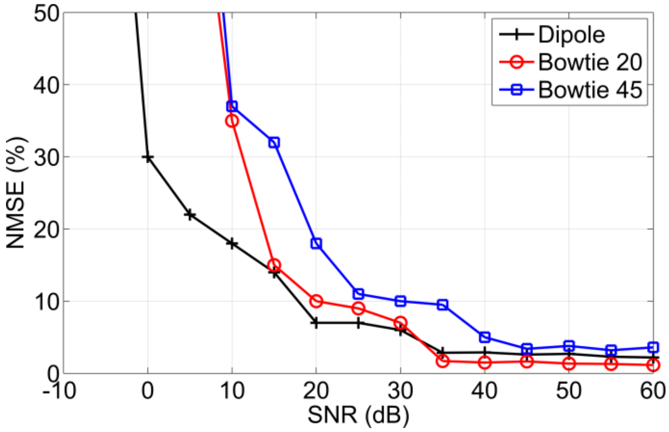
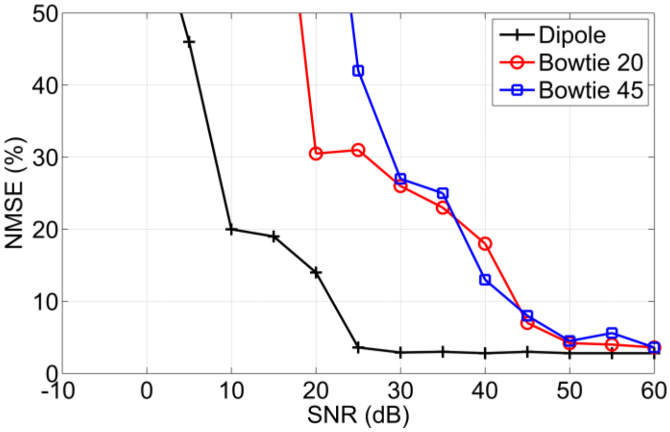


Fig. 17. The flowcharts of the two approaches (1: transient, 2: frequency).

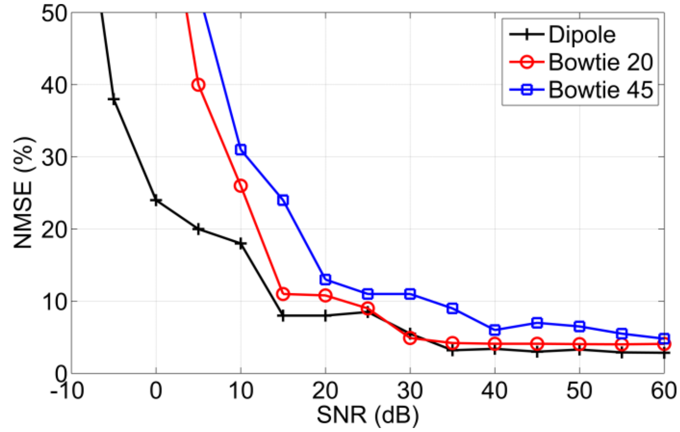


a. Matrix Pencil method

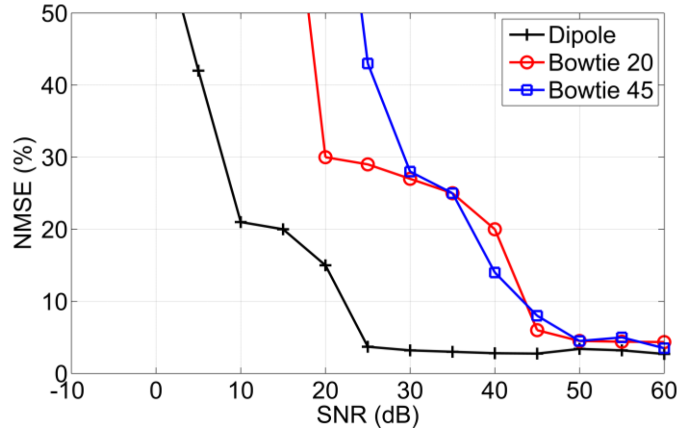


b. Cauchy method

Fig. 18. NMSE of the reconstructed fields versus SNR for approach 1 and for white Gaussian noise.



a. Matrix Pencil method



b. Cauchy method

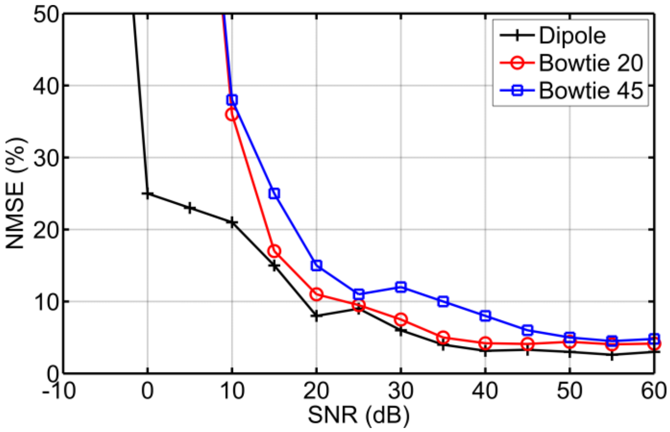
Fig. 19. NMSE of the reconstructed fields versus SNR for approach 2 and for white Gaussian noise.

"mathematical" because they do not have a physical sense. Since the aim of this study is to define which method is the best to extract physical poles of an antenna, we only kept poles close to those extracted from the noiseless field. We consider a pole well extracted when its resonant frequency is around 5 % of the original one, i.e. poles extracted from noiseless data, and damping coefficient around 30 %. Using these poles extracted from noisy data, the radiated field is reconstructed and the NMSE compared to the simulated field is computed. These operations (noise addition, pole extraction, field reconstruction and NMSE computation) are repeated for each SNR value from -10 to 60 dB with a 5 dB step. For each SNR value, these operations are repeated 100 times to limit the random effect of the noise. An average NMSE is then computed using the 100 NMSE values.

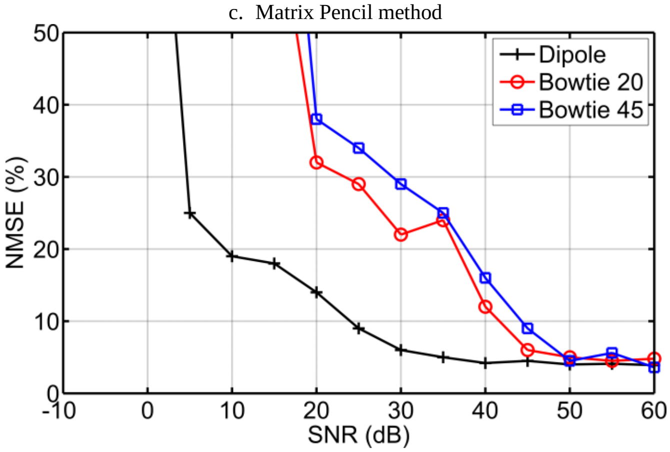
Results for the WGN case for the first and second approach are given in Fig. 18 and Fig. 19, respectively. The two approaches provide very similar results. In fact, adding the WGN in the frequency or the transient domain does not change the results. Therefore, the FFT and the IFFT seem to not disturb pole extraction with the two methods. We can notice that, for a given SNR, the NMSE is generally more important for the 45° bowtie than the 20° bowtie and the dipole antennas. Therefore, we may conclude that for wideband antennas, it is difficult to extract poles with a good accuracy whatever the method used. For the dipole antenna case, the difference between the two methods is around 5 dB to obtain the same NMSE. However, for the two others antennas, the difference is much higher. In fact, for the 20° bowtie, one needs a 45 dB SNR to obtain a 10 % NMSE with the Cauchy's method, unlike 20 dB sufficient with the MP method. For the 45° bowtie, there is a 20 dB difference. So the MP method allows dealing with signals with SNR 5 to 25 dB lower than using Cauchy's method.

The same analysis is done using the mixed noise. Since the two approaches give very similar results, only those obtained from approach 1 are presented in Fig. 20. As for the previous analysis, it is easier for both methods to extract poles from the dipole antenna response than for the bowtie antennas. Otherwise, even if the results are close between MP and Cauchy methods for the dipole response, the MP algorithm allows obtaining a lower NMSE than the Cauchy's method for the two bowties. As an example, the dipole response, noiseless and in presence of mixed noise for SNR = 10 dB, and the reconstructed field are presented in Fig. 21. The NMSE of the reconstructed field is 10 %.

From these analyses, we can conclude that the MP method is less sensitive to noise than the Cauchy's one, especially when applied to wideband or UWB antenna responses. Nevertheless, the SNR needed to obtain poles with accuracy is quite high. During measurements, it will be necessary to increase the SNR as much as possible using especially a time gating to filter unwanted echoes [7].



c. Matrix Pencil method



d. Cauchy method

Fig. 20. NMSE of the reconstructed fields versus SNR for approach 1 and for mixed noise.

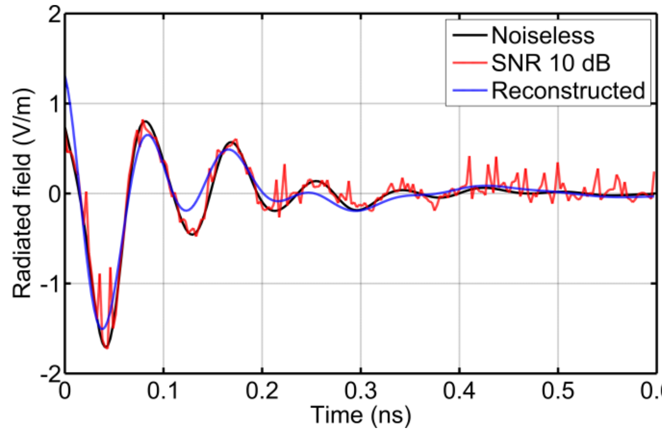


Fig. 21. Field radiated by the dipole antenna, noiseless, in presence of mixed noise for an SNR of 10 dB and reconstructed with NMSE of 10 %.

V. Conclusion

The well-known limitation of the SEM is that pole extraction is high sensitive to noise. Therefore, one has to extract poles very carefully in order to obtain physical ones. In this article, two of the best numerical methods of pole extraction, MP and Cauchy, have been applied to fields radiated by three different antennas, a dipole and two bowtie antennas with different flare angles. In the noiseless case, both methods allow extracting physical poles with a good accuracy. It follows that the late time response of these antennas can be reconstructed by using only a 6-poles set. We also compared the robustness of MP and Cauchy methods in the presence of two different kinds of noises. For the simple case of the dipole antenna, results are close but the MP method is more accurate than Cauchy's method when SNR becomes low. For the two bowtie antennas, the difference between the two methods is more significant. Indeed, the MP method allows dealing with signal with a SNR 20-25 dB lower than that needed for Cauchy's method.

Appendix I
Matrix Pencil Method

The data samples y_k are defined as:

$$y_k \approx \sum_{n=1}^{M} R_n z_n^k, \tag{4}$$

where $k = 0,1,\ldots,K$, K being the sample's number, $z_n = e^{s_n T_s}$ and T_s is the sampling period. A data matrix $[Y]$ is constructed from the data samples.

$$[Y] = \begin{bmatrix} y_0 & y_1 & \cdots & y_L \\ y_1 & y_2 & \cdots & y_{L+1} \\ \vdots & \vdots & \ddots & \vdots \\ y_{K-L-1} & y_{K-L} & \cdots & y_{K-1} \end{bmatrix} \tag{5}$$

L is called the Pencil parameter and is very important to filter noise. It is usually chosen between K/3 and K/2 [11]. In fact, the variance of the extracted poles is the lowest for these values. Then, a Singular Value Decomposition (SVD) is applied to this matrix as $[Y]=[U][\Sigma][V]^H$ where H defines the Hermitian transpose, $[U]$ and $[V]$ are unitary matrices, composed of the eigenvectors of $[Y][Y]^H$ and $[Y]^H[Y]$, respectively, and $[\Sigma]$ is a diagonal matrix containing the singular values of $[Y]$. In the noiseless case, the matrix $[Y]$ contains exactly M nonzero eigenvalues corresponding to the M poles of the system. However, in the noisy case, the other eigenvalues are not exactly equal to zero. So it is necessary to filter these eigenvalues. The smallest ones, minor to a threshold ε, are set to zero. A new matrix $[Y']$ can be written as $[Y']=[U'][\Sigma'][V']^H$ where:

$$[U'] = [u_1 \quad u_2 \quad \cdots \quad u_M]_{(K-L)*M}, \tag{6}$$

$$[\Sigma'] = \begin{bmatrix} \sigma_1 & 0 & 0 & 0 \\ 0 & \sigma_2 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & \sigma_M \end{bmatrix}_{M*M}, \tag{7}$$

$$[V'] = [v_1 \quad v_2 \quad \cdots \quad v_M]_{(L+1)*M}. \tag{8}$$

From $[V']$, it is possible to define two submatrices $[V_1']$ and $[V_2']$ as:

$$[V'] = \begin{bmatrix} l_1 \\ V_1' \end{bmatrix}_{(L+1)*M}, \tag{9}$$

and

$$[V'] = \begin{bmatrix} V_2' \\ l_{L+1} \end{bmatrix}_{(L+1)*M}, \tag{10}$$

where l_i is the ith line of $[V']$. Using the TLS approach [21], poles are obtained from the eigenvalues of $\{[V_1']^H\}^+ [V_2']^H$, where $\{[V_1']^H\}^+$ is the Moore-Penrose pseudo-inverse of $[V_1']^H$. It is now possible to compute residues from (4).

Appendix II
Cauchy's Method

The main idea of this method is to approximate the transfer function $H(s)$ of an antenna into a ratio of two polynomials P and Q as:

$$H(s) \approx \frac{P(s)}{Q(s)} = \frac{\sum_{n=0}^{P} a_n s^n}{\sum_{n=0}^{Q} b_n s^n} \tag{11}$$

Equation (14) can be rewritten as:

$$\sum_{n=0}^{Q} b_n s^n H(s) - \sum_{n=0}^{P} a_n s^n \approx 0. \tag{12}$$

It is possible to write (12) as:

$$\left[\begin{array}{ccc|ccc} H(s_1) & \cdots & s_1^Q H(s_1) & 1 & \cdots & s_1^P \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ H(s_K) & \cdots & s_K^Q H(s_K) & 1 & \cdots & s_K^P \end{array}\right] \begin{bmatrix} b_1 \\ \vdots \\ b_Q \\ -a_0 \\ \vdots \\ -a_P \end{bmatrix} \approx 0. \tag{13}$$

A QR decomposition of (13) is made such as:

$$\begin{bmatrix} R_{11} & R_{12} \\ 0 & R_{22} \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} \approx 0. \tag{14}$$

So these two following equations are obtained:

$$R_{22} b \approx 0 \tag{15}$$

$$R_{11} a + R_{12} b \approx 0. \tag{16}$$

A SVD of R_{22} is done such as:

$$[R_{22}] = [U][\Sigma][V]^H, \tag{17}$$

Using a TLS approach [21], coefficients b are obtained from $b = [V]_{Q+1}$. Coefficients a are then computed using (16). Poles are now found by computing squares of the Q polynomial.

$$\sum_{n=0}^{Q} b_n s^n = b_Q \prod_{n=1}^{Q} (s - s_n) \tag{18}$$

where s_n are the poles and s the Laplace variable. The transfer function can now be written as:

$$H(s) \approx \frac{\sum_{n=0}^{P} a_n s^n}{b_Q \prod_{n=1}^{Q} (s - s_n)} = \sum_{n=1}^{Q} \frac{R_n}{s - s_n}. \tag{19}$$

Residues R_n are obtained from:

$$R_n = \sum_{k=0}^{P} a_k s_n^k \Bigg/ \left(b_Q \prod_{k=1, k\neq n}^{Q} (s_n - s_k) \right). \tag{20}$$

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